reSolve MATHS BY INQUIRY Number Sequences

Y3

Summary of learning goals

- This sequence builds students' algebraic thinking through explorations of additive number patterns. As students solve the problems to generate the patterns, their strategies for addition and subtraction are applied and developed.
- A key focus is to build students' ability to reason why patterns occur and to generalise patterns to fit various situations.

Australian Curriculum: Mathematics (Year 3)

ACMNA055: Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation.

ACMNA060: Describe, continue, and create number patterns resulting from performing addition or subtraction.

Summary of lessons

Who is this sequence for?

- This sequence explores additive number patterns; some experience with number patterns involving addition will help students solve the problem and reason as to why the patterns occur.
- Students are required to add and subtract one- and two-digit numbers and will need to have or be developing flexible strategies for performing these operations.

Lesson 1: Counter Toss

Students toss four two-colour counters to explore number patterns. One colour is allocated a value of 2 and the other 5. Students are asked to consider all possible scores that can be generated by tossing the counters and why the resulting number pattern occurs. Students then explore the different number patterns generated, using different values for the colours on the counters.

Lesson 2: Letterbox Numbers

This task uses the context of house numbers on a suburban street to explore number patterns. Students add together varying sequences of house numbers to reveal different patterns. Students are asked to explain why these number patterns occur. The final investigation asks students to explore and find their own patterns and to reason mathematically as to why they occur.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Reflection on this sequence

Rationale

The *Australian Curriculum: Mathematics* highlights number patterns as an important area of algebra in the primary years. This sequence presents two lessons that explore different number patterns, using particular contexts. A key feature of these lessons is asking the students to explain *why* a number pattern occurs. Students are often able to identify a number pattern, but the real challenge is presented when they are asked to explain why it happens. Explaining why helps students to generalise, which is a critical part of any mathematics lesson.

reSolve mathematics is purposeful

- These tasks focus on substantial mathematical ideas presented in the *Australian Curriculum: Mathematics*. Students work algebraically to explore number patterns. They are asked to identify the pattern and to explain why it occurs.

reSolve tasks are inclusive and challenging

- These tasks are both challenging and inclusive. Students are able to draw on their existing knowledge of numbers and patterns to develop new knowledge and explore relationships as they work on meaningful tasks.
- To complete the tasks, students are engaged in sustained inquiry, problem-solving, decision-making and communication. They are challenged to reason mathematically as to why the patterns occur.

reSolve classrooms have a knowledge-building culture

- The active role of the student and the teacher promotes a knowledge-building culture through these lessons. The teacher is encouraged to monitor and question the students to promote deeper inquiry and valid mathematical reasoning. As the students listen to the solutions and reasoning of others, they can modify their solutions and refine their own thinking about the problems.

LESSON 1

re(Solve) MATHS BY INQUIRY

Number Sequences

Counter Toss

Y3

About this lesson

Students toss four two-colour counters to explore number patterns. One colour is allocated a value of 2 and the other 5. Students are asked to consider all possible scores that can be generated by tossing the counters and why the resulting number pattern occurs. Students then explore the different number patterns generated, using different values for the colours on the counters.

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ACMNA055: Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation.

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Mathematical purpose

- To build students' algebraic thinking through an exploration of additive number patterns. Students' reasoning develops as they explain why a pattern occurs and then as they modify aspects of the task to generate different additive number sequences.

Learning intention

- To explore number sequences to work out rules for number patterns and why the patterns occur.

Time

A lesson of approximately 1 hour.

Vocabulary

- additive

Resources

- two-colour counters (four per student)
  - ◊ If two-colour counters are not available, different coloured sticky dots can be placed on either side of a counter or students can draw four cubes/counters from a bag containing at least four cubes/counters of each colour.

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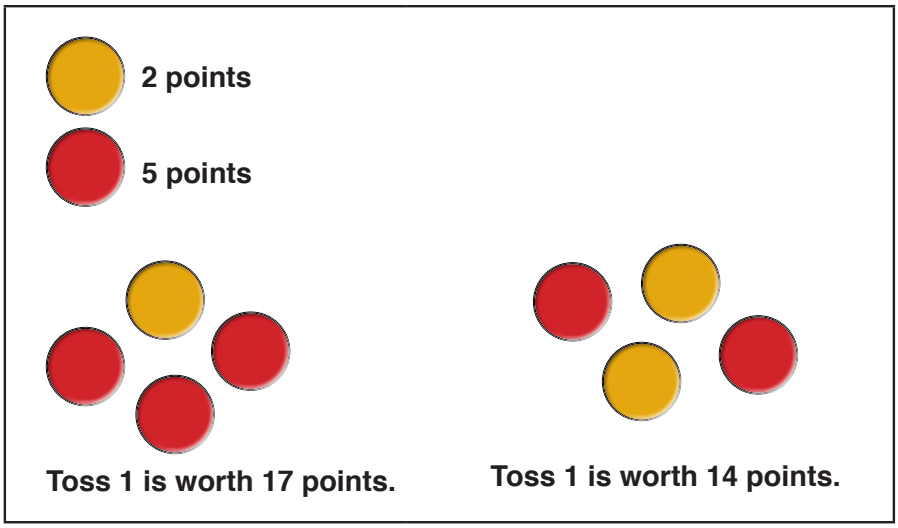
Calculating counter scores

Resources: Provide students with four two-colour counters.

Explain that one of the colours on the counters is worth 2 points and the other colour is worth 5 points. Students toss the counters and calculate the total score based on the colours revealed. For example:



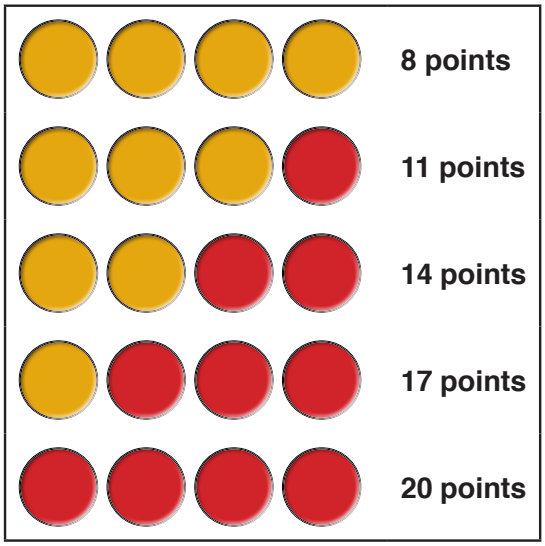
Allow the students to toss the counters and calculate some different scores.

Pose the question: *What are all the possible scores that can be obtained by tossing the counters?*

Exploration

Allow students time to explore all possible scores. Encourage them to work systematically.

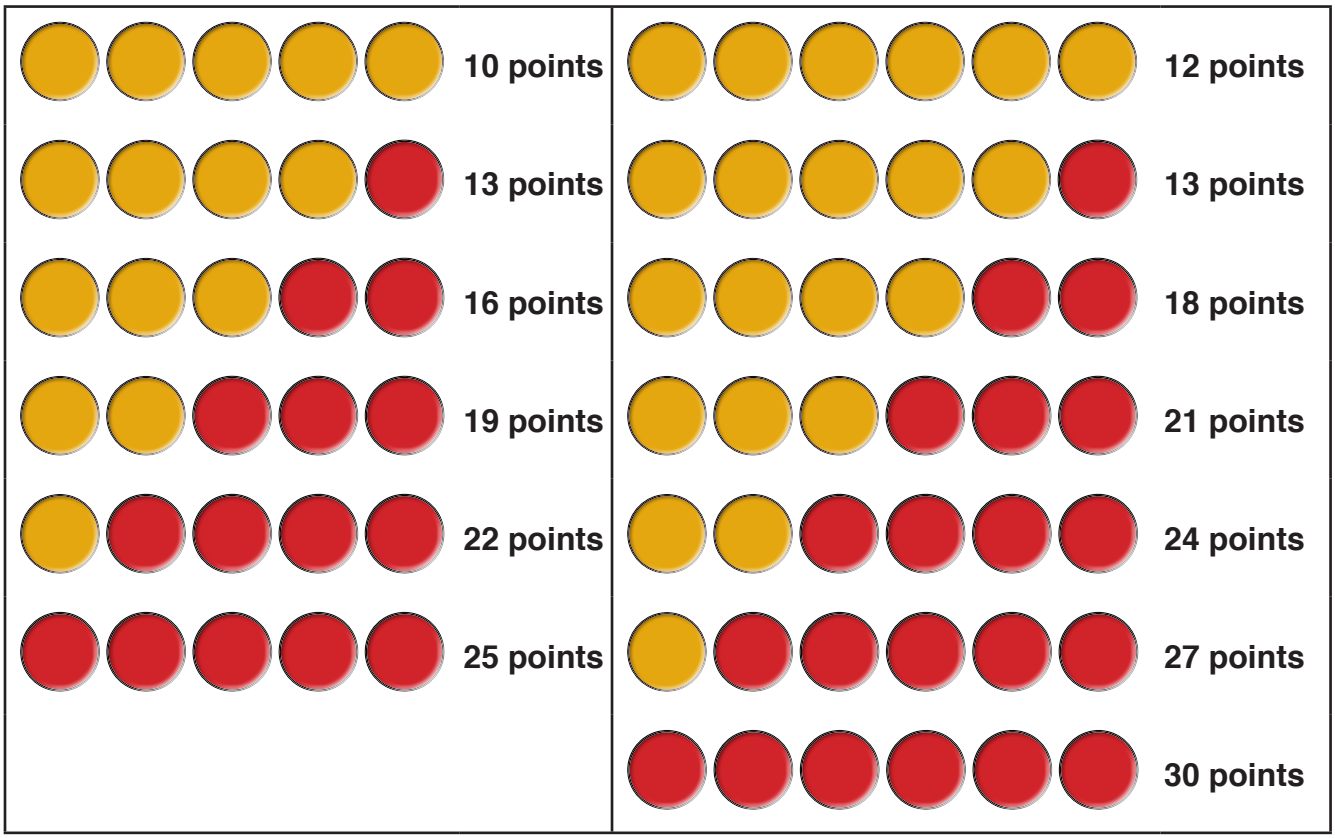
Possible student response:



As students solve for four counters,

Pose the questions: *What if there were five counters? Six counters? In what ways could you work out the possible combinations for any number of counters?*

Teacher notes:



- The smallest score for any number of counters is generated when every counter is worth 2 points. This means that the smallest possible score is the number of counters multiplied by 2.
- The largest score for any number of counters is generated when every counter is worth 5 points. This means that the largest possible score is the number of counters multiplied by 5.
- Starting with all yellow counters, the score increases by 3 points each time a yellow counter is changed to red.

Pose the questions: *What do you notice about the totals? Do you see any patterns? Why do the patterns occur?*

- In each sequence a red counter worth 5 points is added, and a yellow counter worth 2 points is subtracted. The value of the score changes by 3 because the difference between 2 and 5 is 3.

Extending prompts:

- *Does this pattern apply to any number of counters? How can you be sure?*
- *How many different totals could 100 counters have?*

Reflection

Have students share the different sequences that they generated and have them explain their reasoning for why the sequences always increase by 3s.

Pose the question: *Can you change the value of the colours on the counters so that the score increases by 6?*

To get a sequence of scores that increase by 6, students will need to choose two numbers with a difference of 6. Ask students to think of some different combinations that are possible. Working systematically, students can show many options. For example:

- 7 and 1
- 8 and 2
- 9 and 3
- 10 and 4
- 11 and 5
- 12 and 6

Where to next?

Lesson 2: Letterbox Numbers is the second activity in this sequence. It uses the context of house numbers on a suburban street to generate and explore number sequences.

LESSON 2

re(Solve) MATHS BY INQUIRY Number Sequences

Letterbox Numbers

Y3

About this lesson

This task uses the context of house numbers on a suburban street to explore number patterns. Students add together varying sequences of house numbers to reveal different patterns. Students are asked to explain why these number patterns occur. The final investigation asks students to explore and find their own patterns and to reason mathematically as to why they occur.

Australian Curriculum: Mathematics (Year 3)

ACMNA055: Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation.

ACMNA060: Describe, continue, and create number patterns resulting from performing addition or subtraction.

Mathematical purpose

- To generate and explore number sequences. A key focus is to build students' skills in reasoning and generalisation as they explain why different patterns occur.

Learning intention

- To explore different number sequences and why they occur.

Time

A lesson of approximately 1 hour.

Vocabulary

- compensation
- equivalence

Resources

- Student Sheet 1 – Letterbox Numbers Investigations (one per student)
- reSolve PowerPoint *2a Letterbox Numbers Pictures*

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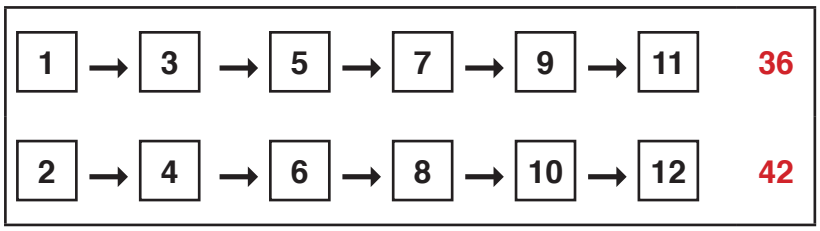
Teacher background information

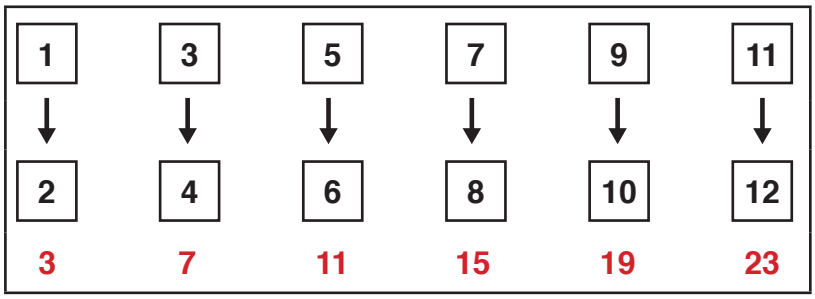
Here we explore the algebra underlying many of the questions in this task. The algebra does not need to be made explicit to students and is provided here primarily for teachers' understanding and reference.

Investigation 1



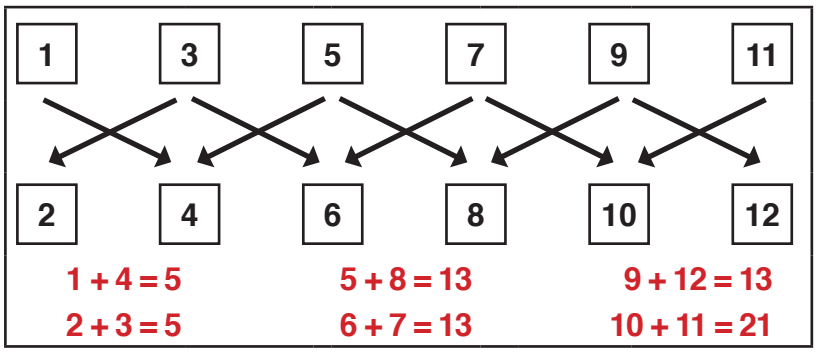
As long as houses are numbered sequentially, the difference between the sums of the two rows will always be equal to the number of houses in each row. This is because each number in the bottom row is one more than the number above it in the top row.

Investigation 2



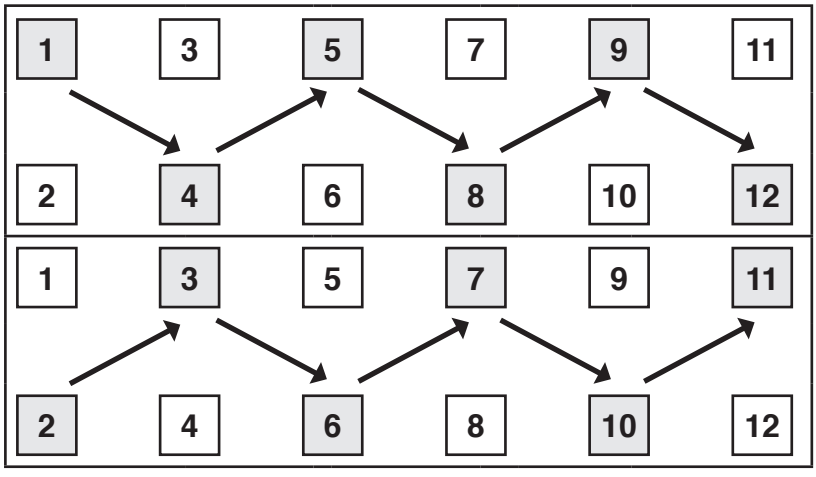
There is a difference of 2 between 1 and 3, and there is also a difference of 2 between 2 and 4. This pattern continues along the rows. The sum will increase by 4 each time because $2 + 2 = 4$.

Investigation 3



This is an opportunity to explore **equivalence** and **compensation**. Using the square formed by the 5, 6, 7 and 8 as an example: $6 + 7 = 13$. Taking 1 from the 6 and adding it to the 7 will give the same value: $(6 - 1) + (7 + 1) = 13 = 5 + 8$

Investigation 4



Both sequences add to 39. Adding the numbers like this will always lead to the same total, as long as the same amount of numbers are added in each row.

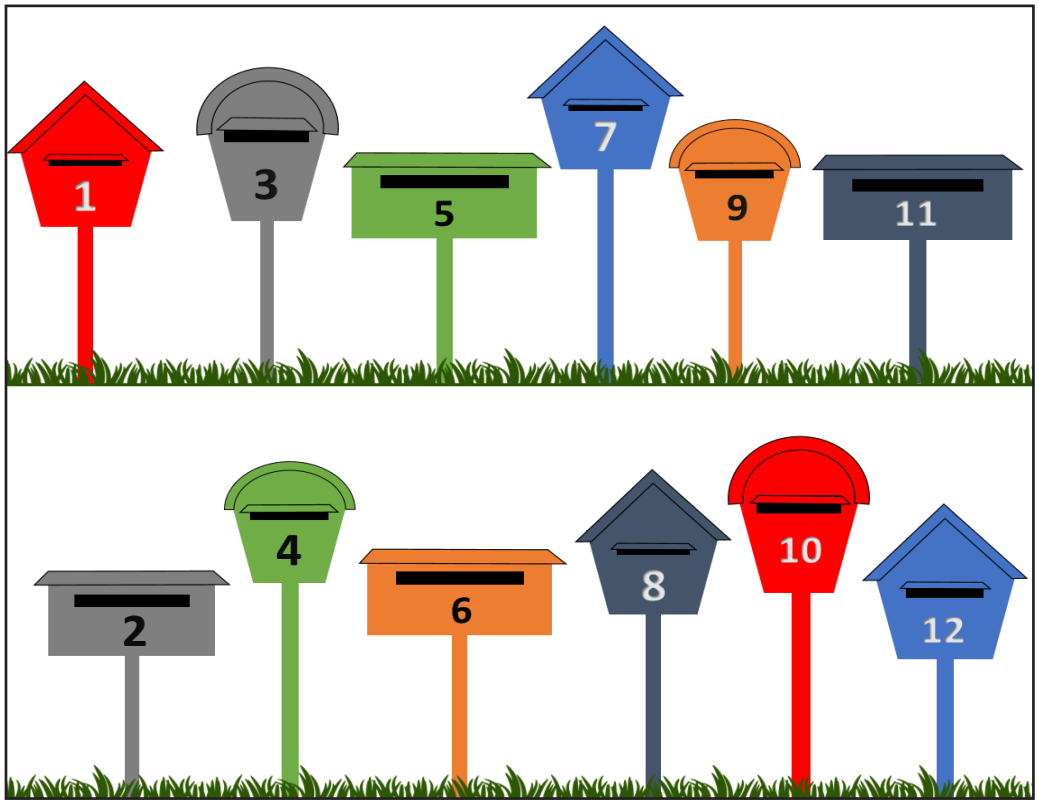
The previous investigation showed that diagonally opposite numbers in a square arrangement add to the same total.

$$(1 + 4) + (5 + 8) + (9 + 12) = 39$$
$$5 + 13 + 21 = 39$$
$$(2 + 3) + (6 + 7) + (10 + 11) = 39$$
$$5 + 13 + 21 = 39$$

Exploring letterbox numbers

Introduce the context of a suburban street with houses on both sides of the road. Each house has a letterbox and the house number is displayed on the letterbox.

Resources: The reSolve PowerPoint *2a Letterbox Numbers Pictures* might be used.



To make an exploration of the numbers easier, the following diagram will be used.

1	3	5	7	9	11
2	4	6	8	10	12

Explain to the students that they are going to explore some patterns formed by the letterbox numbers. The challenge will be to explain why these patterns occur.

Pose the questions:

- *What is the sum of the top row of numbers?*
- *What is the sum of the bottom row?*
- *What is the difference between the sums of the two rows? Why does this happen?*

Investigation

Resources: Give each student a copy of Student Sheet 1 – Letterbox Numbers Investigations.

Investigation 1

Allow the students to explore the sum of the rows and why there is a difference of 6 between the sums of these two rows.

Further exploration:

- Assume that there are the same number of houses in each row and that the house numbers increase in order from one side of the street to the other.
- In the top row the houses go from 1 to 17 and in the bottom row they go from 2 to 18. *What would be the difference between the sums of the two rows?*
- In the top row the houses go from 5 to 21 and in the bottom row they go from 6 to 22. *What would be the difference between the sums of the two rows?*
- There are 15 houses in each row. *What would be the difference between the sums of the two rows?*
- *Explain how you could work out the difference for any number of houses in each row.*

Investigation 2

Look at the sequence that is formed by adding the numbers vertically. *Why does this sequence of numbers grow by 4 each time?*

Investigation 3

Group the numbers in a 2×2 square and add the numbers that are diagonally opposite. *Why do the diagonally opposite numbers in a square add to the same total? Why do the totals increase by 8 each time?*

Extending prompt:

- The diagonally opposite numbers in a square add to 41. *What number are the letterboxes?*
  - Letterboxes 19 to 22.
- *Is it possible to make 51? Or 57?*
  - No. The totals must be 1 more than a multiple of 4. Each set of four numbers contains a multiple of 4. The number diagonally opposite it is 1 more than a multiple of 4 (either from that set of four numbers or the previous set). So, these two diagonally opposite numbers must add to 1 more than a multiple of 4, which means that both pairs of diagonally opposite numbers add to 1 more than a multiple of 4. That is, the totals must be of the form $4n + 1$. Note that 51 and 57 are 3 more than a multiple of 4.

Investigation 4

This investigation looks at adding numbers in two different zigzagging patterns. *What do you think might be the difference between the two totals if you were to add together the numbers in each sequence? Try to answer the question without adding! Use what you have learnt so far to justify your thinking.* Students can check their reasoning by adding up the sequences.

Investigation 5

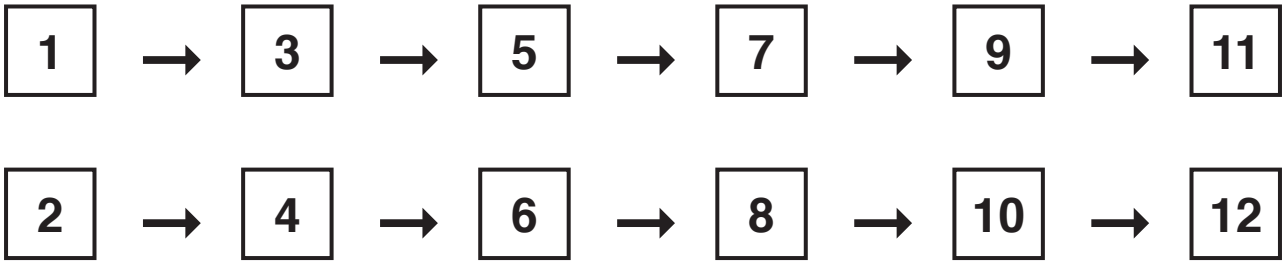
What other patterns can you find? Allow students time to explore and find their own patterns. It is important that students explain why the patterns occur.

Reflection

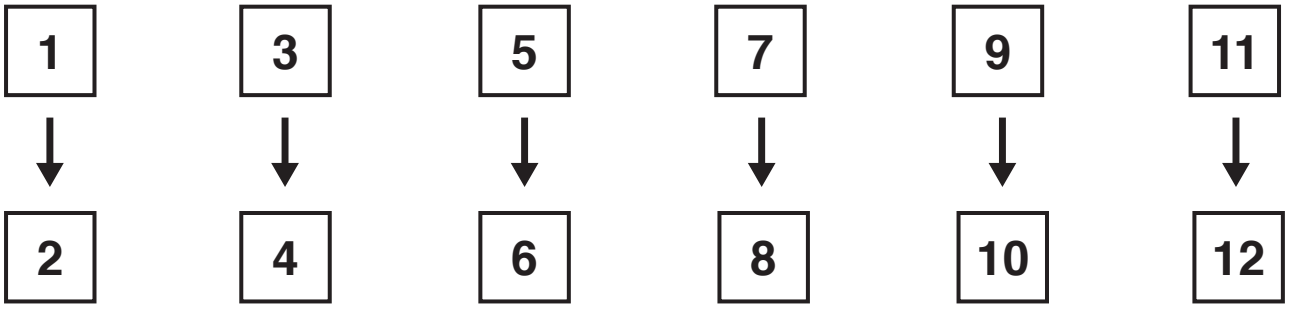
Select students to share their work from the different investigations. The focus of the discussion is to explore why the patterns in the different sequences occur. See Teacher background information for detailed explanations.

Letterbox Numbers Investigations

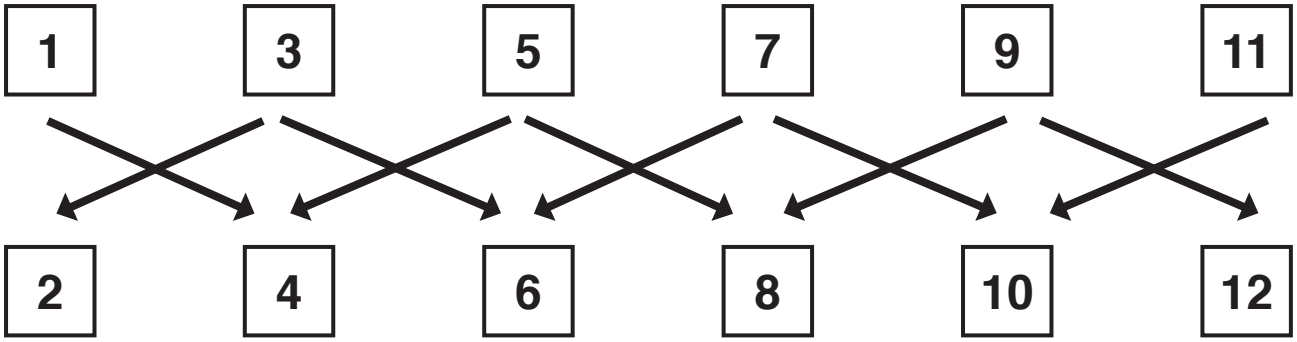
Investigation 1



Investigation 2

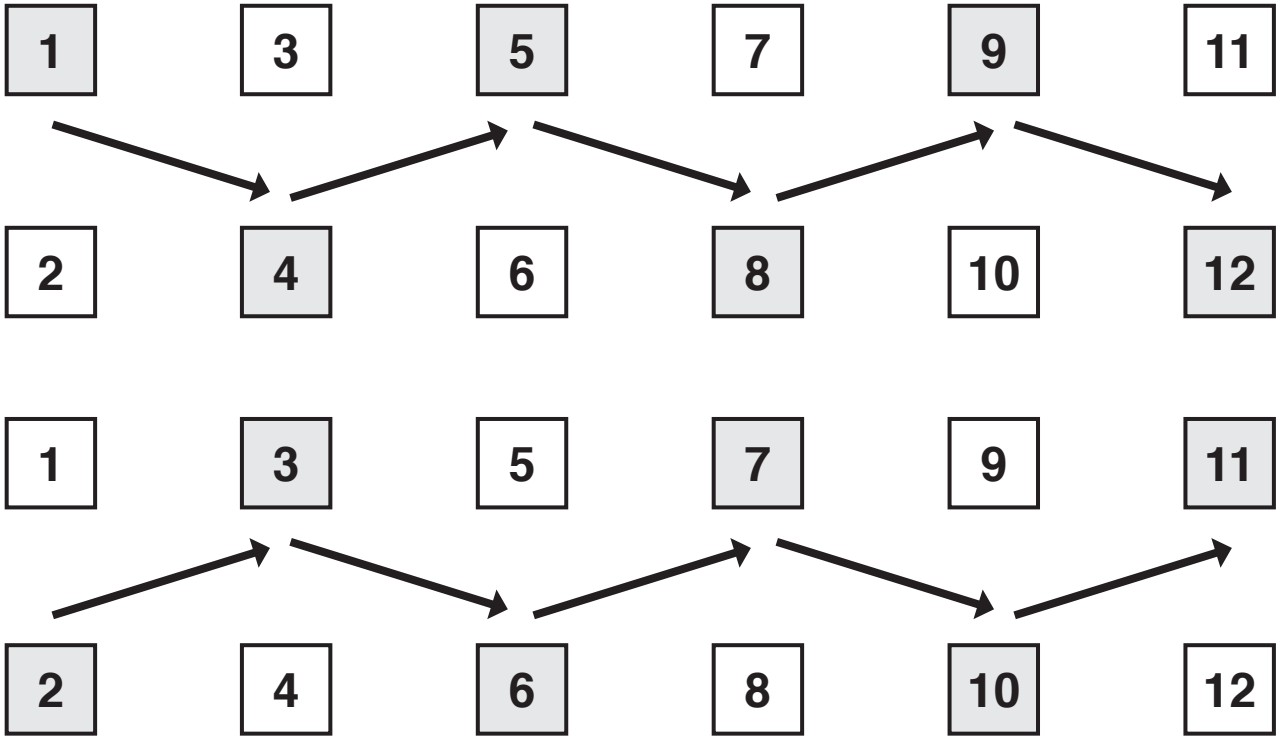


Investigation 3



Letterbox Numbers Investigations

Investigation 4



Investigation 5