Expanded Square Designs

Lesson 1: Discover

Australian Curriculum: Mathematics (Year 4)

ACMMG091: Create symmetrical pictures and shapes with and without digital technologies.

ACMMG088: Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies.

- Identifying common two-dimensional shapes that are part of a composite shape by re-creating it from these shapes.

Lesson abstract

Students are introduced to expanded squares (based on the Eastern art of Notan) and explore their characteristics. They construct their first expanded square and discuss its features, including symmetry.

Mathematical purpose (for students)

When a shape is flipped or turned it has the same shape and size, but flipping gives it a different orientation.

Mathematical purpose (for teachers)

Prior knowledge of one step transformations (flips and turns), symmetry and the features of a square assist to create an artistic expanded square design. Mathematical terminology is used to describe the two-dimensional shapes included in designs and to justify whether designs are symmetrical or asymmetrical.

At the end of the Discover phase, students will be able to:

- Share their initial expanded square by describing the two-dimensional shapes used in the design and the symmetry in the whole design.
- Appreciate the interaction of coloured and white space created by simple transformations in these designs, also known as Notan.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- Symmetrical, asymmetrical, geometric, polygon, curved, transformation, flip-(reflection), horizontally, vertically, expand, orientation, position

Lesson Materials

- Scrap coloured paper or light card (1 sheet per student, approx. A6 with straight edges)
- Student workbooks or sheet of A4 paper
- 16 centimetre coloured paper squares (1 sheet per student, **same colour both sides**)
- Art block paper or A3 paper (white, 1 piece per student)
- Slide show: *ST8_Expanded_Square_1a_Making_Designs.pptx*

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Expanded Squares

What is an expanded square?

This is an example of an 'expanded square'. It is make by cutting out parts of the square and flipping them to the outside. The term expanded square is not formally introduced until after students have made their first design. The design criteria for an expanded square are introduced or hinted at in this lesson, with a full list of criteria given in the next lesson (Devise phase).

If you are unfamiliar with expanded square designs, you may like to view this article by Jane Dunnewold or alternatively search the internet for "expanded square images" before beginning the unit.
http://www.janedunnewold.com/s/ExpandedSquare-JDTutorial.pdf

The slide show *ST8_Expanded_Square_1a_Making_Designs.pptx* contains the images in this lesson.

Introducing the inquiry context

1. Promote engagement in this Inquiry by presenting the following context to students.

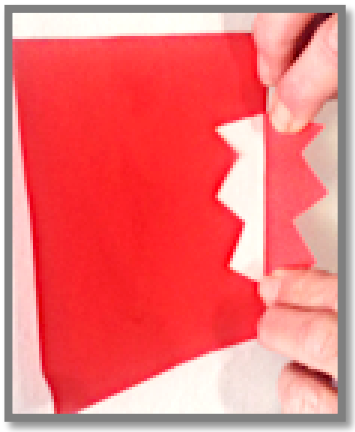
   *I have a great idea for making new card designs using **only** a square piece of paper.*

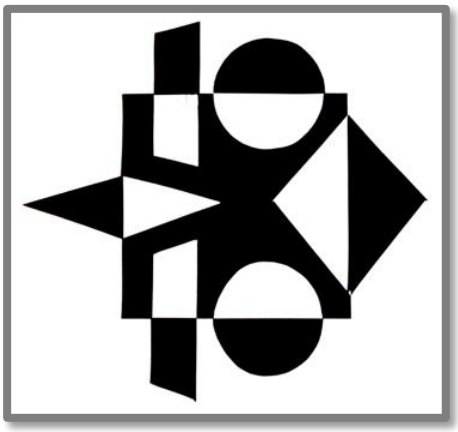
   Hold up a square piece of paper. Ask students if they have any suggestions about how the square could be made more interesting.

   Teacher: *What could I do to with this square to make it more interesting?*

   Suggestions may include:
   - *Draw patterns on the square and colour them in.*
   - *Fold it into a 3D object or turn it into a pop up card.*
   - *Paste pieces onto it.*
   - *Cut pieces out of it.*

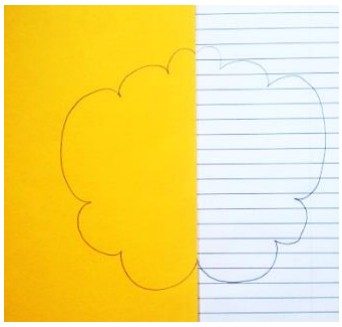
   Value all responses but focus on any responses that move students towards the mathematical intent (**to cut out areas and flip them to the outside of the square**). If flipping (reflecting) cut out shapes is not suggested, suggest it. Show slides 2-3 of *ST8_Expanded_Square_1a_Making_Designs.pptx*. *We could flip the cut-out shapes and paste them right next to the edge of the square. We will call this 'expanding' the square. We can cut out whatever shapes we want. What shapes can you see in these designs?*



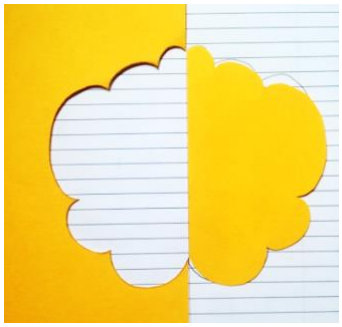


What does a flip do to a shape?

2. Provide students with some scrap coloured paper or light card. Tell them that they will be exploring what happens when we flip a shape to the outside of the paper. This will help later when they are planning their designs. Ask them to:



   - Draw an interesting shape on the edge of the paper. It is easiest if one side of the shape lines up with the edge of the paper. (Encourage composite shapes with asymmetry, but keep in mind that they will need to be cut out as one piece.)
   - Place the paper on a page in their workbook, and draw what they think the shape will look like if it is cut out and flipped along the edge of the coloured paper. The edge of the paper also needs to be traced onto the workbook, as this will be the mirror line. (Students can position their paper so that the line of reflection is either vertical or horizontal.)

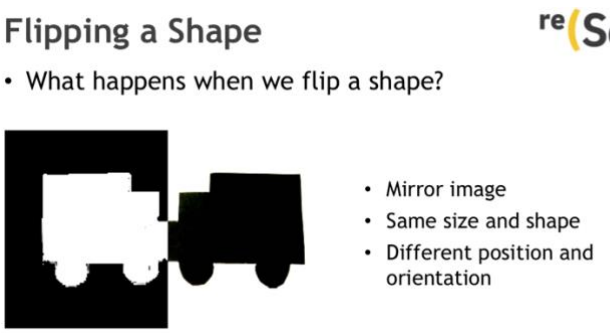


   - Pick up the coloured paper, and cut out the shape.
   - Paste the sheet of coloured paper into the workbook where it was before.
   - Place the cut-out shape in the workbook over the predicted drawing, trace around it, and compare.

3. Ask students: *Does the flipped shape look the same as you predicted? What can you tell me about the flipped shape?*

   - *The size of the flipped shape is exactly the same as the size of the white space.*
   - *The flipped shape is the mirror image of the white space - it has a different orientation. What is on the left of the white space is on the right of the flipped shape. If a picture of your face is flipped around a vertical line, the left side is now on the right side, just like it is in a mirror.*
   - *I placed my shape differently, and flipped it around a horizontal line. What is on the top of the white space is on the bottom of the flipped shape.*

   To summarise, show Slide 4 of *ST8_Expanded_Square_1a_Making_Designs.pptx:*



Expanding a square

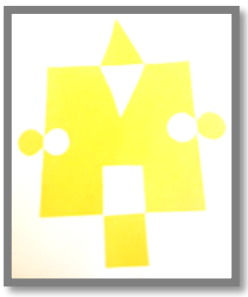
4. Show examples of expanded squares from Slide 5 of *ST8_Expanded_Square_1a_Making_Designs.pptx*, or if desired use your own examples.

   Have students describe what they notice about the expanded squares. Draw their attention to the original square shape. It is important they realise that:

   - The side of the square becomes the mirror line (line of reflection). If each shape is flipped back to the original square it will fit exactly and make a complete square.

- The shape of the original square is still visible in the design. The four corners are not cut.
- All four sides of the square have been used.

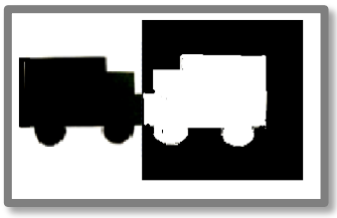
5. Provide each student with a 16 centimetre coloured square (same colour both sides). Ask them to expand the square on their desk by cutting out pieces and flipping them. Tell them that this will be their first design and that they will be planning another design in the next lessons.
6. Allow about fifteen minutes for students to work on their designs before having them paste them onto a sheet of art block (or A3 paper).



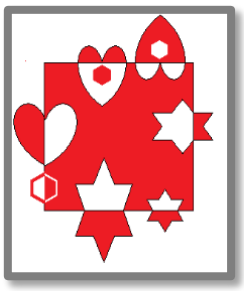
Checkpoint

7. Before designs are shared, write on the board or a poster some mathematical language to describe possible cut-out shapes and their features. Include curved, geometrical (e.g., polygons and circles) and composite shapes. Have students share their designs by describing the two-dimensional shapes used and showing how they have been flipped. Highlight contributions which demonstrate:
    - Accurate flipping from the side of the square (flipped shape is placed on the side of the square and touches along the edge).
    - Use of all four sides.
    - Keeping the original square easily identifiable.
    - Interesting, detailed designs which do not leave large coloured regions in the original square.

    As designs are shared, encourage students to respectfully comment on the designs, looking for things to compliment and aspects to improve. Slide 6 has the following examples which could offered to the class for comment:



Cut-out shape is only on one side:
Student: I like the composite shape you have made. The truck has been flipped correctly but an expanded square looks better with shapes cut out on all four sides. And it is also nice to have smaller cut outs which mix black and white more.



Some shapes are not flipped correctly:
Student: The two hearts on the top edge of the square and the star on the right have not been flipped around the edge of the square: the part outside the square doesn't match the part just inside.
The hexagon on the bottom left is very interesting and has been flipped correctly. Just the outline has been flipped. How did you cut this out? (It is possible to cut out the trapezium and flip it, and then cut and flip back the smaller trapezium.)

Cut out shapes moved not flipped:
Student: Your design is interesting and you have used geometric shapes to create it. It is really hard to see the shape of the original square. I can see shapes have been pasted to the outside of the square but they do not look like they have been flipped.

Some cut out shapes have been cut and flipped a second time:
Student: This is a creative design. I can see triangular shapes have been cut out and flipped. Then it looks like the tip of the triangle has been cut and flipped again. The cut-out shapes outside the square do not mirror the cut-out space in the original square.

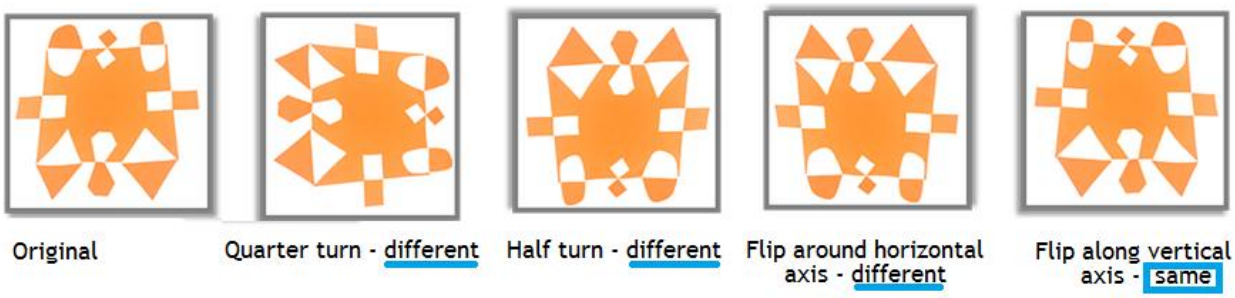
Exploring symmetrical and asymmetrical designs

8. This is another way that students can think more deeply about their designs. Introduce the idea of symmetry in the whole design. Look for three types of symmetry:

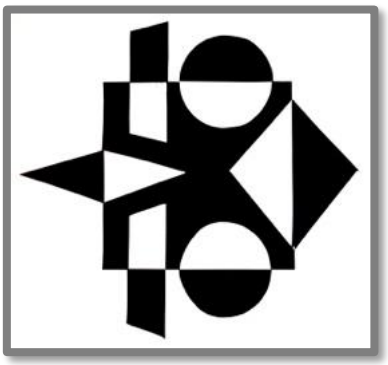
    - Does the expanded square look the same if it is:
        - given a quarter turn? (quarter turn symmetry)
        - given a half turn? (half turn symmetry)
        - flipped over? (mirror symmetry)

    The example below is on Slide 7 of *ST8_Expanded_Square_1a_Making_Designs.pptx*.

    The expanded square on the left of the image below does not have quarter turn or half turn symmetry. It does have mirror symmetry with a vertical fold line.



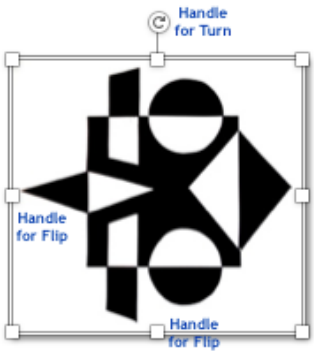
    Slide 8 of *ST8_Expanded_Square_1a_Making_Designs.pptx* has designs for students to discuss.



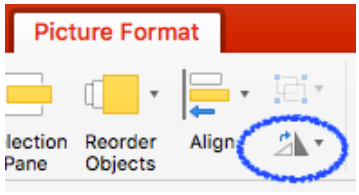
Mirror symmetry around horizontal axis

No symmetry: asymmetrical

Quarter turn and half turn symmetry



If you would like to show students the images flip and turn, you will need to exit the slide show mode, then click on the image to select it. To turn, click and drag the 'handle'. To flip, click and drag one of the squares on the side. The image will get thinner and then flip. Alternatively, use the Rotate/Flip tool in the Picture Format menu (be aware that "Flip Vertical" flips around a horizontal line, and vice versa).



9. Have students look at the design they have created and decide what sorts of symmetry it has, if any. Direct students with symmetrical designs to form one group and students with asymmetrical designs (no symmetry of any sort) to form another group. Within the groups have students check that all the designs meet the criteria (symmetrical or asymmetrical) for their group and if not, have them explain why.

Conclusion

10. Inform students the design they have created today is what artists call an *expanded square* – based on the principle of *Notan* (Japanese word meaning *dark-light)* which focuses on the interaction between the white and coloured spaces in the design. Notan designs generally use only black and white. Across all aspects of life, Eastern culture seeks a balanced view of the world (as depicted in the famous yin-yang principle and symbol), so a design which balances the dark and light spaces is desirable.
11. To familiarize students with expanded square designs, use internet images of expanded squares or *Notan* designs to **discuss** the symmetry in each design (or asymmetry), the shapes used, and the amount of coloured and white space. Challenge them to find examples that do not strictly match the criteria for an expanded square, and explain why. Depending on the availability of digital devices or computers, this could be done either as a whole class or as a partner activity.