Bunches of Balloons
Lesson 2: Devise Phase

Australian Curriculum: Mathematics (Year 2)

ACMNA031: Recognise and represent multiplication as repeated addition, groups and arrays.

- Representing array problems with available materials and explaining reasoning.
- Visualising a group of objects as a unit and using this to calculate the number of objects in several identical groups.

ACMNA032: Recognise and represent division as grouping into equal sets and solve simple problems using these representations.

Lesson abstract

Students confirm that 29 balloons cannot be divided into equal groups, so will not evenly decorate the room. They do this by making arrays to show the counters in groups with left overs. Students refer to the arrays and consider the potential attractiveness of the decoration to choose a good bunch size. Using arrays or otherwise, students record some total numbers of balloons that allow for bunches of that size.

Mathematical purpose (for students)

Grouping objects allows us to calculate totals more efficiently.

Mathematical purpose (for teachers)

Using concrete materials to visualise a group of objects as a unit enables students to calculate a total more efficiently using repeated addition or skip counting. Arrays provide a clear representation, highlight remainders and clearly show the different ways a collection can be divided into equal sized groups. They also support discussion of the commutative property of multiplication (e.g. 3 groups of 4 or 4 groups of 3 both require 12 balloons but the resulting decoration is different). Recording and representing ideas provides clear visual evidence of thinking.

Lesson Length 60 – 90 minutes (may be split into two sessions)

Vocabulary Encountered

- recording/representing
- evidence
- rows
- multiply
- array

Lesson Materials

- Word wall from previous lesson
- Student workbooks
- Device for photographing and recording (optional)
- Counters (at least 29 per group)
- At least 29 Magnetic counters to display arrays
- Chart paper for class posters to use in next lessons

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Which Groupings Have Been Investigated?

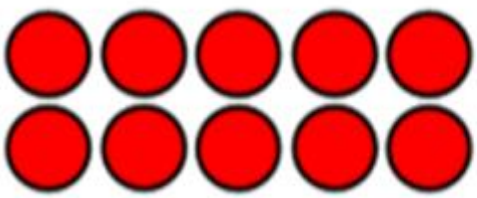
Inquiry Question: What is the best way to evenly decorate the room with 29 balloons?

Introduce Arrays

1. **Recap** on previous lesson to highlight that 29 balloons could not be put into equal bunches.

2. **Introduce arrays.** Compare the groupings that have been investigated. Have students share their groupings to ensure all possibilities have been investigated. As students share their drawings, represent each grouping as an array (and leftovers) on the board/chart for all students to see (see figure below). For example, a child chose to represent groups of 5. Explain how drawing an array makes it easy to see if the groups are all the same and reduces the need to continually count the balloons. Draw the first group of 5 in a row and emphasise how it is easy to see and draw. Then draw another row of 5 to continue the array. Emphasise how the circles are lined up both across and down. Comment on how this is an array: the counters are arranged in both rows and columns. 2 rows of 5 is 10.

   Ask the question: *How many rows of 5 will I need to draw to show all 29 balloons? Will it be the same as the number of bunches you have? How many will be left over?* Model labelling using 'groups' or 'rows of.'



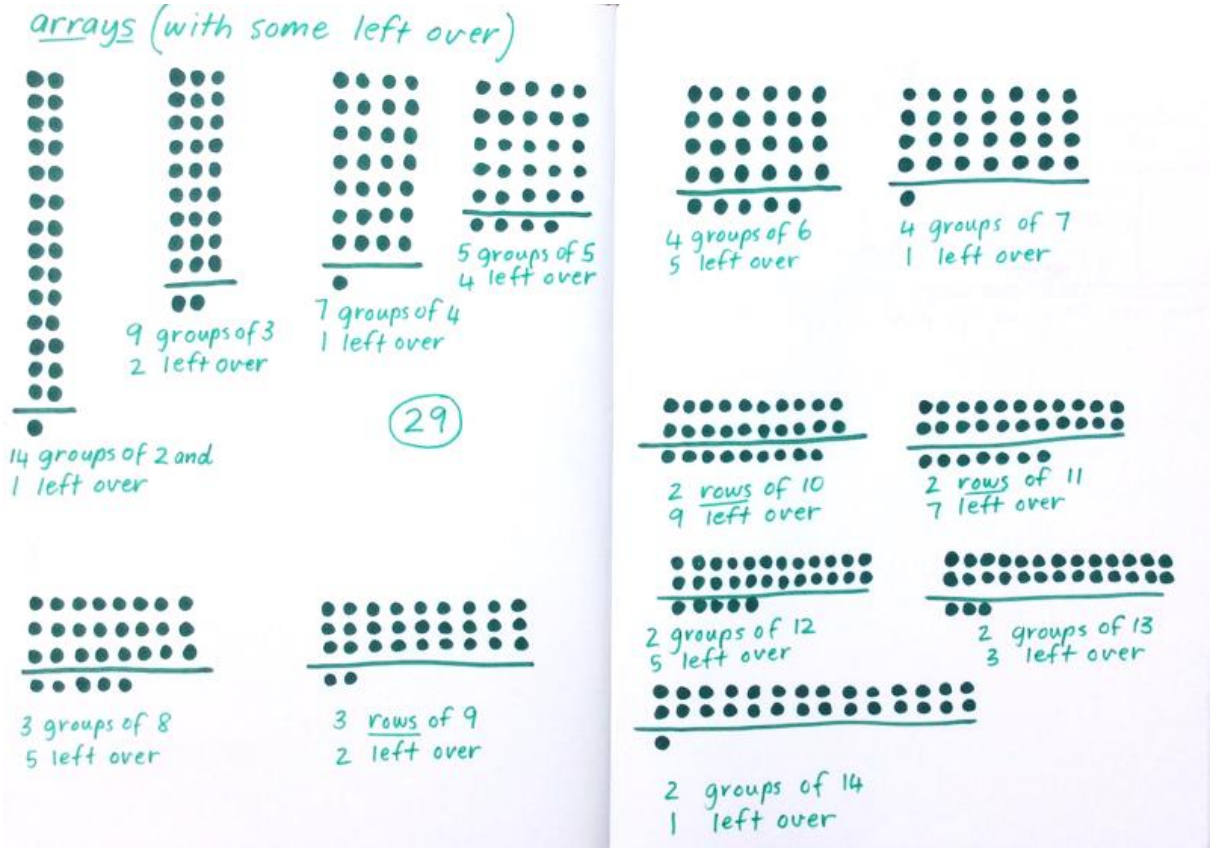
2 rows of 5 is 10

Check all possibilities have been investigated

3. **Record group sizes.** Make a list on the board (or chart for later reference) the group sizes that students have investigated. Work systematically up from bundles of 2 until all have been discussed and listed on the board. Have we checked all the different numbers?
   - Who tried to put 2 in each group? Show me your evidence. How many groups did you have? How many left over? Who tried 3 in each group? Who tried 4,5, etc.?
   - Compare students' representations of the same number to check their evidence.
4. **Draw arrays** on the board or chart until all possibilities are drawn (see example below). This representation will be referred to throughout the Inquiry.
   - Do we need to go beyond groups of 14? (Perhaps some students can say what happened when they tried bigger numbers).
5. **Look for patterns in the arrays**. Ask students to look carefully at the arrays and leftovers. Allow them time to discuss with a partner and then share what they notice with the class. Remind them to use the words on the word wall from the Discover Phase to share observations.

   Possible teacher prompts:

   - *Which arrays have none left over?* (none)
   - *Which arrays are very close to having none left over?* (Those with one or two missing, those with one or two as a remainder.)
   - *Are there any arrays that look the same?* (Yes: 14 x 2 with one left over and 2 x 14 with one left over, etc. Although the arrays look the same if one is rotated, the decoration of the room is very different because there would be 2 large bunches in one case and 14 small ones in the other. Note: The fact that 14 x 2 = 2 x 14 is an example of the commutative principle of multiplication.)



6. **Focus on the different arrays within the arrays.** Use 5 rows of 5 as an example. Explain that arrays are drawn as rows.
    - *Can we make other arrays within this array*? Circle 1 row of 5 to show 1 row of 5 is 5. Circle 2 rows of 5 to show 2 rows of 5 is 10
    - *Could we make an array from 20 counters?*
    - *What other numbers can we make equal bundles of 5 from?*

7. **Revisit the issue of not being able to divide 29 evenly**. *We now have the evidence that 29 cannot be shared into equal groups*. The only possibilities to decorate the room with equal sized bunches are having one bunch of 29, or 29 tiny bunches of 1 balloon each.

Change the Number of Balloons

Generating possibilities for the total number of balloons

8. **Go back to the Inquiry question.** What is the best number of balloons to evenly decorate the room?
    - Draw attention to 'decorate the room evenly' and the need to determine how many balloons would be needed. What do we need to do so we can make equal groups? 29 balloons will not work. We must change the number of balloons and perhaps buy more balloons.
    - Blow up some of the balloons and show students what different groupings of balloons look like.
    - Link the mathematics and counter representations back to the balloons by reiterating that we used the counters to represent the balloons.

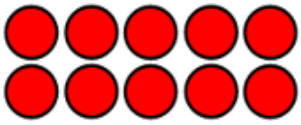
9. **Have the students** consider which size group of balloons they think would best decorate the room.
    - Ask: *What is the best number of balloons to put in each bunch/group to decorate our room? Are there any group sizes which wouldn't look good?* (E.g. If we only have 2 balloons in a corner, that will not be enough decoration.)
    - Discuss totals that are too small or large. (e.g.*80 balloons will be too expensive*; we want at least 4 bunches.)
10. **Go back to the arrays drawn on the board.** *Maybe you could use some of these numbers to help you select how many balloons you may need.*
11. Instruct students to discuss group sizes with their partner/s and decide on a group size they would like to use. *Why have you decided on this number of balloons in each group?*

Creating multiples

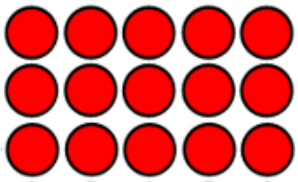
12. When students have decided on a group size, **model for them** (below) how to investigate the possible total number of balloons they might need to make equal groups of that size. They may use the arrays on the board to help decide on a total number of balloons which may be suitable but can also use the following method to explore all numbers which could be possible.
13. **As a class, look at one grouping** e.g. 5 balloons in a group. Ask students the possible numbers of balloons that could be used if groupings of 5 were used. 1 group – 5 balloons, 2 groups – 10 balloons, 3 groups – 15 balloons etc.

1 group of 5 is 5

2 groups of 5 is 10

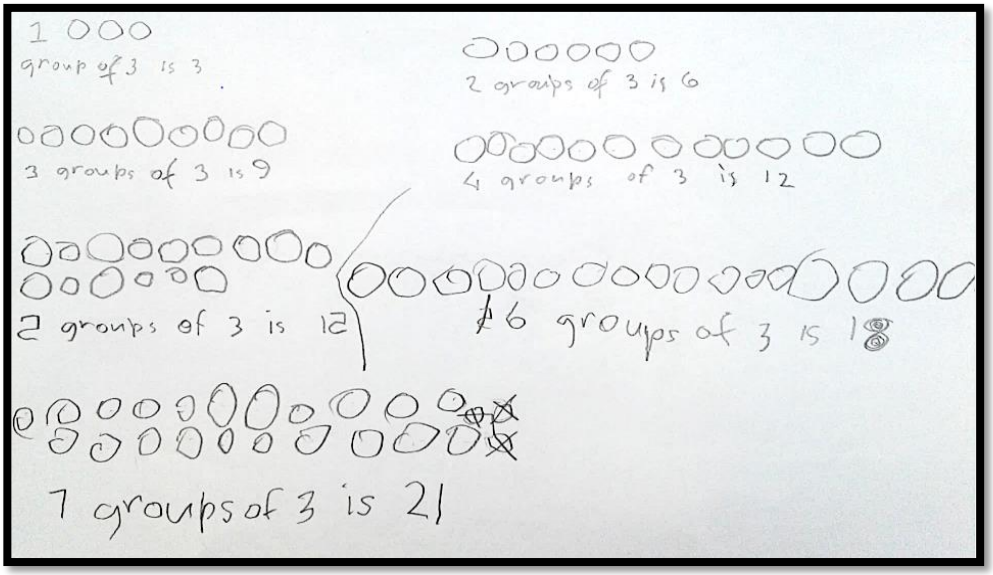


3 groups of 5 is 15

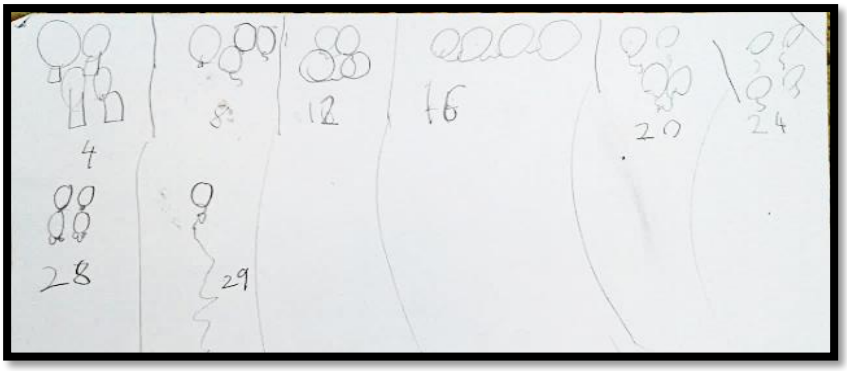


14. **Using their selected group size, have partners explore** how many balloons they might need if they used groups of that size. Ensure teams draw arrays or use counters to represent and record their thinking and the possible numbers of balloons for that group size. This will be referred to in the Develop phase when they decide which number would best decorate the room. The focus at this stage is not on students coming up with a specific number of balloons but on recording several different multiples for their grouping. Again, expect clear labelling.

15. The following work samples show emerging representations of groupings.



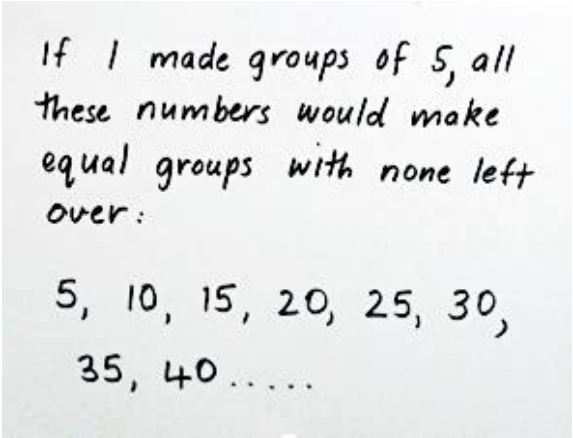
This example, although disorganised, has begun to show thinking through labelling of the representation, but groups of 3 are not identified. Have the student circle groups before moving to arrays. *I can't see your groups of 3. Circle them to make them more obvious.*

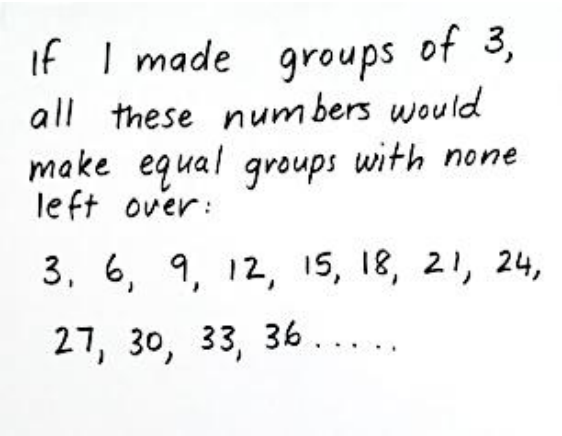


These examples show the multiples and number of balloons in each group, but lack clear labelling of how many groups are needed to create each total. They do not clearly distinguish between balloons and groups of balloons. They demonstrate skip counting and/or repeated addition. They also demonstrate the groups' uncertainty on how to proceed past 29. Move these students towards drawing arrays.

Conclusion

16. **As a class, share findings.** As partners share, record the multiples for each grouping on the board or on a poster so they can be referred to in the Develop phase.





17. **Reiterate that now they have found possible totals that** will not have any left overs. However, they still need to work out how many balloons would be best. This will be the task in the next lesson.