re(Solve) MATHS BY INQUIRY

MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS

10 000 Centicubes

Lesson 1: Discover

Australian Curriculum: Mathematics (Year 4)

ACMNA073: Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems.

ACMNA076: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and division where there is no remainder.

Lesson abstract

The challenge is to make the best container to hold 10 000 centicubes. Students review mathematical vocabulary, and possible shapes, then predict the size of a suitable prism by looking at just one centicube as a guide. They use the idea of stacking layers of equal size to suggest some container dimensions arising from multiplicative partitioning of 10 000. They record using number sentences and pictures.

Mathematical purpose (for students)

Doubling and halving and using extended number facts can help us work out dimensions for the container.

Mathematical purpose (for teachers)

Students see that a prism can be thought of as a base layer, repeated multiple times to develop the height. They see that the total number of centicubes in the prism depends on the number of centicubes in the base and number of layers. The application of place value understanding, extended number facts and multiplicative thinking (doubling and halving and generalisations of this to shifting factors such as 10 or 5) makes it possible for students to generate multiple ways to arrange 10 000 cubes in equal layers.

At the end of the Discover phase, students will be able to:

- Provide at least one multiplication number sentence or picture to make 10 000.
- Recall that prisms have a base and a height, and can be constructed by stacking layers of equal size.

Lesson Length 90-120 minutes

Vocabulary Encountered

- partition, doubling, halving
- number sentence
- base, height, length, width
- edge, vertex, face
- layer
- dimensions

Lesson Materials

- about ten 3D containers, and 30 centicubes
- student workbooks, tablets for photos (optional)
- 3 sets of 36 square ‘Salada’ crackers (pre-arranged in layers of 4, 2 and 1; optional))
- MAB (1000, 100, 10, 1 blocks) sufficient for all students
- 1 cm grid paper (2 or 3 sheets per student)

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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aamt

Exploring the Mathematics in Containers

Inquiry Question: What is the best container to hold 10 000 centicubes?

Teacher Note

A centicube is a small brightly coloured plastic cube with dimensions
1 cm x 1 cm x 1cm.
This is exactly the same size as a mini MAB 'one' block. Explain this to students at a convenient time.

Explore the inquiry question

1. To promote engagement, pose a problem about educational suppliers who are planning to sell centicubes in containers of 10 000 to schools. The suppliers are keen to know the best container size to package the centicubes in, and have asked our school for suggestions. Inform students that in this unit they will be constructing a container to hold 10 000 centicubes and so they will need to work out mathematically how big their container will be.
2. Display the inquiry question, '*What is the best container to hold 10 000 centicubes?*' and have students record it in their workbooks. Ask students to highlight the key mathematical information in the question.

   e.g. '*What is the best container to hold 10 000 centicubes?*'

   (This lesson is concerned with the size of the container. In later lessons, both size and shape matter.)

Review three dimensional objects and associated mathematical language

3. Brainstorm with students some possibilities for the shape of the container and record their suggestions on the board (e.g. rectangular prism, cube, triangular prism, cylinder, or everyday names).
4. Provide containers (e.g. margarine container, storage boxes, ice-cream containers, tissue boxes, biscuit tin, Pringles tin, Toblerone box) for students to look at as they share with a partner what they know about such a container (shape, size, key features, name, use). Students record their ideas in their workbooks.
5. Move between partners listening to students' thinking. Be alert for partners using terminology for 2-dimensional shapes when describing 3-dimensional objects e.g. "*This container is a rectangle and it has six sides*" instead of "*This container is a rectangular prism and it has six rectangular faces*". Lead students to make the distinction between two dimensional and three dimensional terminology. Note common issues to address in the class discussion.
6. Allow **five** minutes for the task before having some partners share ideas with the class, prompting accurate mathematical language if not provided. Ensure the following terminology is covered in the discussion: base, height, length, width, edge and face. Possible prompts:

   Teacher: *Can you suggest a mathematical term you could use to describe this face that is down on the table?* (ANSWER: that face is the base now)

   Teacher: *Let's look at this edge of the rectangular prism. If we were measuring this edge, we would be measuring the ________ of the container.* (ANSWERS: some of length, width, breadth, depth, height)

Predict container size

7. Refer back to the inquiry question, and focus students on the number of centicubes that are to be held in the container. Provide partners with one centicube and have them draw a container (including dimensions if possible) that could hold 10 000 centicubes. Ensure students understand that a prediction requires them to use background knowledge to make an educated guess. Don't worry about correcting dimensions or calculations at this stage. Encourage students to consider box shapes first.

8. Have several partners share their drawings. Prompt partners to explain the thinking they used to make the prediction (e.g. *How did you decide how high your container would be?*). As it is an ideal lead in for the next step, note any partners who explain their thinking using a base layer *(e.g. We thought we could have 1000 on the bottom and have 10 layers)*.

9. As a class, discuss why there is such a large variation in drawings. Ask the students what they found difficult. Discuss these challenges. Students may see having only one centicube as a referent as a difficulty. If not, suggest it.

Layering a base to develop height

10. This section focuses on the doubling and halving relationship for multiplication.

11. Establish that a box (rectangular prism) is likely to be a good shape. If possible, link to the previous discussion of predictions to go through the following example:

    Teacher: *To think about the size of the container, Tom visualised some centicubes making the base layer and having more layers of the same size on top of that. Let us use this idea of layers to help us answer the question.*

12. Show students a packet of 36 crackers (a suitable alternative such as connecting cubes can be substituted here). Discuss how layers have been used to package the crackers. Focus the language on the idea of a base being layered to develop height.

13. The photo shows a packet of 36 crackers with 4 crackers on the bottom row (base layer) and there are 9 layers, making 36 crackers. On the right, the 36 crackers are shown arranged with 2 per layer, and 1 per layer.

    With the 4 cracker layers, ask students to provide a matching number sentence showing the total number of crackers:
    4 x 9 = 36 (prefer in this context to 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 36).

14. Ask students to indicate with their hands the height of the packaging if the same number of crackers (36) were packaged with only two crackers on the base layer. As they explain their thinking, draw out the doubling and halving relationship, spending time on this relationship, as it is essential for later learning: 2 x 18 = 36.

15. Ask students to indicate with their hands how high they think this package would be if there was only one cracker in each layer.

    $4 \times 9 = 36$

    ÷2 x2

    $2 \times 18 = 36$

    If we halve the number of crackers on the base (2 crackers on the base), we need to have twice as many layers so the packaging would be twice as high.

    If we halve the number of crackers on the base again (1 on the base) then the package will be four times as high as the original packaging. We are dividing the number of crackers on the base by 4 so we need 4 times as many layers, and that would increase the height to 4 times as much.

Layers of 4

Layers of 2

Layers of 1

Partitioning 10 000 Multiplicatively

Varying the base layers

1. Ask students to use the doubling/halving relationship to consider different base layers for their containers.

   Use 10 MAB ‘thousand blocks’ to confirm that 10 groups (layers) of 1000 equals 10 000. At this point, ensure students recognise that 10 000 can be partitioned multiplicatively in various ways: 10 x 1000, 2 x 5000, 5 x 2000 etc.

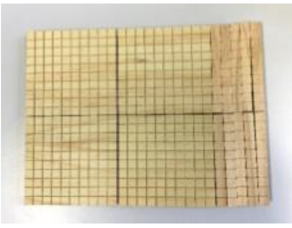
   Model doubling and halving (and the generalization of multiplying and dividing by other factors such as 5, 10 or 20) with 1000 in the base layer.

1000 x 10 = 10 000 If I double the number in the base layer, how many layers high would I need? 2000 x ? = 10 000	1000 x 10 = 10 000 If I halve the number in the base layer, how many layers high would I need? 500 x ? = 10 000

2. Partner students and have them explore possible base layers for a stack of blocks that could help in the visualisation of 10 000 centicubes (MAB ‘ones’), recording number sentences and pictures of considered partitions.

   Provide as many MAB blocks as possible for this, and supplement with photocopied sheets of 1 cm grid paper. Use MAB blocks for at least part of the structure so students can see the heights of their suggestions.

Expected Response

	Base layer has 500 (50 x 10) = 500	To get 10 000 cubes, we need 20 layers of 500 500 x 20 = 10 000
We put 500 MAB ‘ones’ on our base. We had 50 long and 10 wide. 50 x 10 = 500. This is really long, so we doubled the 10 and halved the 50 to get a base of 25 x 20 which must also equal 500.		
	Base layer has 500 (25 x 20) = 500 To get 10 000 cubes, we need 20 layers of 500 500 x 20 = 10 000	

3. Allow partners 15-20 minutes to explore several possible ways to partition 10 000 multiplicatively. Impress on students the importance of recording their mathematical thinking when working on an inquiry. They will need a record of the work they have done to share their thinking with others.

4. Move between partners listening for students’ thinking, prompting partners where necessary to think about how extended number facts and/or representations might assist them.

   T: You have used 2 layers of 5000 to make 10 000. What would happen if you want to use 20 layers?

   S: I would have less centicubes on the base.

   T: How many would you have on the base?

   S: I don’t know.

   T: What mathematics did you use to increase 2 layers to 20 layers?

   S: I multiplied it by ten so now I know. I will divide the number in my base layer by 10 and get 500.

   If evidence of student flexible multiplicative partitioning is not evident, then explicit teaching will be needed for potential combinations (generalising halving and doubling, and drawing on number facts).

5. As partners are working, pause the class and share a few examples of multiplicative thinking that use non-standard partitioning *(I know 500 x 20 = 10 000 (20 layers of 500 = 10 000) so if we double the number of groups, we halve the number in each group which also means 250 x 40 = 10 000 (40 layers of 250 = 10 000)*.

   Some students may generate pictures that rely solely on additive thinking. Spend time with these students to review the relationship between repeated addition and multiplication. For example:

   Teacher: *I see you have recorded 2000 + 2000 + 2000 + 2000 + 2000 = 10 000.*

   Can you think of another way to write this?

   How many times have you added 2000?

   Some students may also need support to identify the fact that all the layers have the same number of blocks.

Conclusion

6. Allow a further five minutes for students to review the partitions they have recorded, checking them for accuracy (calculators can be used here) and ensuring they have written number sentences. These will be shared in the next lesson (Devise Phase).