re(Solve) MATHS BY INQUIRY

MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS

Bunches of Balloons
Lesson 3: Develop and Defend Phases

Australian Curriculum: Mathematics (Year 2)

ACMNA031: Recognise and represent multiplication as repeated addition, groups and arrays.
- Representing array problems with available materials and explaining reasoning.
- Visualising a group of objects as a unit and using this to calculate the number of objects in several identical groups.

ACMNA032: Recognise and represent division as grouping into equal sets and solve simple problems using these representations.
- Dividing a collection of objects into equal-sized groups.
- Identifying the difference between dividing a set of objects into three equal groups and the same set of objects into groups of three.

Lesson abstract

In the Develop phase, students test different room arrangements using counters with chosen bunch size. They analyse arrays to decide on the best number of balloons to decorate the room. They represent their arrangement using a diagram, array, number sentence and written justification. In the Defend phase, students present their representations to the class and justify why their arrangement is best. They actively listen to others, ask clarifying questions and provide feedback. The class chooses the best option and they decorate the room.

Mathematical purpose (for students)

Representing our arrangement using a diagram, array, number sentence and written justification will help convince others that our arrangement of balloons is the best way to decorate the classroom.

Mathematical purpose (for teachers)

In the Develop Phase, students represent arrangements in a variety of ways that connect repeated addition and skip counting to early multiplication (*m* groups of *n*) and division (how many groups of *n* can I make?). Providing a written justification for why their arrangement is best develops mathematical reasoning and involves both mathematical and everyday decisions. In the Defend Phase, students further develop their mathematical reasoning as they explain their justified solution and provide feedback for others.

Lesson Length 90 minutes approximately (may be split into two sessions)

Vocabulary Encountered
- arrangement
- reason
- effective

Lesson Materials
- Student workbooks
- Device for recording (optional)
- About 30 counters per group
- Student Sheet 1 – My Bunches of Balloons (optional, 1 per student)
- Array chart and lists of multiples poster from the Devise phase

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Develop Phase

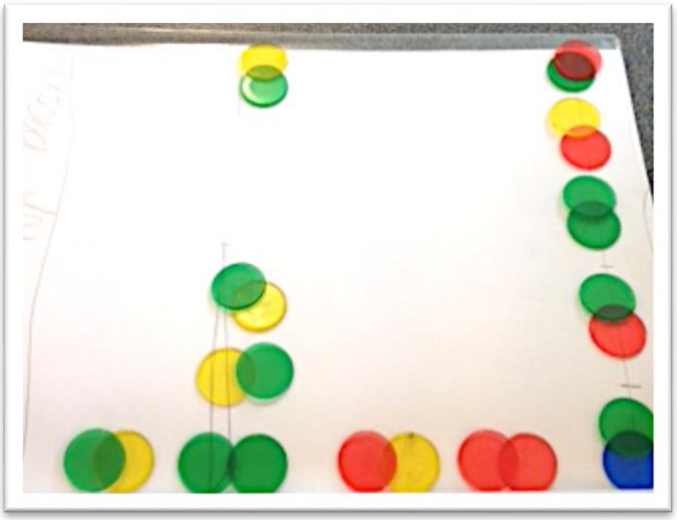
Considering possible totals

1. **Revisit the multiples** listed on the poster in the Devise phase to review the possible numbers of balloons in one grouping, such as all groups of 5. Tell students that all these numbers allow us to make groups of 5 balloons to decorate the room, with none left over. We could use just one bunch of 5 balloons, but that wouldn't decorate the room evenly. We could use 10 balloons. How many bunches would that give us? Could that decorate the room evenly? Repeat with another grouping to reiterate that the multiples they recorded yesterday will help them work out the number of balloons they might use.

2. **Distribute counters (at least 30 per group).**

3. **Students now explore** how many groups of balloons **would best** decorate the room. With their partners, students **draw** a bird's eye outline of the room. They can use this test different arrangements, placing the counters to show where the bunches will be (such as in the four corners of the room). The number of balloons per bunch should be that chosen at the end of the previous lesson.

4. Reiterate the need to record and label **all** arrangements as evidence to be referred to in the Defend phase. Students can label their arrangements in multiple ways: in groups, arrays and number or word sentences (5 + 5 + 5 + 5 + 5 = 25, 5 groups of 5 = 25)



Student: *Yesterday we decided to have 2 balloons in each group. I have 12 groups of 2*

Teacher: *How many balloons will you need to decorate the room with this arrangement?*

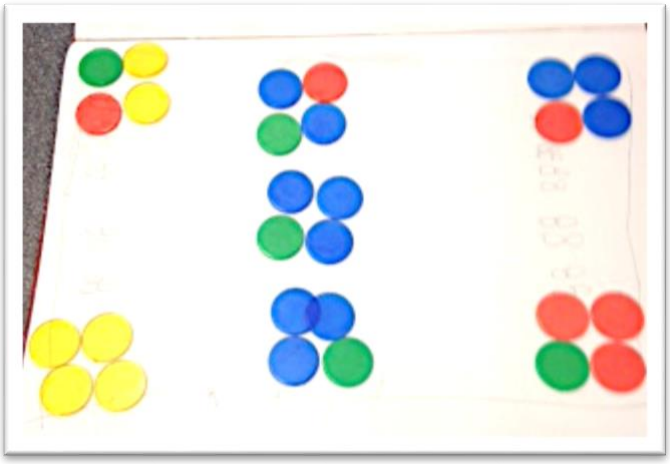
Student: *24*

Teacher: *Does this arrangement decorate the room evenly?*

Student: *No*

Teacher: *What could you do to make sure the room is all decorated?*

Student: *I could put some more bunches of 2 or I could move some of these a bit*



Student: *I have 7 groups of 4. I have a bunch in each corner and three in the middle. They can tie onto the string (the display wire running across the room.) The bunches are even around the room.*

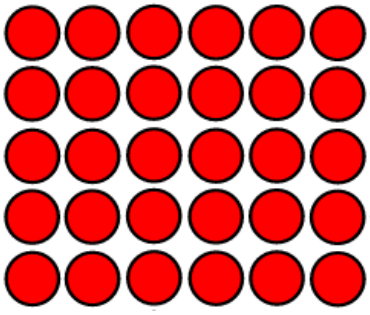
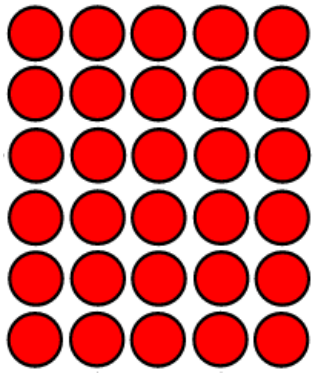
7 groups of 4 is 28. I will need 28 balloons. I just took 1 off 29 and had 28.

5. **As partners work,** visit each group. Ask questions such as "*Where in the room could the balloons go?*" "*How many balloons would you need if you arranged the balloons this way?*" "*Can you think of another arrangement?*" Observe different ideas students have for placing the balloons and the reasons they are giving when they discuss arrangements.

Checkpoint

6. **Emphasise mathematical concepts surrounding multiplication** with the whole class.

   At a convenient time, stop students to share their initial arrangements with the class. Record their groupings on the board as arrays. This will allow the students to look for patterns, see if other groups have the same findings and provide further opportunities to compare and discuss the mathematics.

	
5 groups of 6 I have 30 balloons	6 groups of 5 I have 30 balloons

7. **Reinforce the value of arrays** as an efficient and quick way to organise the groups so totals are easily calculated. Demonstrate the difference in how numbers "look" when they can be arranged in equal groups (such as 30 balloons above) compared to when this is not possible (such as 29 balloons: compare with the 'arrays with left overs,' chart from the Devise phase).

8. Look for student representations that can demonstrate the following for discussion with the class.

   - **Common total, different arrays.** There are some numbers which have several groupings e.g.: *We can make many different arrangements from 24 balloons. Are there other numbers of balloons which can be organised in many different arrays?*
   - **Commutative property.** Although they use the same number of balloons, 5 groups of 6 do not look the same in the room as 6 groups of 5. *Can you think of other examples like this?* Focus students on the fact that although the total is the same, the arrangement is different.
   - **Same number in groups, different numbers of groups.** *Sammy has made groups of 5 and her total number of balloons is 25. Lyn has made groups of 5 and her total is 30. How many groups have they each made? Which will decorate our room better? Why? Where will you put your bunches?*
   - **Skip counting to check totals.** *How can we quickly check if the total is correct without counting balloons one by one?*
   - **Repeated addition.** *Tom's group has recorded his arrangement as 5 groups of 6. How can he write this as a number sentence?* (6 + 6 + 6 + 6 + 6 = 30)
   - **Quotition and Partition. Most of the division situations above are 'quotition', e.**g. *How many bunches of 5 can I make from 30 balloons?* Also include some 'partition' situations: *I have 20 balloons and I want one for each corner of the room. What size bunches should I use?*

Teacher Note

The tasks in this inquiry allow the teacher insight into students' understanding of the processes of multiplication and division (repeated addition, skip counting and arrays) and provide opportunities for diagnostic interactions. Writing number sentences with the multiplication symbol is not specified in the Australian Curriculum: Mathematics until Year 3.

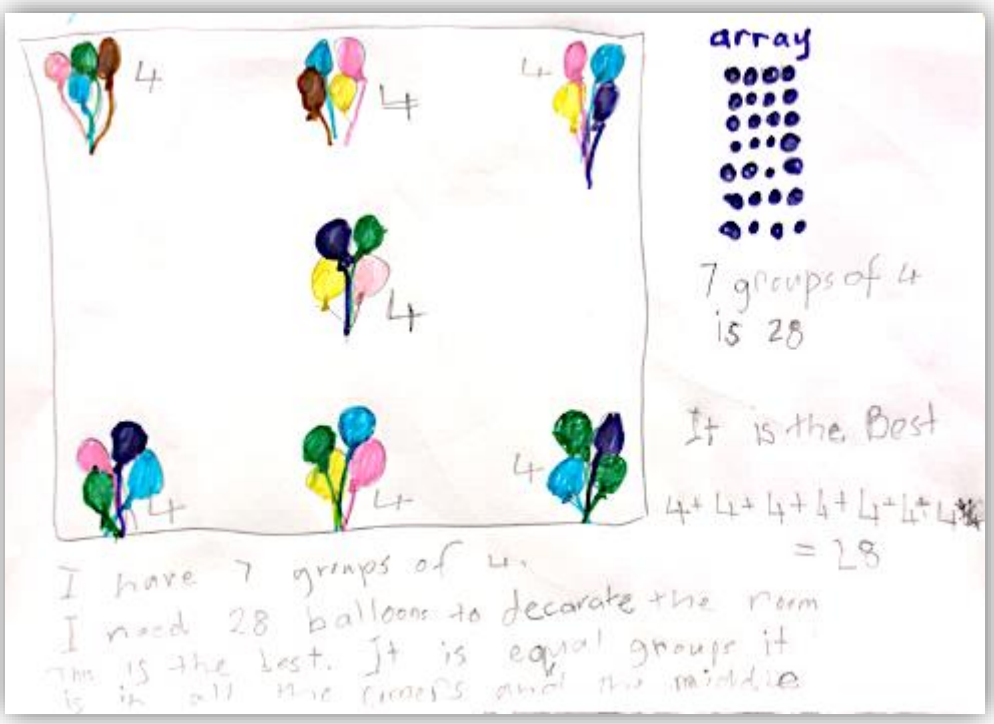
Refining Plans

9. Revisit the Inquiry question. '*What is the best way to evenly decorate the room?*' and direct partners to revisit their representations and decide which total they like best and why.

10. Students prepare to present and defend their choice. Write the checklist below on the board and/or hand out Student Sheet 1 – My Bunches of Balloons to guide students in this preparation. The student sheet may be especially helpful for any students working with a teacher aide.

    - A clear picture showing where the balloons will go
    - Write the number of balloons in each group
    - An array that matches the groups (model how to represent and label arrays in words and numerals if required).
    - A number sentence
    - A statement explaining why you chose to decorate the room this way. Why did you choose that number in each bunch and that total number of balloons?

Expected Student Response



Defend Phase

Presenting findings

1. **Partners present their ideas to the class.** Encourage students to offer feedback, using the checklist and Student Sheet as a guide. Help students to respectfully listen to and ask questions of others.

2. **Display all final arrangements.** Ask students to observe whether any ideas are similar.

3. **Have a whole class discussion** on the best way to decorate the room. Is there a common solution that the class can agree upon – a preferred total number of balloons and a preferred bunch size?

4. Decorate the room for your special event!!

Extensions

- What might the room look like if we doubled the number of balloons we have? (Make sure to highlight that the number of balloons in each bunch could be doubled, or the number of bunches could be doubled – but not both at the same time).
- How many balloons would we need if we decorated all our Year 2 classrooms in the same way?
- Can you find any other numbers like 29 that cannot be divided into equal groups? (These will be prime numbers.)
- Make a list of numbers that can be divided into equal groups in a lot of different ways. (e.g. 24).

My Bunches of Balloons

Name: _____

Draw your solution for decorating the room.
Label the groups you make.

	Draw as an array: Number sentence:

I need _____ balloons.

This is the best number of balloons because

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