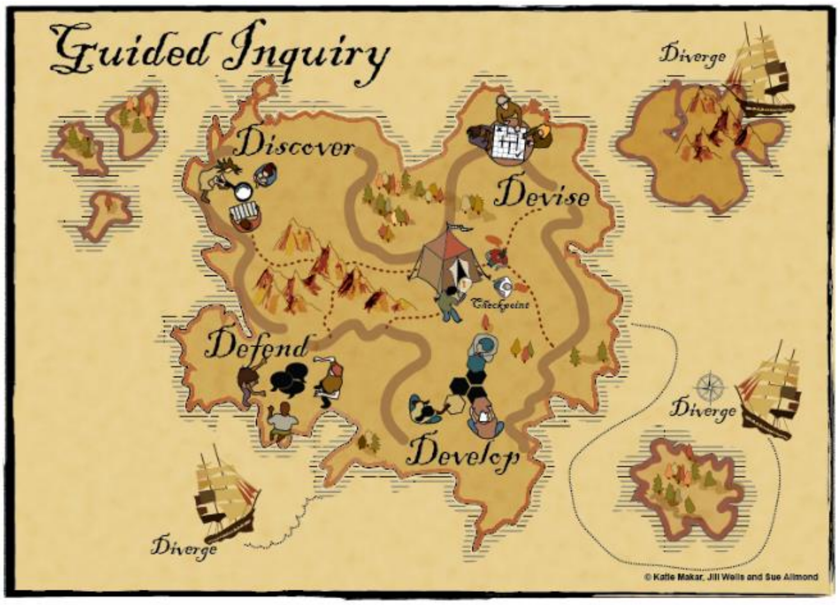
MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS
TEACHERS' GUIDE

re(Solve) MATHS BY INQUIRY

Contents



This guide introduces you to Special Topic 8: *Mathematical Inquiry into Authentic Problems* and provides you with general guidance on teaching the units. Inquiry covers a range of practices that incorporate open-ended problems. This Special Topic focuses on *extended* mathematical inquiry problems that: (1) are driven by an inquiry question; (2) contain ambiguities that require negotiation; and (3) require mathematical evidence to address the problem and justify a proposed solution.

The units have been developed around a 4D Guided Inquiry model with four phases—*Discover, Devise, Develop and Defend*. Each unit aims to develop content knowledge as well as assisting students to understand the process of inquiry. Mathematical inquiry is not intended to replace other teaching approaches, but rather complement them; the aim is to rebalance students' experiences so that they build procedural skills and conceptual understanding, but also learn to apply mathematics to solve and provide insight into messy, everyday problems. Teaching mathematics through inquiry can be quite different than conventional approaches to teaching mathematics. This Guide briefly provides advice for getting started and a list of resources for further development.

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

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Unit Summaries

All units are presented as four sessions, written around the four phases: Discover – Devise – Develop – Defend.

Unit	Year Level	Brief Description of Activities	Australian Curriculum Links
Tea Party	Early Foundation	Children plan a tea party relying on representation, counting to 10 and beyond, and one-to-one correspondence.	ACMNA001, ACMNA002, ACMNA003, ACMNA289
Grandma's Soup	Year 1	To improve a failed recipe for soup, children estimate how many macaroni are in a handful using various methods of skip counting and grouping strategies to count 100 macaroni.	ACMNA012, ACMNA013, ACMNA014
Target Ball	Year 1	Informal measurement strategies are developed to select and test the best ball to use for a game of Target Ball and to decide where to place the target. Measurements are represented on a number line as mathematical evidence to justify findings.	ACMNA013, ACMNA019, ACMSP262, ACMSP263
Bunches of Balloons	Year 2	Partitioning a packet of balloons into equal groups is investigated to decorate the room. Arrays are introduced as a method to find numbers that can be put into groups with no left overs.	ACMNA031, ACMNA032
What's for Lunch?	Year 2	Prior to using standard multiplication, students develop and represent strategies with larger numbers to estimate how many sandwiches (or other lunch items) they eat in a year.	ACMNA027, ACMNA028, ACMNA030, ACMNA031, ACMSP049
Bottle Flipping	Year 3	Children use unit fractions to compare the amount of water that is best for bottle flipping. They support findings with evidence using tallies from trials of flips.	ACMNA058, ACMMG061, ACMSP068, ACMSP069, ACMSP070
10 000 centicubes	Year 4	A box to hold 10 000 centicubes is designed using smaller numbers as referents and breaking down large numbers into possible dimensions of length, width and height.	ACMNA073, ACMNA076, ACMMG079, ACMMG084, ACMMG290
Expanded Square Designs	Year 4	An expanded square is an art technique that flips cut-outs of shapes inside a square to the exterior. Children estimate irregular areas to design an expanded square that flips half the area to the outside.	ACMNA077, ACMMG087, ACMMG088, ACMMG091
Reaction time	Year 5	Students work with decimals to record, compare and analyse reaction times of students in their class.	ACMNA104, ACMNA105, ACMSP118, ACMSP119, ACMSP120
Pyramids in a Box	Year 6	Nets and features of 3D objects are explored to design and construct a box that can hold two pyramids.	ACMMG140, **ACMMG141**

Why Inquiry?

What is Guided Inquiry?

Foundation children record their count in Tea Party.

Inquiry is a pedagogical approach which uses problems that are more open than typical problems in mathematics classes (Artigue & Blomhøj, 2013). In contrast, most traditional mathematics problems have one correct answer and children are taught a method (perhaps just one method) to solve the problem. Questions such as “What is the area of a rectangle with sides 4 cm and 7 cm?” are closed-ended (one correct answer) and usually closed-method (a single solution method is taught in advance). By contrast, an inquiry question such as “How far is it from our classroom to the Tuckshop?” could be answered using several different methods (e.g., counting paces, using a trundle wheel, comparing multiple routes through the school), and could have a range of reasonable answers. Mathematical inquiry requires the teacher to support and scaffold students through each phase of the process.

This Special Topic focuses on mathematical inquiry problems with three characteristics:

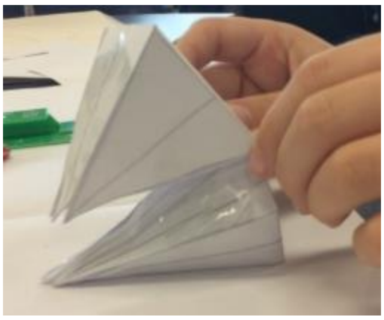
1. Problems are driven by an inquiry question to be addressed (as opposed to a task or an activity);
2. There are ambiguities in the inquiry question and/or method of solution that require negotiation;
3. Mathematical evidence is needed to address the inquiry and persuade an audience of a proposed solution.

For example, a mathematical inquiry could be **driven by the inquiry question**, “Do the children in our school eat a healthy lunch?” The class might then **negotiate the ambiguities** of both what is meant by “a healthy lunch” and a method for how they would find out. They could devise a method for collecting evidence from a few students in each grade level and then analyse their findings to come to a conclusion, **supported by mathematical evidence**.

Why is guided inquiry needed?

Imagine learning spelling and grammar in English class, but never getting to read or write. Or practising soccer skills, but never playing a game. This is what children experience when they learn mathematical skills but don’t get to put the mathematics to *use* in an authentic context. When students only learn mathematical skills, they come to see mathematics as a set of rules to be memorised. Students can develop a fixed mindset where they are not willing to try something new for fear of getting it wrong. Or they may believe that only some people can be good at mathematics. This can promote disengagement and lack of persistence. To learn more about this, we encourage you to investigate Jo Boaler’s work on developing students’ Mathematical Mindsets (see additional readings and resources). The reSolve protocol has been designed to support a growth mindset, which encourages students to embrace struggle as a natural part of learning (Thornton, 2017).

Understanding in mathematics relies on a multitude of skills, concepts, practices and dispositions, only partially addressed by traditional approaches. Problems outside the classroom can be messy and ambiguous with few direct cues about which mathematics to use or how to go about solving problems. They require a flexibility and willingness to apply incomplete ideas without the fear of lacking a complete, correct approach from the start.



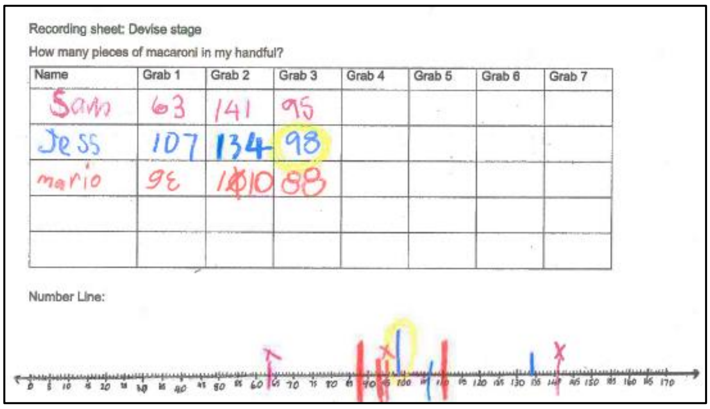
Year 6 students assemble two pyramids to go in a box (Pyramids in a Box)

Research in mathematics education demonstrates that using conventional mathematics problems alone (such as those in most textbooks) can also de-emphasise the usefulness of mathematics. If students do not see how mathematics is useful, they may find it irrelevant to their lives and become discouraged from pursuing further mathematics (AAMT, 2008). This is a critical problem because mathematics is needed for everyday life, engaged citizenry and national innovation in society and the sciences.

Aim of Special Topic 8: Mathematical Inquiry into Authentic Problems

Authentic problems in this Special Topic refer to complex, non-routine problems with a purpose (for students), set in a context that is accessible and potentially familiar. In many cases, these problems address both problems from "real life" and utilise solution methods that align with ways that would be used outside of school (Palm et al., 2008). Authentic problems emphasise the purpose and utility of mathematics (Ainley, Pratt, & Hansen, 2008), a key element of the reSolve protocol.

The aim of this Special Topic is to develop and extend skills, conceptions, practices and dispositions appropriate to addressing such authentic problems. These inquiry-based units have been designed to involve students for an extended time in solving problems that authentically connect with their knowledge and practical experience, emphasise mathematical evidence, improve their engagement and confidence in mathematics, and build 21st Century Skills of collaboration, communication, and creative and critical thinking. Building these skills takes time, and inquiry-based approaches can provide an excellent environment for this (Thornton, 2017).



Year 1 students record their evidence of how many macaroni are in a handful (Grandma's Soup).

Inquiry in the reSolve project

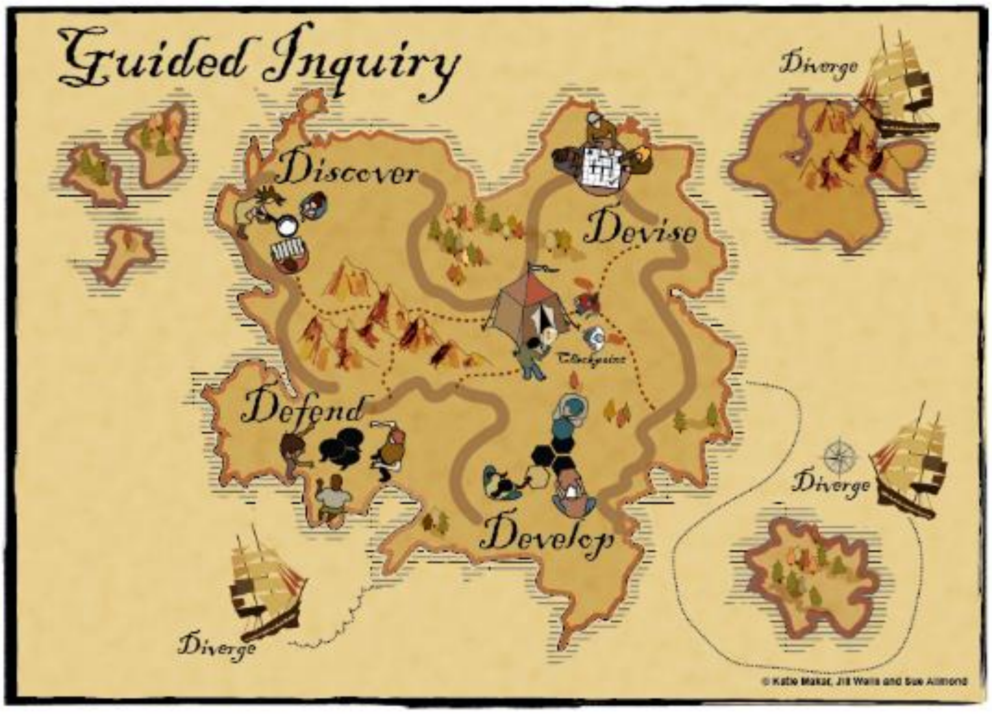
'Promoting a spirit of inquiry in school mathematics' is the by-line of the whole reSolve project. This spirit of inquiry is intended to be evident in all resources; those which explicitly subscribe to Inquiry as an overarching pedagogical approach as these units do, and those that do not.

The units of this special topic are structured around "an inquiry", with a main question which is to be answered as the culmination of a set of lessons. Fortunately, however, the spirit of inquiry can also be evident in any lesson when a teacher (or indeed a student) poses a question that demands thought and exploration, even for just a couple of minutes. In this wider reSolve sense, inquiry can be stimulated anywhere and anytime. Questions may be open-ended or closed in the sense of having one correct or many justifiable answers. The question might be authentic in the sense that may arise in someone's everyday life, or completely abstract about mathematical objects only. It may be a specially selected question or it might arise from a student's observation about a routine exercise in a school textbook. Anything that sparks investigation.

A good mathematics education needs to offer students a wide variety of experiences and attend to conceptual understanding, fluency, reasoning and problem solving. Solving a substantial problem over some lessons, as characterises these units, plays an important part of developing students' mathematical capacity. Crucially, however, the spirit of inquiry should be evident in thinking mathematically in all lessons.

Using the 4D Guided Inquiry Model and Evidence Triangle

The units in this Special Topic rely on a 4D Guided Inquiry model and Evidence Triangle. Although the units can be used without these, they are recommended to assist helping students understand what is expected in each phase of the inquiry. They are also useful for designing your own guided inquiry units. Free downloadable copies of the 4D Guided Inquiry map and the Evidence Triangle for the classroom or for students to paste into their maths inquiry books are available in A5, A4, A3, or A2 sizes at www.mathsinquiry.com.au.



The phases of the 4D Guided Inquiry model

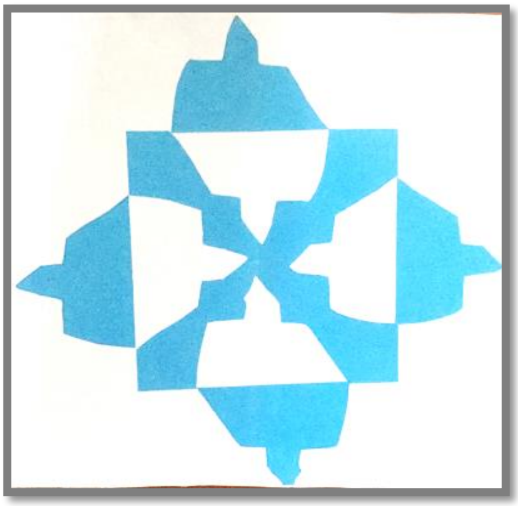
The mathematical inquiry units are structured around the 4D inquiry model. The 4Ds consist of four phases — *Discover*, *Devise*, *Develop* and *Defend* —that describe the process students go through. Guided inquiries offer optional opportunities to *Diverge* (an optional 5th 'D') from the specific inquiry to integrate or connect with other content areas, or to explore further questions that emerge. Another feature of the 4D model is the *Checkpoint*, where the class pauses to share interim ideas and challenges. Each unit in this Special Topic explicitly describes how the phases are applied to that unit. We hope that you will find this model useful to design your own mathematical inquiries. Some resources are suggested at the end of this Teachers' Guide.

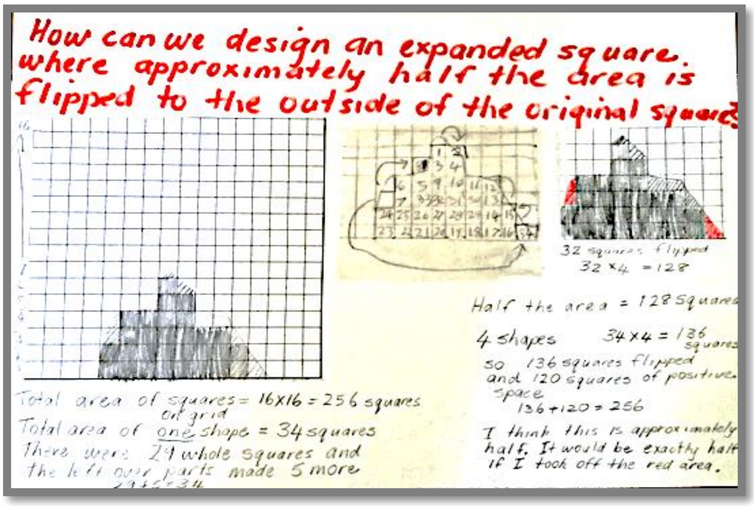
Discover

The *Discover* phase is the "hook" that engages students and builds excitement in the inquiry unit. The inquiry question is introduced, and potential mathematical connections are highlighted. This is a very important phase: as students are immersed in the problem, they begin to connect the inquiry context with their prior knowledge. These connections are revisited and emphasised throughout the unit. The teacher can then gain insight into students' background knowledge and create common understanding of the big ideas in the inquiry. As a phase of 'discovery', the focus of the *Discover* phase is exploratory in nature.

Devise

In the *Devise* phase, students build on their everyday knowledge to begin devising a method (planning) to address the inquiry problem. *Devise* can be an opportunity to recognise, negotiate and reject impulsive approaches to solving problems. Evidence therefore plays a critical role in the *Devise* phase to build good habits of problem-solving, as students will deliberate the information (evidence) they need to address the problem and the evidence that will be needed to defend their solution (including a process to get there). Students may use the *Devise* phase to trial ideas and use their observations to improve their method (a key element of the reSolve protocol's knowledge-building culture). In this phase, the teacher may identify and address concepts that students need to learn to generate evidence and develop their solution. In this way, mathematical inquiry and more conventional teaching approaches can be used together to effectively support student learning. Another important aspect of the *Devise* phase is that it can build students' belief and confidence that mathematics is a human endeavour—and that they can tackle complex, non-routine problems; they also learn the value of planning rather than simply 'jumping in' to try to solve a problem quickly.





Develop

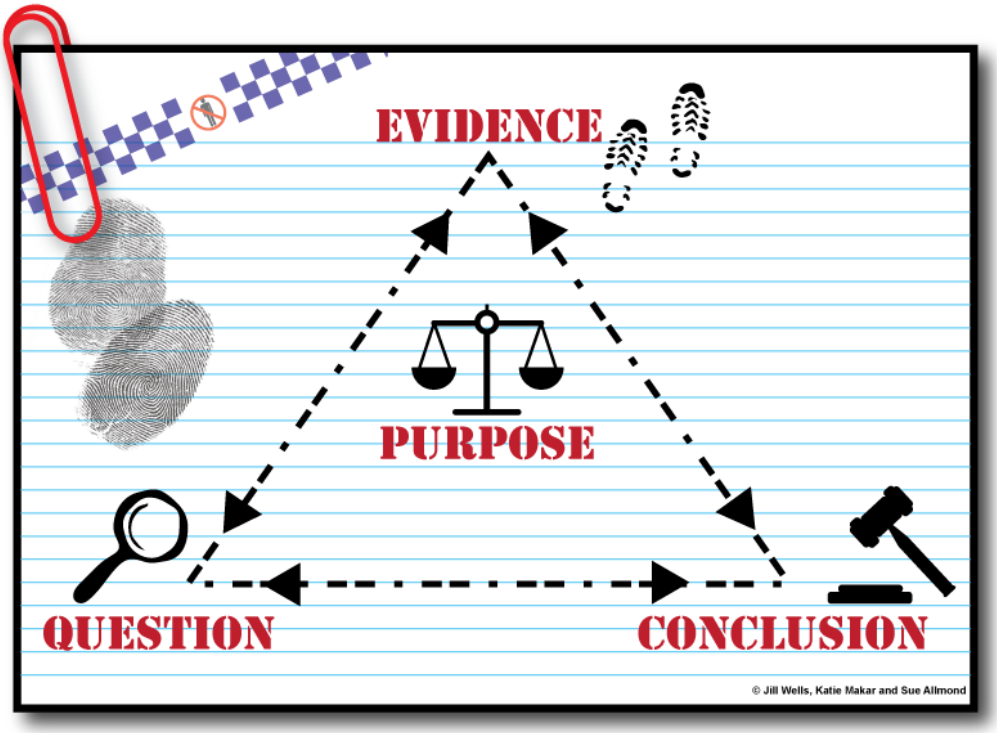
The *Develop* phase is where students put their planning into practice. The focus is to generate or collect evidence, analyse it, document and improve on the solution process, and come to a preliminary conclusion. Evidence again plays a key role in that students rely on evidence to generate a solution, but also think ahead to ensure that they have evidence to defend their solution in the next phase. Once again, teachers may find opportunities in the *Develop* phase to teach, revisit or revise mathematical concepts as students wrestle with adapting and applying their mathematical knowledge and processes to create a solution. Because students are interested in solving the problem, they may see practising mathematical skills as both relevant and useful.

Defend

Students prepare their answer to the inquiry question in the *Defend* phase. They use the evidence they have gathered to support, justify and convince their peers that their conclusion answers the inquiry question. It is critical to ask students to communicate their solution so that they make explicit the connections between the inquiry question, the conclusion and the mathematical evidence that supports their conclusion. The communication of findings in the *Defend* phase can be in many forms, for example, an oral presentation, a poster, a digital report, or a video diary. This is an opportunity to connect with the *Literacy: Interacting with others* sub-strand of the Australian Curriculum: English. Sharing in the Defend Phase provides an opportunity for feedback on the conclusion to the inquiry question. Feedback should focus on:

- the use of appropriate mathematical terminology
- clear mathematical thinking including accurate calculations and detailed representations
- reasoning that justifies the conclusion with mathematical evidence.

The Role of Evidence



One way to support the students through the inquiry process is to use a criminal investigation analogy. A jury can't find someone guilty of a crime (accept the conclusion) unless they have sufficient evidence. So we ask the students to consider what the question is (who did the crime), what evidence the 'jury' might need to be convinced of a particular outcome, and what the conclusion (outcome of the trial) is. Displaying the evidence triangle poster in the classroom enables you to continually refer to each of these components.

Once an inquiry question has been posed with the students, their attention can be drawn to the evidence that needs to be gathered, recorded or created to address the question (make a conclusion) and to convince others that their solution answers the inquiry question. Prompts can assist students to make connections between the inquiry question, the evidence and the conclusion. For example:

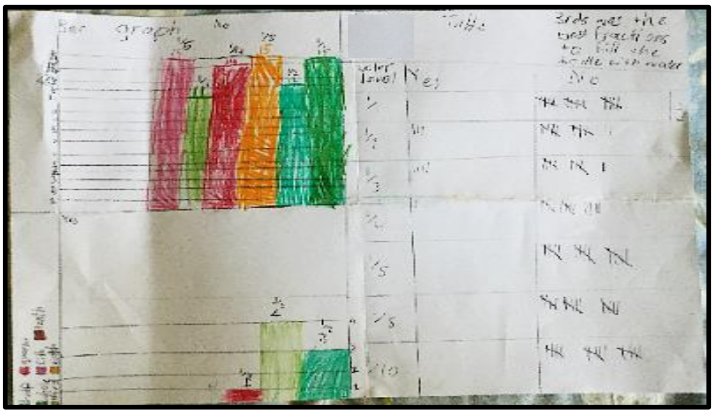
- What evidence would we need to collect to be able to answer the question?
- How can we collect the evidence?
- How could we record the evidence as we collect it?
- Would your evidence convince someone of your conclusion?
- Does your conclusion answer your question?

Advice on Teaching Mathematical Inquiry

Teaching mathematics through inquiry requires a supportive classroom environment. Scaffolding by the teacher assists students to focus on the mathematics, improve their ideas, productively collaborate, and maintain engagement and momentum. The reSolve protocol emphasises a knowledge-building environment, where students actively explore and collaborate on problems, consider multiple perspectives, take intellectual risks and use mistakes as productive avenues for improving ideas.

If you are new to teaching mathematics with inquiry, we suggest that you start small and explicitly teach students over time to be active listeners and to explain their thinking. We discuss these below.

Develop classroom talk: Encourage active listening and explaining thinking

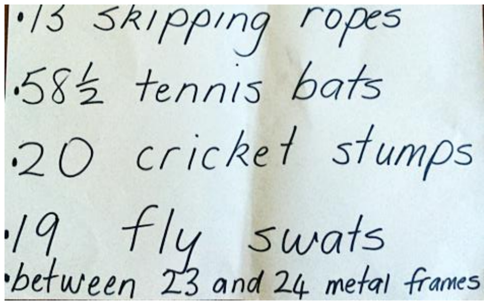


In Bottle Flip, Year 3 students discuss and compare evidence for which unit fraction is the best to fill a bottle for flipping.

As emphasised in the reSolve protocol, a key skill of mathematical inquiry is to collaborate and communicate ideas. To build this, encourage students to explain their thinking and listen to others. These are skills that need to be explicitly taught and practised. Providing a positive and supportive environment allows students to focus on the reasoning behind problems and not fear getting something wrong. Wrestling with ideas is a normal part of an inquiry classroom. Asking questions such as "tell me more about that" instead of correcting students can support them to be more willing to explain their thinking. If students struggle with getting an idea out, you can try to revoice (restate) their idea in clearer way or use more fluent mathematical terminology. Building on students' ideas also acknowledges the seeds of their thinking. Once students feel more comfortable with explaining ideas, you can help them to build on one another's thinking. While students are working, select a few with different approaches and ask them to share their ideas. This provides an opportunity to highlight benefits and drawbacks of different methods, making multiple approaches a normal part of solving problems. Sharing ideas can also rebuild momentum, assist groups who are stuck, and encourage students to use peers as resources.

Start small and build slowly

Teaching mathematics through inquiry may involve developing new skills, both for the teacher and the students. We therefore encourage you to start small and build your confidence slowly. For example, the units in this Special Topic assume you are familiar with navigating class discussions, teaching students to work collaboratively, and creating a classroom environment that embraces student mistakes. It can be daunting to try to take on multiple new skills at once. If you are new to teaching with these skills, it may be advisable to start with smaller reSolve tasks. Choose a problem that can be solved in a single lesson and use these shorter problems to teach students how to share and justify their ideas.



In Target Ball, Year 1 students recognise a need for a common unit of measure when they try to compare informal measurements of how far their ball rolled.

Developing inquiry skills

There are many useful resources on websites and available from booksellers and educational suppliers that are useful for developing students' skills in collaboration, communication and working with open-ended problems. A list of favourites is given below.

Differentiate

All students are capable of participation in guided mathematical inquiry, as emphasised in the reSolve protocol. Inquiry problems are "low floor, high ceiling" (Boaler, 2015) in that they broaden the range of opportunities for students at a multitude of performance levels. Students who have difficulty in traditional mathematics often find mathematical inquiry to be more accessible as they can use their personal experience to connect with the inquiry problem, and the context can act as a scaffold to support the mathematical concepts. Teacher scaffolding and prompts allow for lower performing students to gain support and adjust the level of challenge as they access a method of solution that makes sense to them. For students who often excel in mathematics, mathematical inquiry allows them to be extended by deepening the quality of the evidence and devising more challenging pathways through the problem. For example, if Year 5 students have been challenged to design a one-litre container, some students can use a simple rectangular prism while more advanced students may elect to use a cylinder or more complex mathematical object.

Checkpoints

As students are working, Checkpoints can be used to pause the class to share interim progress and address issues early that several groups may be facing. In a Checkpoint, students account for their progress by explaining what they have done and any challenges they are encountering, and see ways that other groups have been tackling the task. Checkpoint as a whole class activity enables a teacher to emphasise and build on good practices and good mathematical ideas. Revisiting the inquiry question in a Checkpoint prompts student to consider what they will need to do next to work towards a solution; this ensures that momentum in the inquiry is maintained. Regular sharing of progress (checkpoints) during the Devise and Develop phases provides many teaching opportunities.

Sharing progress in a Checkpoint class activity enables a teacher to showcase the ideas that different groups are exploring and build on productive practices. This builds a sense of contributing to a collective understanding and progress towards a solution. It is not necessary for *all* groups to share in each session: select groups who demonstrate a range of approaches. If groups are not moving in a desired direction, the teacher can plant the 'seed' of an idea with a group to be able to build on in the whole class.

Checkpoints allow students to:	Checkpoints allow teachers to:
• Account for their progress by explaining what they have done and answering any requests for clarification. • See how other groups have worked on the task • Present any challenges they are facing and seek ideas to help them move forward. • Revisit the inquiry question to consider what they will need to do next. • Ensure sufficient appropriate mathematical evidence is being gathered. • Analyse other groups' ideas and provide constructive feedback, which has the potential to improve thinking and ideas.	• Highlight ideas which have the potential to improve the quality of the mathematical ideas. • Model clarifying questions and feedback that focuses on the mathematics. • Encourage students to refine plans or build further on their ideas. • Validate challenges as a normal part of problem solving. • Prompt students to consider an alternative pathway if the current one is unproductive. • Refocus the inquiry to maintain momentum.

Notes from field trial teachers

Each of the units went through field trialling with teachers, many of whom were new to inquiry. Below we briefly summarise a few tips for undertaking these units, that were especially highlighted by teachers.

Tea Party (Foundation)

- Explicitly model ways that students could respond (in writing or representations) in each phase.
- Take any opportunity to discuss counting and the final counts students make. Constantly count aloud.

Grandma's Soup (Year 1)

- Remind students to check their own and their peers' counting and grouping skills and techniques. This gives them additional practice and emphasises why some counting methods are more efficient than others.
- Highlight the need for students to record (e.g., photograph) their counting strategies for later comparison.
- Scaffold students' written summary of their findings with modelling and writing prompts.

Target Ball (Year 1)

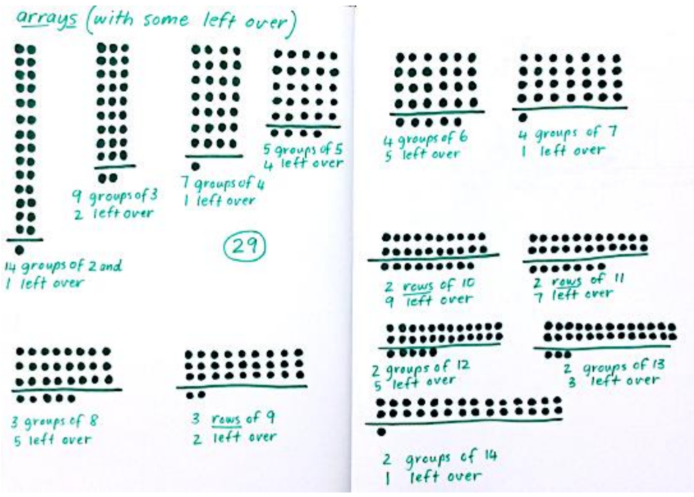
- Explain what it means to be closest to the Target. This helps to instigate the need for measurement.
- Ensure that students collaborate to measure one another's distances so that everyone has an opportunity to develop their skills of measurement and the language of measurement.
- Initially observing students' measurement practices (rather than teaching it first) allows for their productive and unproductive approaches to be used as examples to improve their skills.

Bunches of Balloons (Year 2)

- Highlight the mathematics and need for clearly labelled representations
- Use Checkpoints to guide students and establish and/or clarify expectations

What's for Lunch? (Year 2)

- It can be challenging to support students to invent and improve strategies. Allow opportunities to explicitly share ideas and listen to student reasoning, so that misconceptions can be shifted as they arise.
- Use the multiple examples of student inventions to support student invention.



Can we make equal bunches out of 29 balloons? Year 2 children learn about arrays to find out in Bunches of Balloons.

Bottle Flipping (Year 3)

- YouTube clips build background knowledge of techniques for this popular craze.
- Recording sheets provided guidance on what and how to record data; these improve the momentum of the lessons to keep students engaged.
- Emphasise the *evidence* rather than the *outcome* of their investigation of the best unit fraction of water for flipping a bottle. Evidence can also be used to compare groups with different outcomes.

10 000 Centicubes (Year 4)

- Explicitly focus on students' mathematical explanations, probing incorrect ideas or language.
- Teach and model for students how to give useful or appropriate (respectful) feedback. They need time to develop giving and receiving feedback, so it's worthwhile to take the time to do this.

Expanded Square (Year 4)

- Students tend to rush into creating designs without planning, leading to only simple versions. Examples from the internet can be helpful so students consider designs with greater complexity.
- Consider giving students' creations to another class to check; this can focus their attention on finding multiple accurate ways to find and check areas, as well as providing clear and persuasive evidence.

Reaction Time (Year 5)

- Lots of opportunities for rich discussion in this unit; checkpoints in the unit assist with this.
- Students may forget to convert drop distances (cm) to times (hundredths of a second).

Pyramid Boxes (Year 6)

- Encourage multiple iterations of improvement until students have minimal leftover space in their box.
- If students are fluent with drawing nets for a prism and a pyramid, they may not innovatively look beyond simple designs. Encourage them to consider nets and box designs beyond the traditional ones.

References

Ainley, J., Pratt, D., Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, *32*(1), 23-38.

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM - International Journal on Mathematics Education, 45*(6), 797–810. https://doi.org/10.1007/s11858-013-0506-6

Australian Association of Mathematics Teachers (AAMT) (2008). *Maths? Why not? Report to the Department of Education, Employment and Workplace Relations*. Canberra: DEEWR.

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*, *67*(1), 37–58. *doi*: 10.1007/s10649-007-9083-3

Thornton, S. (2017). Empowering learners through inquiry. In B. Kaur (Ed.), *Empowering learners in mathematics. Yearbook 2017, Association of Mathematics Educators* (pp. 313-332). Singapore: World Scientific Publishing.

Further Information

Below we offer suggestions of some of our favourite readings and resources to extend the information provided in this guide.

Readings for teachers

Bills, C., Bills, L., Watson, A., & Mason, J. (2004). *Thinkers: A collection of activities to provoke mathematical thinking*. ATM (UK).

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

We love this book! It is one of our favourite teacher resources that inspires us to create a positive classroom environment. The book explains why positive mindsets are important and how to move students out of a "fixed mindset". Lots of examples are provided.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009*). Classroom discussions: Using math talk to help students learn (Grades K-6)*. Sausalito, CA USA: Math Solutions.

We highly recommend this book if you want to learn more about developing classroom discussions (in pairs, small groups and whole class). The book is full of tips for starting out, explaining different types of student contributions, tips for keeping the focus on mathematics, teaching students to use respectful talk, etc.

Flewelling, G., Lind, J., & Sauer, R. (2013). *Rich learning tasks in number and algebra for primary students*. Adelaide: Australian Association of Mathematics Teachers.

Fry, K. (2014). Assessing inquiry learning: How much is a cubic metre? Australian Primary Mathematics Classroom, 19(3), 11-15.

This article discusses opportunities for formative assessment in an inquiry mathematics classroom in Grade 6.

Hunter, J. (2009). Developing a productive discourse community in the mathematics classroom. *The New Zealand Mathematics Magazine*, 46(1), 1-12.

This article describes the process a teacher went through to develop her students' mathematical talk and mathematical community. It provides several excerpts from student discussions to highlight the kinds of responses that might be expected as students gain experience over time and with teacher scaffolding.

Lilburn, P. & Sawczak, I. (2011). *Teaching and assessing maths through open-ended activities*. Melbourne: Pearson.

This is a great resource to ease into teaching mathematical inquiry. It provides scores of short open-ended tasks and longer investigations specific to lower, middle and upper primary students in each of the major content areas. General rubrics are provided to assess the tasks at each age level with examples of student responses.

Small, M. (2009). *Good questions: Great ways to differentiate mathematics* instruction (2nd ed). New York: Teachers College Press.

Sullivan, P., & Lilburn, P. (2004). *Open-ended maths activities*. Oxford University Press Australia

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2014). *Elementary and middle school mathematics. Teaching developmentally* (8th int'l ed.). Essex UK: Pearson.

This is a useful and easy-to-read text for teaching specific areas of the mathematics curriculum. As a reference book, use it to brush up on content, better understand student reasoning and briefly overview productive teaching approaches for specific topics in primary mathematics. With many versions of this text (1990-2017), look for a second-hand or library version to keep the price down.

Research articles on mathematical inquiry

Fielding-Wells, J., Dole, S., & Makar, K. (2014). Inquiry pedagogy to promote emerging proportional reasoning in primary students. *Mathematics Education Research Journal*, 26(1), 47-77.

This article uses an inquiry in grade 4 to highlight how inquiry can promote mathematical reasoning. The article outlines research backing on the 4D inquiry model.

Makar, K. (2012). The pedagogy of mathematical inquiry. In R. Gillies (Ed.), *Pedagogy: New developments in the learning sciences* (pp. 371-397). Nova Science.

This article outlines a teacher's experience with her grade 6 class as they design an inquiry to test for the "best" bubble gum. The paper discusses the kind of learning that students gain with inquiry and why it takes time to develop.

Makar, K., Bakker, A., & Ben-Zvi, D. (2015). Teacher's scaffolding over the year to develop norms of mathematical inquiry in a primary classroom. In M. Marshman, V. Geiger, & A. Bennison (Eds.), *Mathematics education in the margins (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia)*, pp. 397–404. Sunshine Coast: MERGA.

This short paper outlines how a teacher gradually developed her students' classroom talk through mathematical inquiry over a year in Grade 4.

Website resources

Inquiry Maths Pedagogy in Action www.mathsinquiry.com.au

This website was developed by several teacher-researcher authors of the inquiry units in this special topic. It includes more resources, additional inquiry units that use the 4D model and tips for teaching mathematical inquiry in the primary years.

Engaging Maths – Dr Catherine Attard's website https://engagingmaths.co/

This website has lots of teacher resources for engaging students in mathematics, including inquiry-based learning.

Jo Boaler's "*YouCubed*" website www.youcubed.com

This site has a substantial set of resources to develop students' "mathematical mindset". We recommend the videos for students and inquiry tasks.

Dan Myer's Blog http://blog.mrmeyer.com

One of the most inspirational mathematics teachers around, Dan Myer focuses on building the story of solving mathematics problems with this "Three-Act Math" approach. His TED-talk is worth watching to understand how he improves engagement in mathematics by allowing students to wrestle with problems.

Bowland Maths www.bowlandmaths.org.uk

This UK website targets students aged 11-14. It contains tasks, assessment and professional development videos to support teachers to use mathematical inquiry in the classroom.

reSolve: Maths by Inquiry Special Topic 7 Mathematical Modelling. www.resolve.edu.au

This series of units for Years 9 and 10 students provides authentic inquiry units for secondary schools. Each 5-lesson unit poses an authentic problem, and takes students through steps to resolve it using mathematics. Lessons draw students' attention to the processes of mathematical modelling, including formulating a model, employing mathematics to find a mathematical solution, interpreting the solution in real world terms and evaluating how well the solution answers the original problem.

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