Expanded Square Designs
Lesson 2: Devise

Australian Curriculum: Mathematics (Year 4)

ACMMG091: Create symmetrical pictures and shapes with and without digital technologies.
ACMMG087: Compare the areas of regular and irregular shapes by informal means.

- Comparing areas using metric units, such as counting the number of square centimetres required to cover two areas by overlaying the areas with a grid of centimetre squares.

ACMMG088: Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies.

- Identifying common two-dimensional shapes that are part of a composite shape by re-creating it from these shapes.

Lesson abstract

Students use fractions to describe the amount of white space in some partially coloured grid squares, and then in some expanded square examples. They consider what fraction of white space is best for visual balance. Students begin to design an expanded square that has approximately half the original square flipped to the outside.

Mathematical purpose (for students)

Half of the area of a square can be made up from several separate pieces.

Mathematical purpose (for teachers)

Areas can be estimated or calculated. The lesson confronts the misconception that a fraction of a square (e.g. one quarter) must be in one continuous iconically shaped piece. A quarter of a square might be made up of several small regions. To create an expanded square design that has approximately half the original square flipped to the outside it is necessary to plan carefully. Some students will estimate the required size of flipped pieces; some may plan to make up the half with equal parts of known size. At the end of the Devise phase, students will be able to:

- Present their plan for an expanded square
- Explain how they determined about one half of the original square has been flipped to the outside.

Lesson Length 90 – 120 minutes

Vocabulary Encountered

- approximately
- visually balanced

Lesson Materials

- Student workbooks or scrap paper (for planning design)
- 16 centimetre coloured paper squares (1 sheet per student, same colour on both sides)
- Envelopes to store cut out shapes (1 per student)
- Art block paper or A3 paper (white, 1 piece per student)
- Student Sheet 1 – A Balanced Design or slide show *ST8_Expanded_Square_2a_Balanced_Design.pptx*
- Student Sheet 2- Design Planner and slide show *ST8_Expanded_Square_2b_Design_Planner.pptx*

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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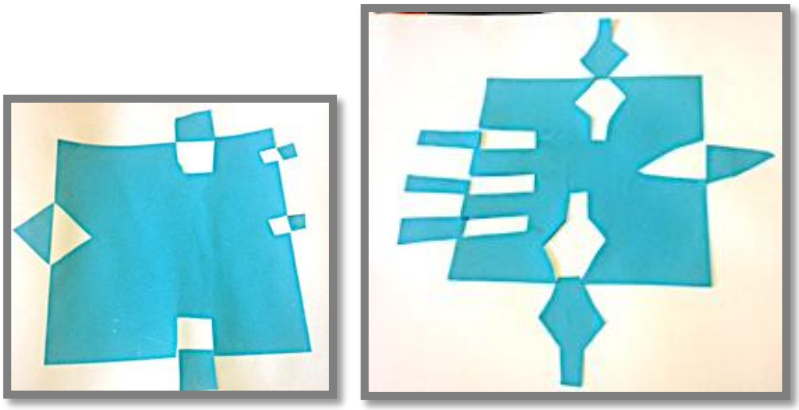
Areas of White and Coloured Space

1. Review with students the work they did in the previous lesson. *Last time you 'expanded a square' by reflecting (flipping) cut out parts to the outside of the original square. Today you will create another expanded square after the class has determined the best amount of the original square to flip to the outside.*

   Display two expanded squares created in the Discover Phase, preferably one that has a small amount of white space and the other that has a large amount of white space. Some student work samples are given here for reference:



2. Teacher: *Look at these **designs**. Are they both expanded squares? How do you know? (Yes. They both have shapes cut out of the sides that have been flipped and pasted right next to the edge. Also, the corners are still there and you can see that shape of the original square.)*

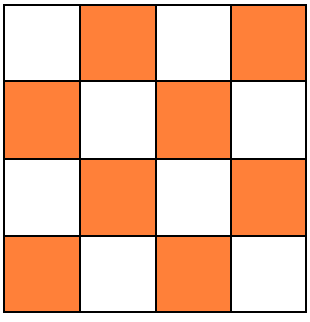
3. Review with students what is meant by **area** (the area of a two-dimensional shape is the amount of space within the shape). Have partners compare the amount of white and coloured space inside the two selected expanded squares. For example:

> Student: *More has been cut out of the second design.*
>
> Teacher: *Can you rephrase that using mathematical language?*
>
> Student: *The white space inside the square in the first design covers a smaller area than the white space inside the square in the second design.*

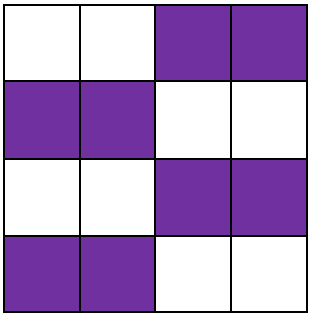
Calculating total area when combining multiple smaller areas

4. Ask students how they might work out the total white space when the bits of white space are separated. Hand out Student Sheet 1 – A Balanced Design or present slide show *ST8_Expanded_Square_2a_Balanced_Design.pptx, slides 2-4.*)

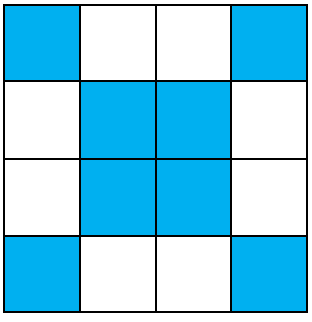
   Look at each of these squares and work out what fraction of the area is white. Tell your partner what fraction you worked out and explain to them how you decided.



Square A

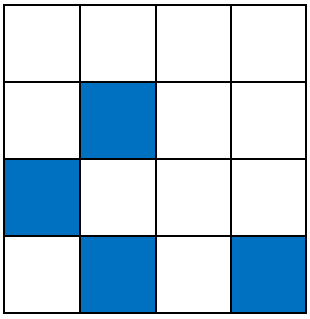


Square B

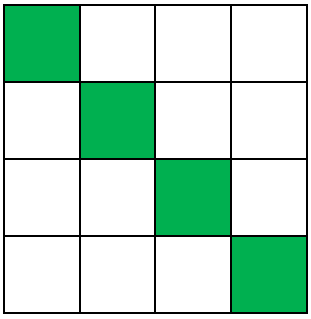


Square C

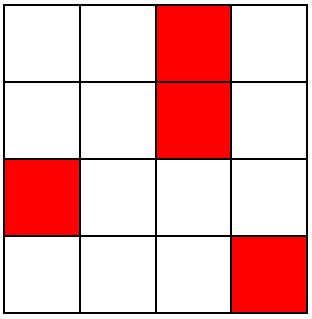
What about these squares? What fraction of each square is white? Explain your answer to a partner.



Square D



Square E

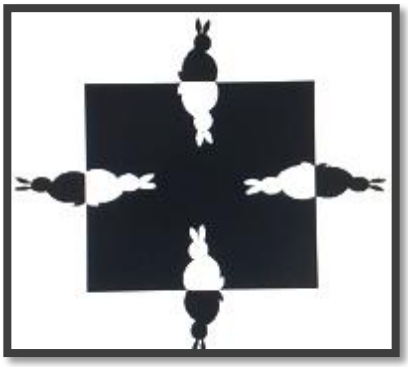


Square F

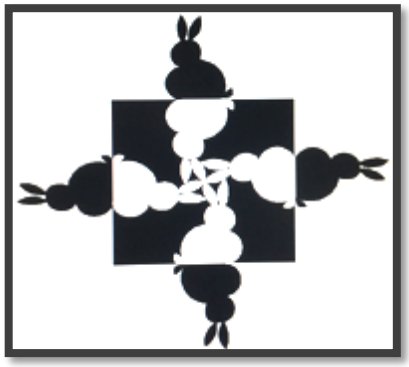
Expected Student Response

- *A, B and C each have half white space. If you count the small white squares there are 8 in each big square of 16.*
- *D, E and F each have three quarters white space. There are 12 small white squares out of 16 in each big square. It doesn't matter how the white squares are arranged. If the big squares have the same total number of squares and the same number of white squares, they will have the same fraction of the total area white.*

What about these expanded squares? Estimate what fraction of the original square is white space. Explain your estimated fraction to a partner.



Design A



Design B

Design C

Expected Student Response

- *The bunnies in Design A are quite small. If they were joined together, I think they would cover about a quarter of the square so a quarter of the original square is white space.*
- *In Design B, each white bunny is about the same size as the coloured space next to it, so I think half of the original square is white space.*
- *Design C is harder to work out but I think that there is more white space in the original square than Design B so I will estimate that three quarters of the square is white.*

Share estimates and reasoning for each example as a class and agree on a good estimate for each.

Deciding the best amount of white space

4. Remind students that artists use areas of dark and light space to create visual balance in their designs. Have them study Designs A, B and C (above) and decide what is the best fraction of white space in the original square to make a visually balanced design.

   Teacher: *Look carefully at each design and the estimated amount of white space. If we want a design that is **visually balanced**, what do you think is the **best fraction** of white space? Remember to provide a reason for your choice.*

5. Have three or four students share their responses and reasoning (prompting where required). For example:

   > Will: *I think Design B is the most visually balanced because it has about half of the original square cut out and flipped.*
   >
   > Teacher: *Why do you think Design B is more visually balanced than the other designs?*
   >
   > Will: *Design A leaves too much coloured space which dominates the design and just draws your eye to the black paper instead of the bunnies. Design C cuts too much from the original square and makes a design that is too busy, too crowded. The design takes over the original square and it looks unbalanced. In Design B the white and coloured spaces balance because they are in equal amounts.*

   If a student's reasoning is based on a preference for a design because it appeals visually (e.g. three quarters, because Design C has four different shapes not just the same shape), reiterate that they are considering only the best fraction of white space to create a visually balanced design.

6. In preparation for the inquiry question, establish that flipping one half of the original square is a good guide to get a balance between coloured and white space. If you wish, conduct a quick poll of students to reach this conclusion.

Introduce the inquiry question and the Design Planner

Inquiry Question: How can we design an expanded square where approximately half the area of the original square is flipped to the outside?

7. Display the inquiry question (found in the slide show *ST8_Expanded_Square_2b_Design_Planner.pptx, Slide 2)*. Inform students that they will need to plan their design before it is created, and hand out Student Sheet 2- Design Planner. Model the type of thinking students could use when planning as you display the slide show. Encourage students to respond to the 'I need to think about..' statements.

8. Record the characteristics of an expanded square on the board as they emerge from the discussion, or display Slide 8 from *ST8_Expanded_Square_2b_DesignPlanner.pptx*.

   - All corners are left intact
   - Pieces are cut from all four sides
   - All cut out pieces are flipped to the outside of the original square and pasted right next to the edge so they are a mirror image
   - The original square can still be seen in the design

9. Students can plan their designs in various ways – e.g. some may sketch the design, others may cut up a square of paper and arrange the cut-out shapes to work out a design. Students may also work through Student Sheet 2- Design Planner in a different order than presented.

10. Circulate and discuss with students how they know that the area being cut out and flipped is approximately half the original square (at this stage they will most likely be making rough estimates). Take note of any interesting designs that will be useful to focus on during the checkpoint.

11. When students complete their design, have them share it with a partner. Partners check the design to see if they agree that approximately one half of the original square will be cut out and flipped. If partners

disagree on the fraction, have them discuss and make the necessary adaptations to the design plan. For example:

Kylie: *I think your design has less than half cut out and flipped. You could make your car a half centimetre bigger all round and that would make it closer to a half.*

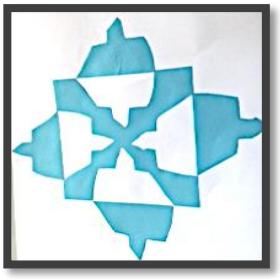
12. After design plans have been checked by a partner (and adapted where necessary), allow students to begin constructing their expanded squares using a 16 centimetre coloured square and the white art block paper.

Checkpoint

13. Seek three or four volunteers to share interesting designs and how they determined the area to be cut out and flipped. Explain to students that Checkpoints are pauses while their work is incomplete to check in on their progress and any challenges they are encountering. Advise students that they will be sharing their designs, even if not finished, to:
    - Explain what they have done so far.
    - Present any challenges they are having.

    Highlight the importance of actively listening and engaging with the sharing of ideas by:
    - Asking questions to help presenters see where they need to make their thinking clearer.
    - Providing feedback and ideas to others to enable them to work through challenges.
    - Valuing the different approaches others use.

    When sharing, have students discuss:
    - The shapes used – composite, geometrical, curved, etc.
    - Symmetry in the design - quarter turn, half turn, horizontal mirror line, vertical mirror line, or none.
    - How they determined they had flipped half of the area of the original square.
    - What makes the design interesting and visually appealing.



Teacher: *How do you know approximately half of the area of the original square is flipped to the outside?*

Fabi: *I think if I placed all the cut-out pieces together they would take up about half the square.*

14. Allow students time to complete their designs. (The next lesson can proceed even if some students have not finished *constructing*, but they need to have finished planning their design.)

Conclusion

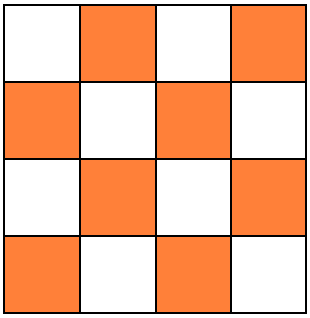
15. Ask students: *Are you confident that your expanded square actually has half the area of the original square flipped to the outside? Why/Why not?* Share responses, eliciting the need to have a way to accurately measure the amount of cut out space. Inform students this will be the focus of the Develop Phase of the inquiry (next lesson) and encourage them to consider different ways to measure the amount of cut-out space before the next lesson.

    Hand out envelopes for students to store any loose pieces. Ensure students retain any templates they have made and used in the design. These will be useful as they check and refine their designs in the Develop and Defend phases.

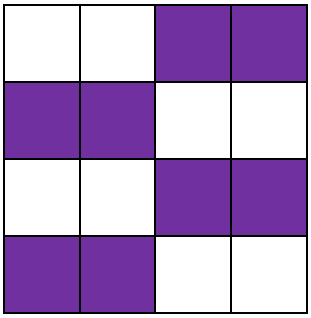
A Balanced Design

Name: _____

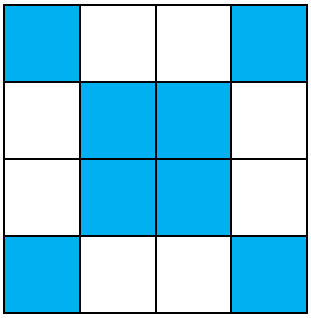
Look at each of these squares and work out what fraction of the area is white. Tell your partner what fraction you worked out and explain to them how you decided.



Square A

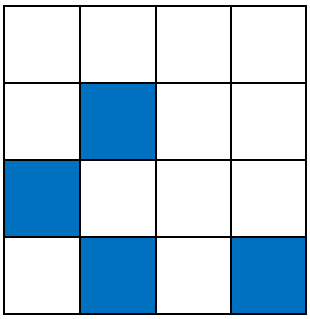


Square B

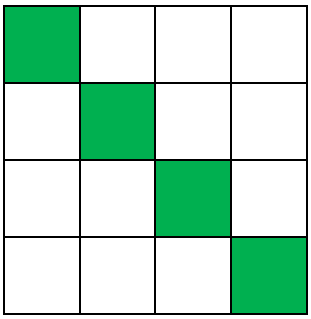


Square C

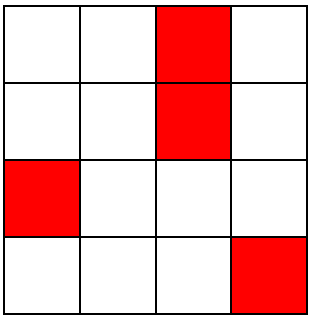
What about these squares? What fraction of each square is white? Explain your answer to a partner.



Square D

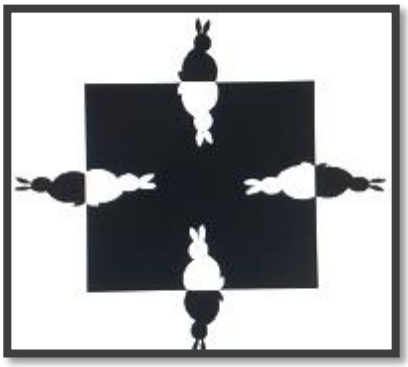


Square E

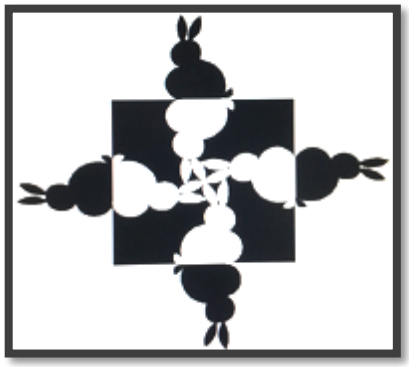


Square F

What about these expanded squares? What fraction of the original square is white space? You will need to estimate. Explain your estimated fraction to a partner.



Design A



Design B

Design C

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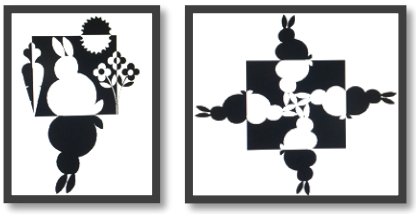
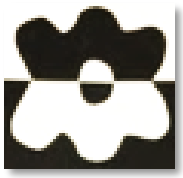
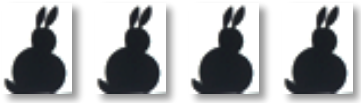
Design Planner

Name: _____

I need to think about…	
	Do I want my design to be asymmetrical or symmetrical: -when whole design turned? -when whole design flipped?
	What shape/s will I use? What geometric shapes will I combine to create my shape/s? Will the whole shape be easy enough to cut out?
	Will I flip the whole shape or half the shape? How can I create a half shape from my whole shape?
	Half the area of the square is to be flipped. How can I work out the size of the shape to be flipped?
	How will I be able to make the shape exactly the same each time?
What are the characteristics of an expanded square that I need to remember?	

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