Expanded Square Designs
Lesson 3: Develop

Australian Curriculum: Mathematics (Year 4)

ACMMG091: Create symmetrical pictures and shapes with and without digital technologies.
ACMMG087: Compare the areas of regular and irregular shapes by informal means.

- Comparing areas using metric units, such as counting the number of square centimetres required to cover two areas by overlaying the areas with a grid of centimetre squares.

Lesson abstract

Students produce mathematical evidence to convince others that approximately half the area of their original square has been flipped to the outside. They find areas by counting grid squares, or covering with 1 cm cubes, and perhaps rearranging shapes. They seek constructive feedback on the method they used to determine the fraction of the area that has been flipped and on the mathematical evidence they recorded.

Mathematical purpose (for students)

The area of a shape can be found by cutting it up and rearranging the pieces into shapes with known area.

Mathematical purpose (for teachers)

Areas can be calculated by counting grid squares and these areas can be expressed as a fraction of the whole grid. Visual and numerical evidence is required to convince others that approximately half of the area of the original square has been flipped and it is necessary to record and clearly display this evidence. In determining the flipped area by placing it over a grid, students will need to account for part grid squares. They may encounter the principle that area is conserved when shapes are cut into convenient subshapes and rearranged into shapes with known areas (e.g. rectangles).

At the end of the Develop phase, students will be able to:

- Describe the method they are using to calculate the area flipped to the outside of the square.
- Provide some visual and/or numerical evidence used to calculate this area.

Lesson Length 90-120 minutes

Vocabulary Encountered

- purpose, question, evidence, conclusion
- equation

Lesson Materials

- one centimetre grid paper (at least one sheet per student)
- tracing paper (at least one sheet per student)
- scrap paper (for cutting up)
- centicubes and/or MAB ones (large containers for sharing)
- student workbook or scrap paper (for planning evidence)
- Evidence Triangle (optional, see *Mathematical Inquiry into Authentic Problems Teachers' Guide*)

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Gathering Evidence

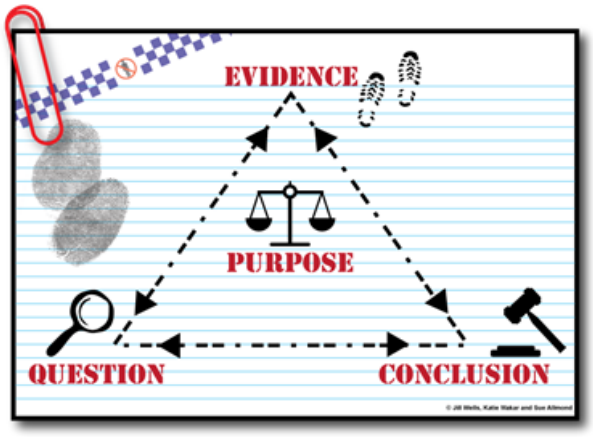
Inquiry Question: How can we design an expanded square where approximately half the area of the original square is flipped to the outside?

1. Revisit the inquiry question. Discuss the uncertainty at the end of the previous lesson about whether the area flipped was approximately half of the original square. Ask students: *How could you convince others that approximately half the area of the original square has been flipped to the outside?*

   Suggestions include:

   - Cut up the pieces that are flipped and paste them on to the remaining space in the original square.
   - Count the total number of squares in a 16 centimetre grid square and work out how many squares make up half the area and then:
     - Then draw your design on the grid square and count the number of squares that cover the shapes to be flipped. Is this the same number, or close to it? OR
     - Use centicubes or MAB ones to cover the cut-out pieces. Count how many. Use centicubes to cover the remaining area. Count how many. See if the numbers match. OR
     - Use tracing paper to trace the cut-out parts and then place over the grid square to count the area. Check that about half has been covered.

2. (Optional) Display the Evidence Triangle (see *Mathematical Inquiry into Authentic Problems Teachers' Guide* for images and explanations on using it) to emphasise that when they answer an inquiry question, students need to provide evidence. The triangle can help remind them that the evidence needs to relate to the question and the conclusion.



The Evidence Triangle

Check designs to ensure approximately half the area is flipped

3. Have students check the fraction of the original square that is flipped to the outside of their design. Remind students that it is necessary to record their thinking to first convince themselves and then others. Circulate and monitor each student's progress. Make note of any difficulties or good to use in the subsequent Checkpoint. Use questioning and prompts to encourage students to persevere through challenges and to guide them towards using a productive method to determine the area. For example:

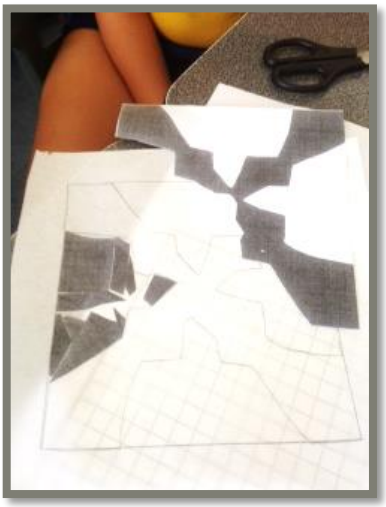
> Teacher: What have you done to determine the area of your cut-out shapes?
>
> Student: I have traced them all on to a piece of tracing paper. I could now paste them onto grid paper and count how many squares they cover.
>
> Teacher: Give it a go.

Checkpoint

4. After students have been working for about **twenty minutes**, bring them together for a Checkpoint. Select a few students to share their progress, particularly the ones noted with challenges or good progress as you circulated. As students share their evidence (refer to the Evidence Triangle in Step 3), model how to provide feedback that focuses on the **mathematical strengths of ideas**. For example:

   Tracing your shape onto grid paper and then counting the squares was a useful method of finding the area of the cut-out shape. You then multiplied this number by 4 because you had four of the same sized shapes in your design. What do others think about that idea?

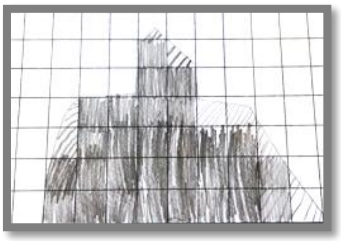
5. Use the sharing to focus on the **informal measurement of area**. Three possible discussion points to raise are provided below:



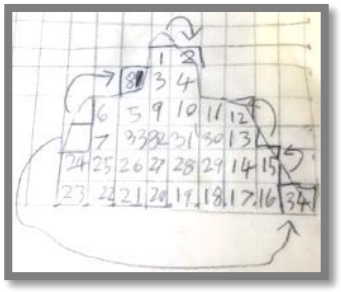
Teacher: Cutting up a part of your coloured space and pasting it on a tracing of your flipped shape was a creative idea. Your visual evidence shows that the pasted coloured area almost covers the area of one of your flipped shapes. What is your next step?

Student: I'm wondering if I need to cut out all the coloured space then paste it on the white spaces in my tracing or if I can just say about half the area of the square has been flipped because all four cut out shapes are the same.

Teacher: What do others think?



Student: I made a symmetrical design because I flipped the same shape from all four sides. I traced the shape onto grid paper and counted the squares to find the area. There were 29 whole squares and when I combined the little parts I got 5 more whole squares. To find the total area flipped, I multiplied 34 by 4 and got 136 squares.



Teacher: I like the way you wrote in the numbers as you counted the squares. It made it easier for me to see how you got an area of 34 squares. Remember to record the thinking you used. You need to make it clear why you multiplied by 4. What do you need to still do to convince others approximately half the area has been flipped?

Student: Find the total area of the square.

Full squares – purple- 49
More than ¾ of square – green -16
About half a square – orange - 28

Teacher: Did anyone use a different way to decide on the area (number of squares) being flipped?

Student: I drew the outline of my design on grid paper. The shapes in my design made it really tricky to count the number of squares in the shapes being flipped. I'm not finished yet but I started with sorting and colouring the squares in the shapes. I coloured the full squares purple, the squares about a half orange and the squares more than ¾ green. I did not worry about the squares than had less than ¼.

Teacher: What will you do next?

Student: I will need to find the total number of squares being flipped by halving the number of orange squares and adding that number to the green and purple squares.

Teacher: Remember to add calculations to your evidence so it is clear for everyone to follow the thinking you used.

Summary and conclusion

6. As a class summarise and record ideas shared and discussed during the checkpoint that have the potential to improve the evidence displayed.

Evidence Ideas
Use a 16 centimetre grid square. Show the calculation of the cut-out area visually – coloured, numbered or shaded grid squares. Add calculations to show how the total area and half the area was determined. Add calculations to explain how the combined white space made approximately half of the total area of the original square.

Remind students that evidence must help answer the inquiry question and also convince others of the solution. Allow time for students to work on completing and improving their evidence.

Tell students that next lesson there will be an opportunity think about refining their design before completing an evidence poster to display. If their flipped area is not approximately one half of the original square, they can still think of ways of adjusting their design. They will be adding to their evidence when they explain any adjustments.