re(Solve) MATHS BY INQUIRY

Reasoning with 2D Shapes

Y2

Summary of learning goals

- This sequence starts with a task that focuses on developing students' knowledge of the properties of shapes, particularly triangles, quadrilaterals, pentagons and hexagons.
- Students apply their knowledge of these properties to identify shapes that might not fit more common representations.
- Students then move on to work with transformations of two-dimensional (2D) shapes, using flips and turns. Students will learn to work systematically to find all the possible solutions to the task.

Australian Curriculum: Mathematics (Year 2)

ACMMG042: Describe and draw two-dimensional shapes, with and without digital technologies.

ACMMG045: Investigate the effect of one-step slides and flips, with and without digital technologies.

ACMMG046: Identify and describe half and quarter turns.

Summary of lessons

Who is this sequence for?

- Year 2 students who have had some experience with common 2D shapes.
- Students should recognise polygons (i.e. 2D shapes) as plane, closed shapes with three or more straight sides and corners (or angles). They should be familiar with the fact that polygons can be classified according to properties, including the number of sides.
- Students should be able to use these properties to recognise and name 2D shapes, including non-typical examples of shapes. Students would benefit from some experience manipulating 2D shapes.

Lesson 1: Shape Makers

Students create 2D shapes by joining pins on a circular geoboard. As they create shapes, they develop an understanding of the properties of triangles and quadrilaterals. They also identify what makes a shape regular or irregular.

Lesson 2: Joining Triangles

Students explore the different shapes that can be made by joining together a set number of identical equilateral triangles. Students first explore combinations of two, three and four triangles. They are then asked to find all possible shapes that can be made from five triangles. They are asked to justify that they have found all possible combinations.

Lesson 3: Combining Squares

Students explore the different tetrominoes that can be made by joining together four squares. The students are asked to justify that they have found all five possible tetrominoes. The students then use two of each tetromino to create two different rectangles.

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Reflection on this sequence

Rationale

This sequence aims to provide a firm foundation for a conceptual understanding of shape, knowledge of shape properties, and skills in manipulating shapes using transformations. This is done by a group of students, acting as anchor points while sitting in a circle, using a ribbon (or similar material) to create shapes. Then, as an individual activity, students use a geoboard to create shapes. The focus is on knowing and understanding the properties of triangles, quadrilaterals, pentagons and hexagons.

In the second lesson, students explore shapes that can be made by joining together five triangles and then combining four squares. This leads to dynamic visualisation through the use of simple transformations to explore the relationships between the shapes constructed. Students are required to manipulate these new shapes using flips and turns to show whether shapes are the same or different, and to justify that they have found all possible combinations of five triangles and four squares.

reSolve mathematics is purposeful

- This sequence uses real-world contexts of games and puzzles to work with 2D shapes, by seating students in a circle for group activities and drawing on references to Tetris. It develops a deep knowledge of shapes and their properties beyond typical classroom activities by focusing on:
  - classifying shapes according to the number of sides and identifying and naming special examples of quadrilaterals
  - defining shapes as regular and irregular
  - the variety of shapes constructed when joining together triangles and squares.
- Students' dynamic visualisation skills are developed through generating and manipulating polygons, using simple transformations. Deep reasoning is also required to classify shapes and explain why they are the same or different.

reSolve tasks are inclusive and challenging

- The introductory activity begins with a common experience accessible to all students. This is done by a group of students, sitting in a circle, taking turns to act as anchor points to make shapes with a ribbon, while the other students identify the shapes created.
- This leads into students completing a similar activity using circular geoboards. The subsequent activities have a low floor and high ceiling. Students are asked to find different ways that five triangles can be joined together and then four squares. The low floor is realised as students find one or two examples of each. The high ceiling on the task is achieved as students apply transformations to show some shapes are the same and then reason mathematically to show that they have found all combinations.

reSolve classrooms have a knowledge-building culture

- This sequence relies on active collaboration to build a robust understanding of shapes, their properties and the effect of transformations. This is seen particularly in the second and third tasks, during which the class works together to find all possible combinations of five triangles and four squares. It is suggested that a class display of different combinations is set up for a period of time. This allows students to visit the display and apply their knowledge of transformations to find shapes that are the same as those featured in the display. In doing so, the class will have worked collaboratively to find all possibilities.

LESSON 1

re(Solve) MATHS BY INQUIRY

Reasoning with 2D Shapes

Shape Makers

Y2

About this lesson

Students create two-dimensional (2D) shapes by joining pins on a circular geoboard. As they create shapes, they develop an understanding of the properties of triangles, quadrilaterals, pentagons and hexagons. They also identify what makes a shape regular or irregular.

Australian Curriculum: Mathematics (Year 2)

ACMMG042: Describe and draw two-dimensional shapes, with and without digital technologies.

Mathematical purpose

- Students will develop a deeper understanding of the properties of 2D shapes, specifically the number of sides and corners. They will learn to identify regular shapes as those having sides all the same length.

Learning intention

- Shapes have properties that can be used to name and classify them.

Time

A lesson of approximately 1 hour.

Vocabulary

- angle
- hexagon
- quadrilateral
- regular and irregular shapes
- triangle

Resources

- 12-point circle geoboards for each student or, alternatively, copies of Student Sheet 1 – Shape Makers: Circles for each student
- one piece of string/ribbon/rope (at least 10 metres long)

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Teacher background information

This task focuses on building students' knowledge of shapes, specifically their definitions. The following are important definitions to be used in this task. Most have been taken from the *Australian Curriculum: Mathematics*.

polygon	A polygon is a plane figure bounded by three or more straight sides. The word derives from the Greek 'polys', meaning many, and 'gonia', meaning angle. (Adapted from ACM.)
regular polygon	A regular polygon has sides and angles that are equal in length and magnitude.
triangle	A triangle is a polygon with three sides. (Not included in the ACM.)
quadrilateral	A quadrilateral is a polygon with four sides.
trapezium	A trapezium is a quadrilateral with one pair of opposite sides.
kite	A kite is a quadrilateral with two pairs of adjacent sides that are equal.
parallelogram	A parallelogram is a quadrilateral whose opposite sides are parallel.
rhombus	A rhombus is a quadrilateral with all sides of equal length.
rectangle	A rectangle is a quadrilateral in which all angles are right angles.
square	A square is a regular quadrilateral. It has all the properties of a rectangle and all the properties of a rhombus. (Adapted from ACM.)
pentagon	A pentagon is a polygon with five sides. (Not included in the ACM.)
hexagon	A hexagon is a polygon with six sides. (Not included in the ACM.)

Outlining shapes

The lesson begins with a circle of students making shapes.

1. Ask 12 students to sit in a circle on the floor. These students are the 'shape makers'. The other students can stand around the outside and look down on the shapes that will be made.
2. Give a long piece of string or ribbon to one student in the circle.
   Ask the students: *How many students will need to hold onto the ribbon to make a quadrilateral?*
3. Ask four students in the circle to hold the ribbon. Make sure these students are not evenly spaced (so as not to form a square).
4. **Pose the questions:** *What other quadrilaterals can we make? Can we make a regular quadrilateral?*
5. Allow the students in the circle to make some different quadrilaterals, including a regular quadrilateral; that is, a square. See if the class can name the quadrilaterals made.
6. Ask the students to consider how many and which students should hold the ribbon to make a triangle. Make some triangles as a class.

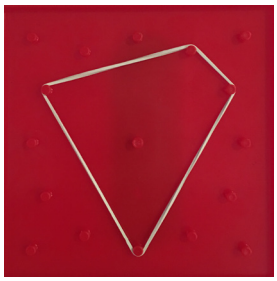
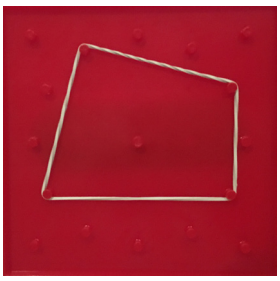
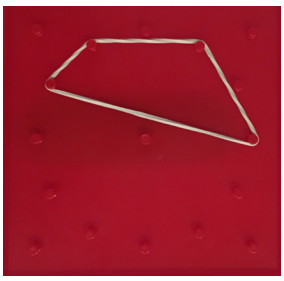
Resources: Show the students a 12-point circle geoboard (or Student Sheet 1 – Shape Makers: Circles).

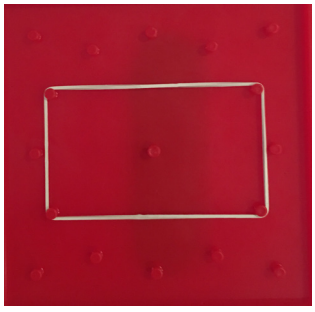
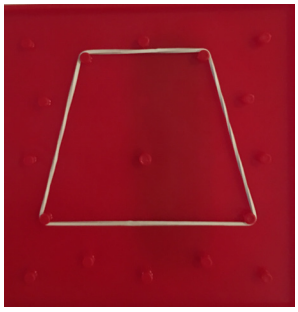
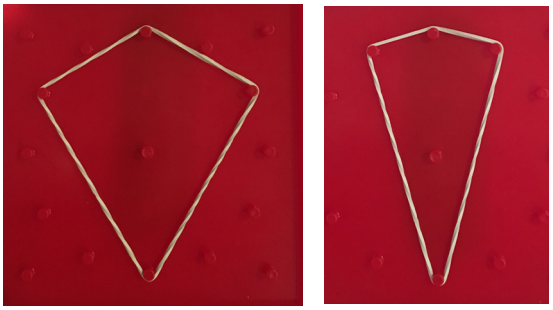
Pose the question: *What shapes can you make on the geoboard?*

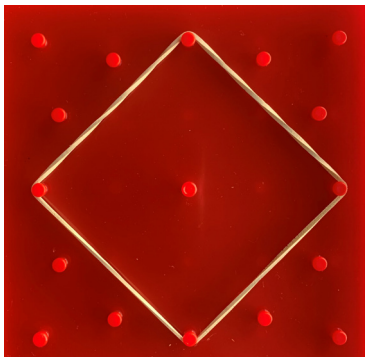
Exploration

Resources: Provide the students with geoboards or Student Sheet 1 – Shape Makers: Circles.

Pose the questions: *What other **quadrilaterals** can you make on your geoboard? Which of these quadrilaterals have special names? Can you make a regular quadrilateral?*

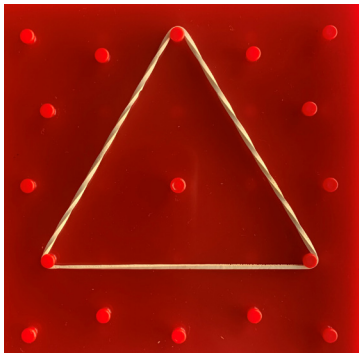
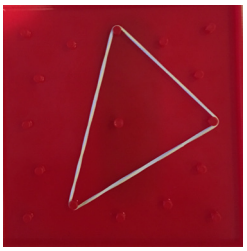
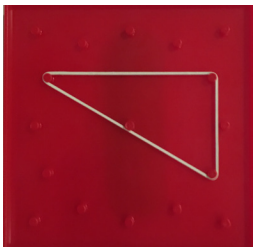
These quadrilaterals do not have special names.		
		

The following quadrilaterals have special names.		
rectangle	**trapezium**	**kites**
		

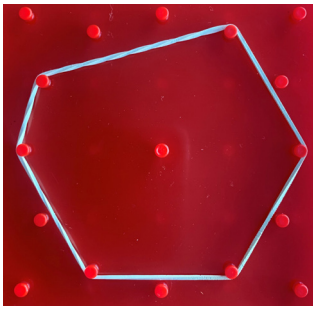
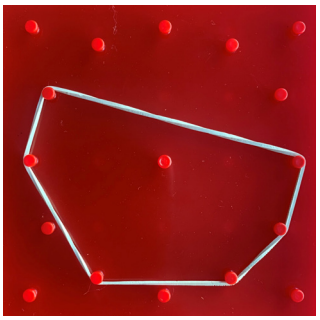
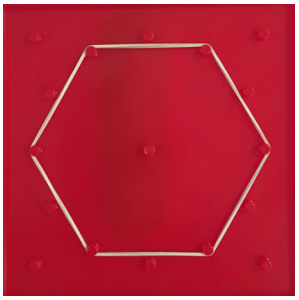
Joining every third pin on the geoboard creates a square.	
	Note: This is not a 'diamond'. A 'diamond' is not a mathematically defined shape. It is important that students recognise a square in different orientations.

Pose the questions: *What **triangles** can you make on your geoboard? Can you make a **regular triangle**?*

Students can join any pins; they don't have to be equally spaced. Students should recognise that any three-sided polygon is a triangle regardless of orientation or side length.

Joining every fourth pin creates a regular triangle.	These are irregular triangles.		
			

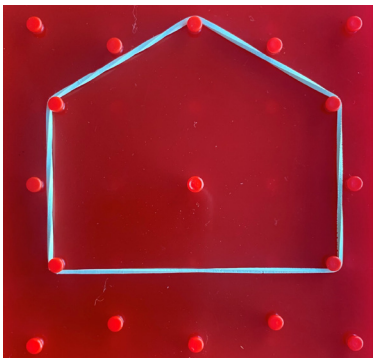
Pose the questions: *What **hexagons** can you make on your geoboard? Can you make a **regular hexagon**?*

These shapes are both hexagons.		Joining every second pin creates a regular hexagon.
		

Pose the questions: *Can you make a **pentagon**? Can you make a **regular pentagon**?*

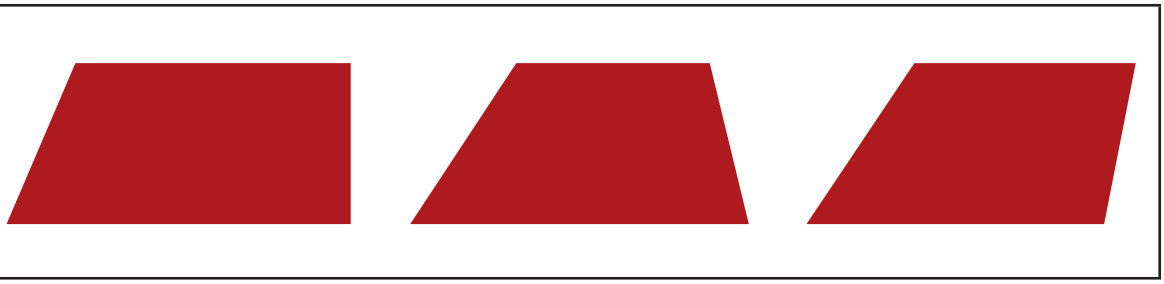
It is not possible to make a regular pentagon on a 12-pin geoboard.
Ask students to consider: *How many pins would be required to make a regular pentagon?*
A 10-pin geoboard, or one with any multiple of 10 pins, would be needed.

A possible pentagon is shown.



Extending prompt:

- Ask: *What quadrilaterals cannot be made on the geoboard?*
  - ◊ Only one parallelogram can be made on the geoboard — a square. A square is classified as a special parallelogram; that is, a rhombus. It is not possible to make any other parallelograms due to the locations of the pins.
  - ◊ The only trapezium that can be made is one in which the non-parallel sides are equal in length. It is important to note that the following are also trapeziums:



Reflection

Look at the shapes that were made and discuss the similarities and differences within each group of shapes.

Construct class definitions for triangles, hexagons and quadrilaterals. Construct definitions for special groups of quadrilaterals.

Further activities

Activity 1

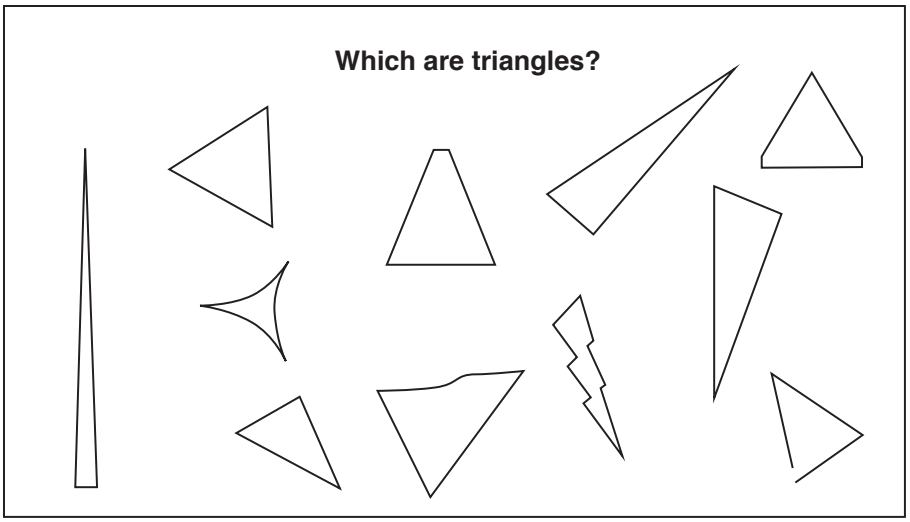
Provide students with a collection of quadrilaterals. Ask them to independently sort the quadrilaterals and then have a friend guess how they have been sorted. Can they find multiple different ways to sort the quadrilaterals?

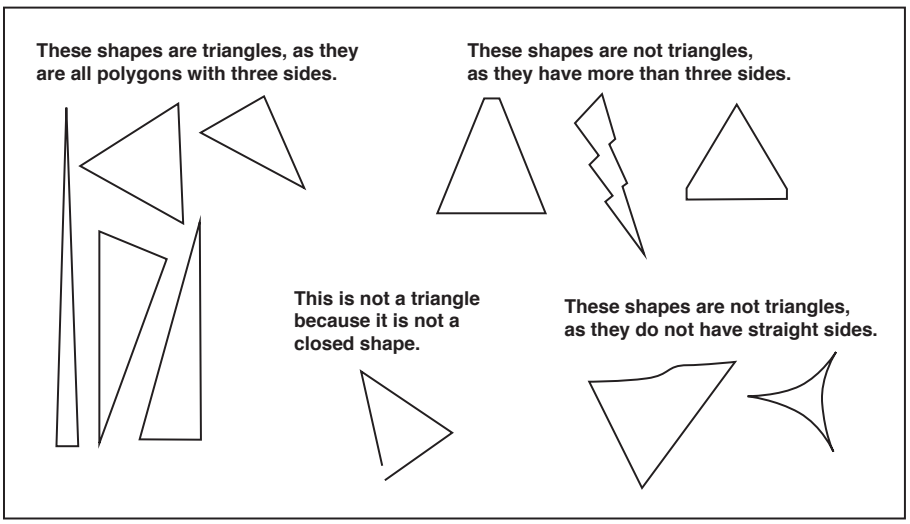
Repeat the activity with other groups of shapes.

Activity 2

Provide students with examples and non-examples of triangles. Ask them to identify the triangles and justify why they are triangles. Ask the students to also justify why the others are not triangles. Repeat with different groups of quadrilaterals, pentagons and hexagons.

For example:

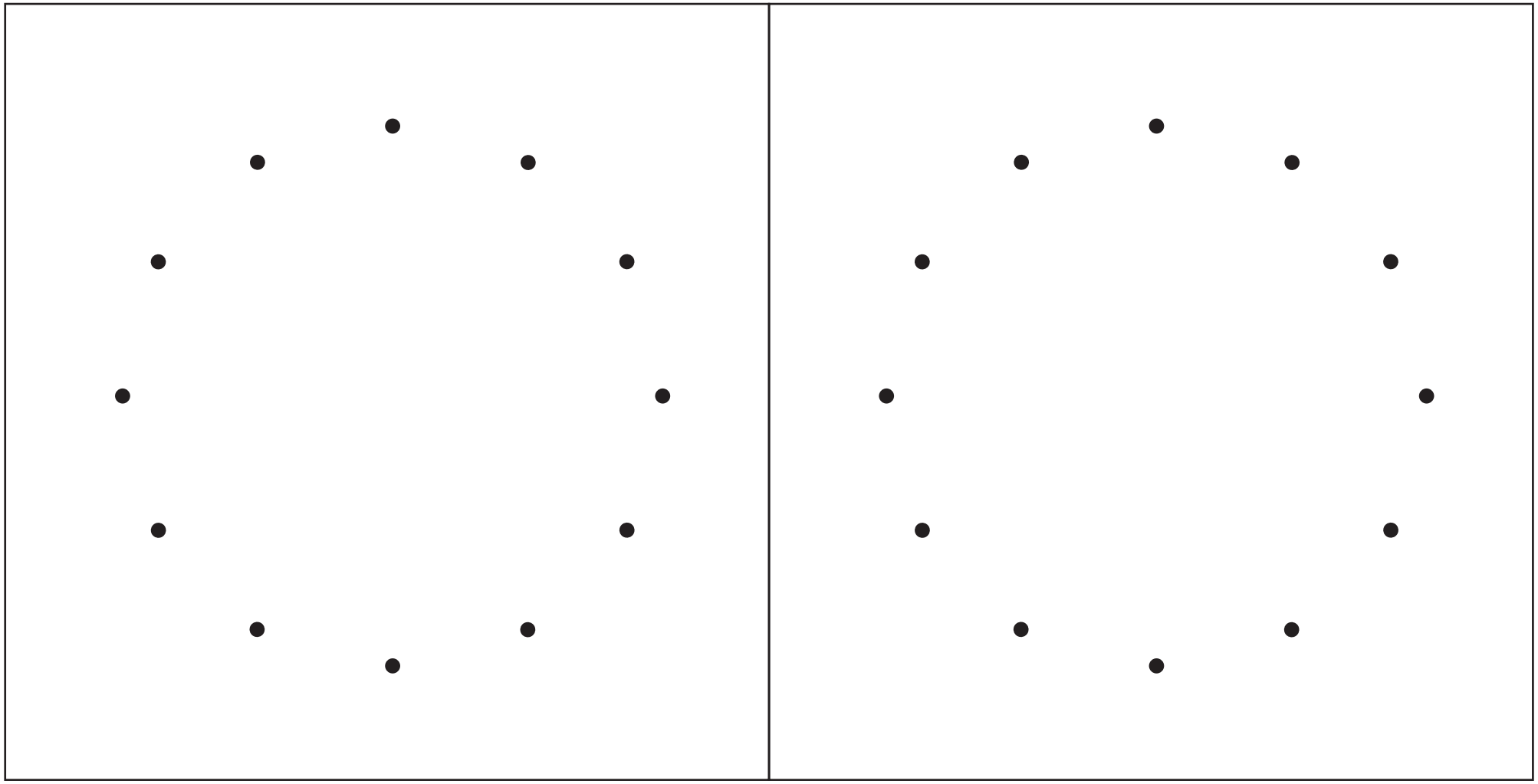




Where to next?

Lesson 2: Joining Triangles is the second activity in this sequence. It explores the different shapes that can be made by joining equilateral triangles together.

Shape Makers: Circles



LESSON 2

re(Solve) MATHS BY INQUIRY

Reasoning with 2D Shapes

Joining Triangles

Y2

About this lesson

This task explores the different shapes that can be made by joining together a set number of identical equilateral triangles. Students first explore combinations of two, three and four triangles. They are then asked to find all possible shapes that can be made from five triangles. They are asked to justify that they have found all possible combinations.

Australian Curriculum: Mathematics (Year 2)

ACMMG042: Describe and draw two-dimensional shapes, with and without digital technologies.

ACMMG045: Investigate the effect of one-step slides and flips, with and without digital technologies.

ACMMG046: Identify and describe half and quarter turns.

Mathematical purpose

- Students will develop their understanding of the properties of shapes, and use flips and turns to show congruency between shapes. They will develop their skills in working systematically as they aim to find all the possible solutions to the task.

Learning intention

- There are multiple ways that five regular triangles can be joined together to create tetrominoes. Flips and turns can be used to show that all possible combinations of triangles have been found.

Time

A lesson of approximately 1 hour.

Vocabulary

- 2-dimensional
- congruent
- equilateral
- flip
- orientation
- rhombus
- rotation
- slide
- trapezium
- turn

Resources

- green triangle pattern blocks (at least 12 per student)
- isometric grid paper for each student (A sample template is included.)

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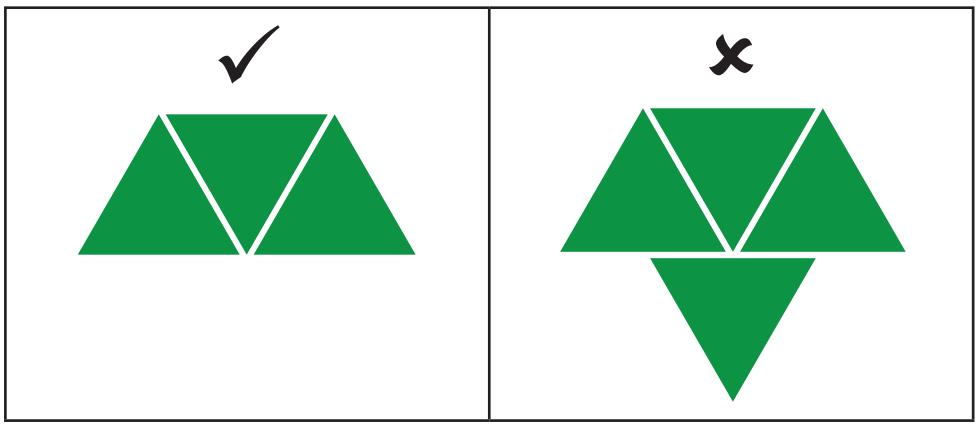
Joining equilateral triangles

Resources: Provide each student with at least 12 green triangle pattern blocks.

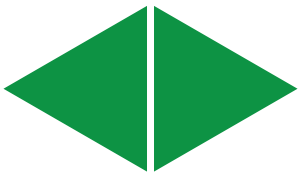
Explain to the students that they will be looking at the different shapes that can be formed by joining triangles together. Triangles must be joined by matching full sides together. You cannot join parts of sides together.



Pose the following questions one at a time, allowing students to explore each one before asking the next:

- *What are the different shapes that can be made by joining two triangles together?*
- *What are the different shapes that can be made by joining three triangles together?*
- *What are the different shapes that can be made by joining four triangles together?*

Teacher notes:

There is only one way that two equilateral triangles can be joined together. As the sides are equal in length, the shape will not change when different sides of the triangle are joined together.			
There is only one way that three equilateral triangles can be joined together.			
There are three ways that four triangles can be joined together.	 A triangle can be added to the top, to create a larger triangle.	 A triangle can be added to one of the sides, to create a parallelogram.	 A triangle can be added to the base of the trapezium.

- These shapes can be cut out of paper so that turns (rotations) and flips (reflections) can be used to show that these are the only options.

These questions lead to our main task.

Pose the questions: *What are the different shapes that can be made by joining five triangles together? Can you find them all?*

A possible context for the task: A secret garden requires a special key to open its gate. The key is made up of five triangles joined together. Can you find the key to unlock the gate?

If using the context of keys, you can **pose the question:** *Can you find all the keys that can be made by joining five triangles together?*

Enabling prompt:

- *Start with the three options made using four triangles. Join an extra triangle to these different shapes. What new shapes can you make?*

Exploring combinations

Students explore the different ways that five triangles can be joined together. Have them record the different options they find. Some options for recording solutions are listed below.

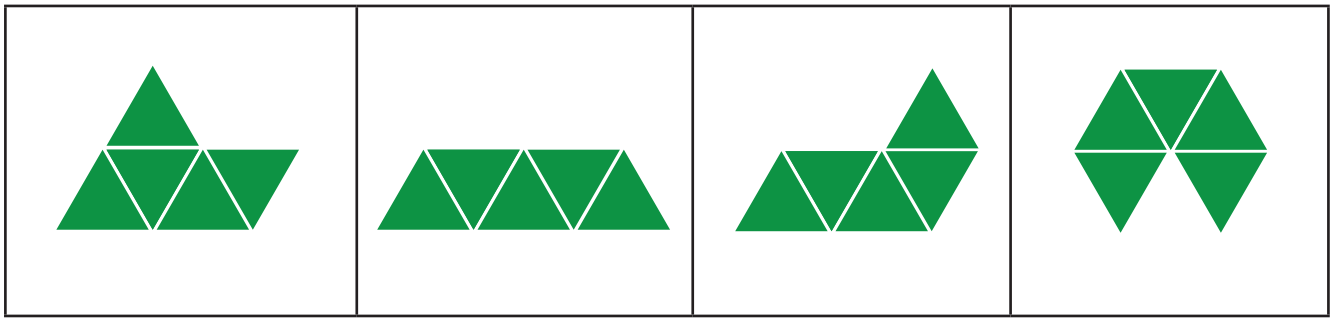
- Use isometric grid paper made up of large equilateral triangles. Students can outline and/or colour the different solutions found.
- Provide students with equilateral triangles that can be cut out and stuck onto paper.
- Take photos of each combination made with the pattern blocks and collect a digital file of the different options.

It is likely that students will collect a large number of different combinations for five triangles. Cutting out their combinations means that they can use transformations to show if they are the same or different to other combinations.

Pose the challenge: *Using flips and turns, can you show which of your shapes are the same and which ones are different? How many unique shapes do you have?*

Possible student responses:

- Only four unique shapes can be found.



- These can be found by starting with the combinations for four triangles, as shown.

Starting with the large triangle, a fifth triangle can be added to the side.

Starting with the parallelogram, a fifth triangle can be added to make two new shapes.

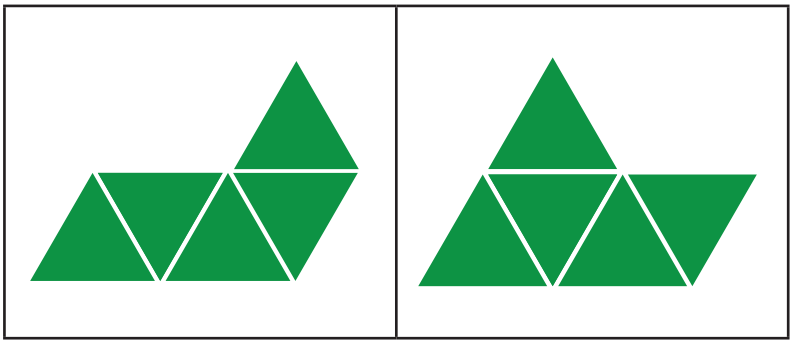
A triangle can be added to the shape that looks like part of a regular hexagon (left), to create the shape shown on the right.

Prompts and questioning to direct the investigation and challenge students' thinking and reasoning:

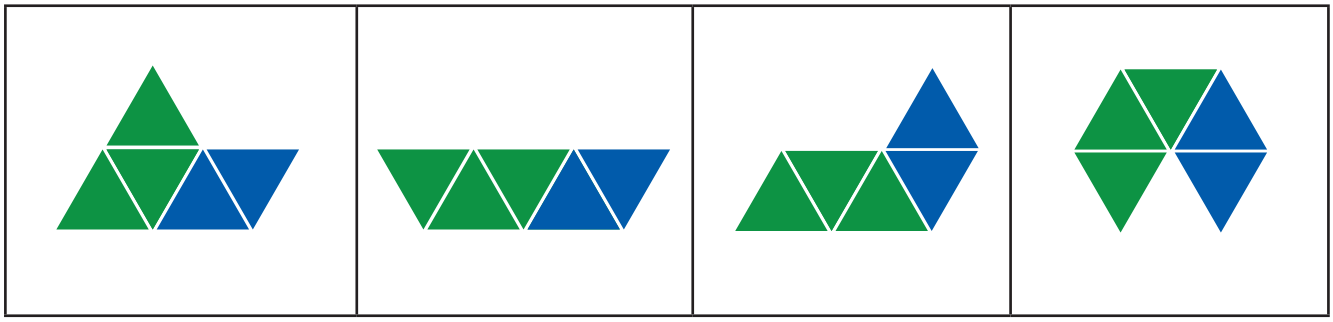
- *Can you sort your shapes to show which ones are the same as each other? Show they are the same, using flips and turns.*
- *In the five-triangle combinations, can you find the shapes made using two and three triangles?*

Teacher notes:

- The following two shapes will be the most complex for students to work with:



- All the shapes can be made by combining two and three triangle shapes.

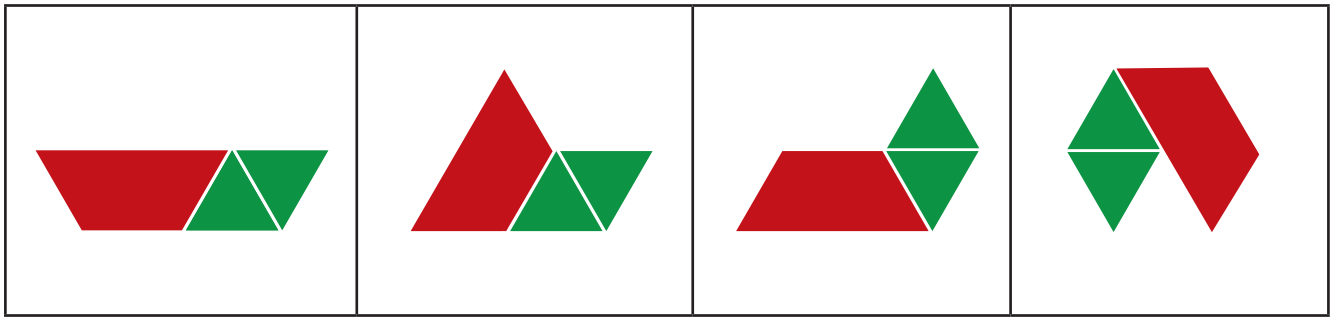


Extending prompt:

- *How many shapes can you find using six triangles?*
  - ◊ Students can combine the shapes made with two and four triangles. They could also look at the combinations that can be made with two of the three triangle trapeziums.
  - ◊ Students can make links to pattern blocks; for example, two green triangles equal the blue rhombus or two red trapeziums equal the yellow hexagon.

Reflection

As a group, discuss ways in which the shapes found are similar and different. For example, you might observe that every shape is made of a trapezium and a rhombus:



Pattern blocks can be used to help students see the four options.

Have some students demonstrate how they used flips and turns to show shapes that are similar and different.

Come to a group consensus that there are only four *unique* options.

Further activities

Activity 1

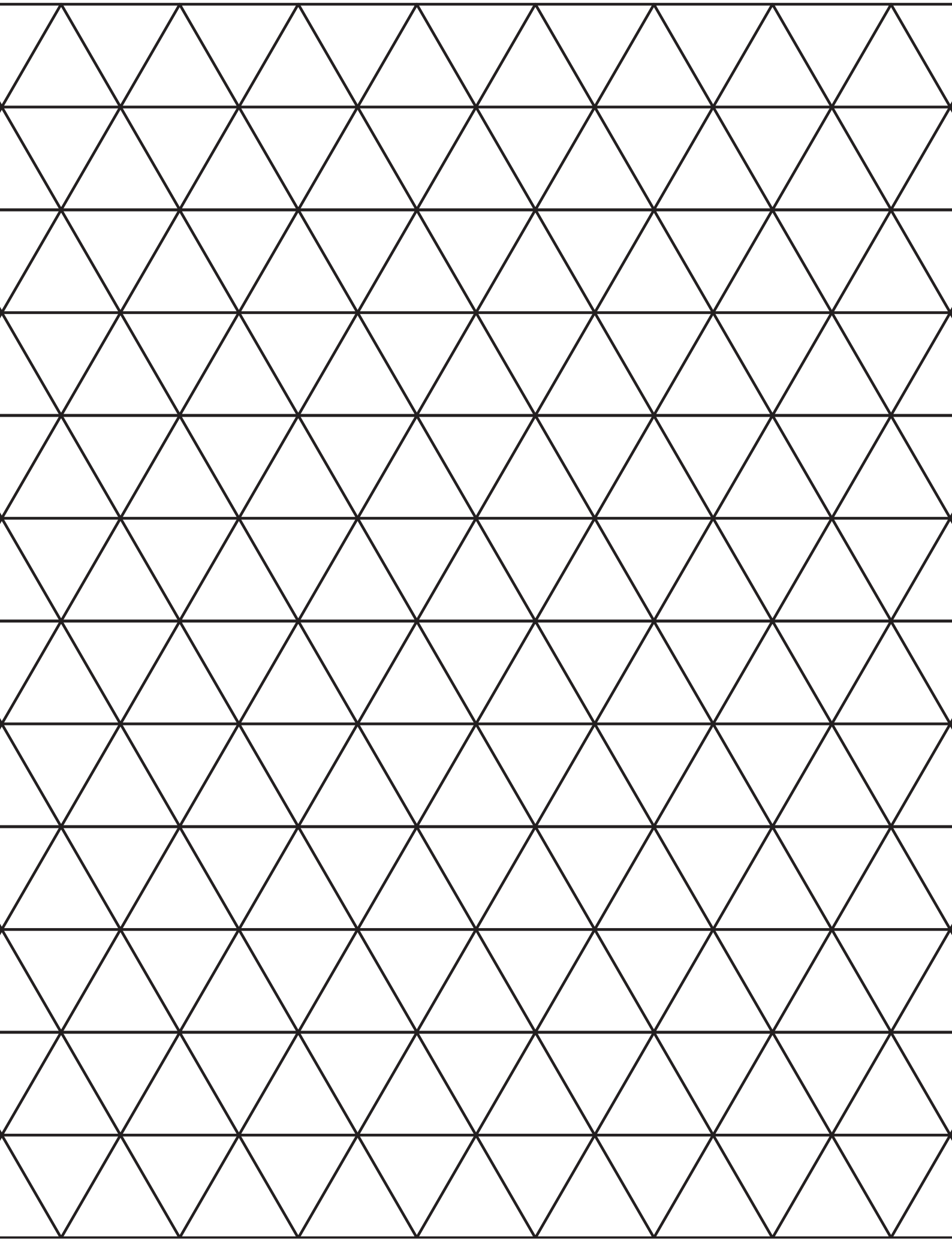
Students create a simple design using combinations of pattern blocks. They must recreate their design to show what it would look like when turned 180° or flipped horizontally or vertically.

Activity 2

Students use pattern blocks to find all the possible ways to make a regular hexagon. They might start by trying to make the yellow hexagon, using different pattern blocks. Students can then see if they can make larger regular hexagons.

Where to next?

Lesson 3: Combining Squares is the third activity in this sequence. It explores all the different shapes that can be made by joining four squares together.



LESSON 3

Reasoning with 2D Shapes

Combining Squares

Y2

About this lesson

This task explores the different tetrominoes that can be made by joining together four squares. The students are asked to justify that they have found all five possible tetrominoes. The students then use two of each tetromino to create two different rectangles.

Australian Curriculum: Mathematics (Year 2)

ACMMG042: Describe and draw two-dimensional shapes, with and without digital technologies.

ACMMG045: Investigate the effect of one-step slides and flips, with and without digital technologies.

ACMMG046: Identify and describe half and quarter turns.

Mathematical purpose

- Students will develop their skills in using flips and turns to show congruence and to fit shapes together. Students will learn to work systematically to find all the possible solutions to the task.

Learning intention

- There are multiple ways that four squares can be joined together to create tetrominoes. Flips and turns can be used to show that all possible tetrominoes have been found.

Time

A lesson of approximately 1 hour.

Resources

- orange square pattern blocks

Vocabulary

- congruent
- flip
- orientation
- rhombus
- rotation
- slide
- tetromino
- trapezium
- turn

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Introduction to tetrominoes

Resources: Provide the students with a collection of orange square pattern blocks.

If the class has completed Lesson 2: Joining Triangles, remind them that there are four possible ways of joining five triangles. Explain that the class will be looking at combining four squares.

Pose the question: *There are four unique solutions for combining five triangles. Do you think there will be more options for combining four squares, or do you think there will be fewer options?*

Ask students to suggest the number of options they think there will be and to provide reasons.

Explain to the students that there are five different options. These five options are called tetrominoes.

Pose the challenge: Can you find all five tetrominoes?

Finding tetrominoes

Students explore the different ways to join together four squares. They record the different solutions they find.

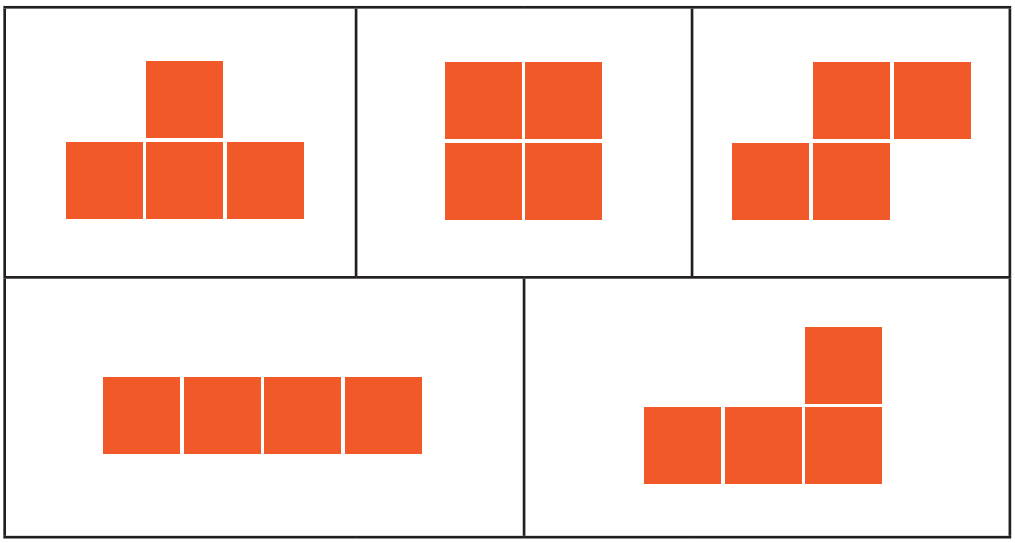
Enabling prompt:

- *What different combinations can you find for two squares? What about three squares? Can you add an extra square to find some combinations for four squares?*

Ask students to apply their understanding of transformations, specifically flips and turns, to show if shapes are the same or different.

Possible student response:

- The five different tetrominoes are presented below.



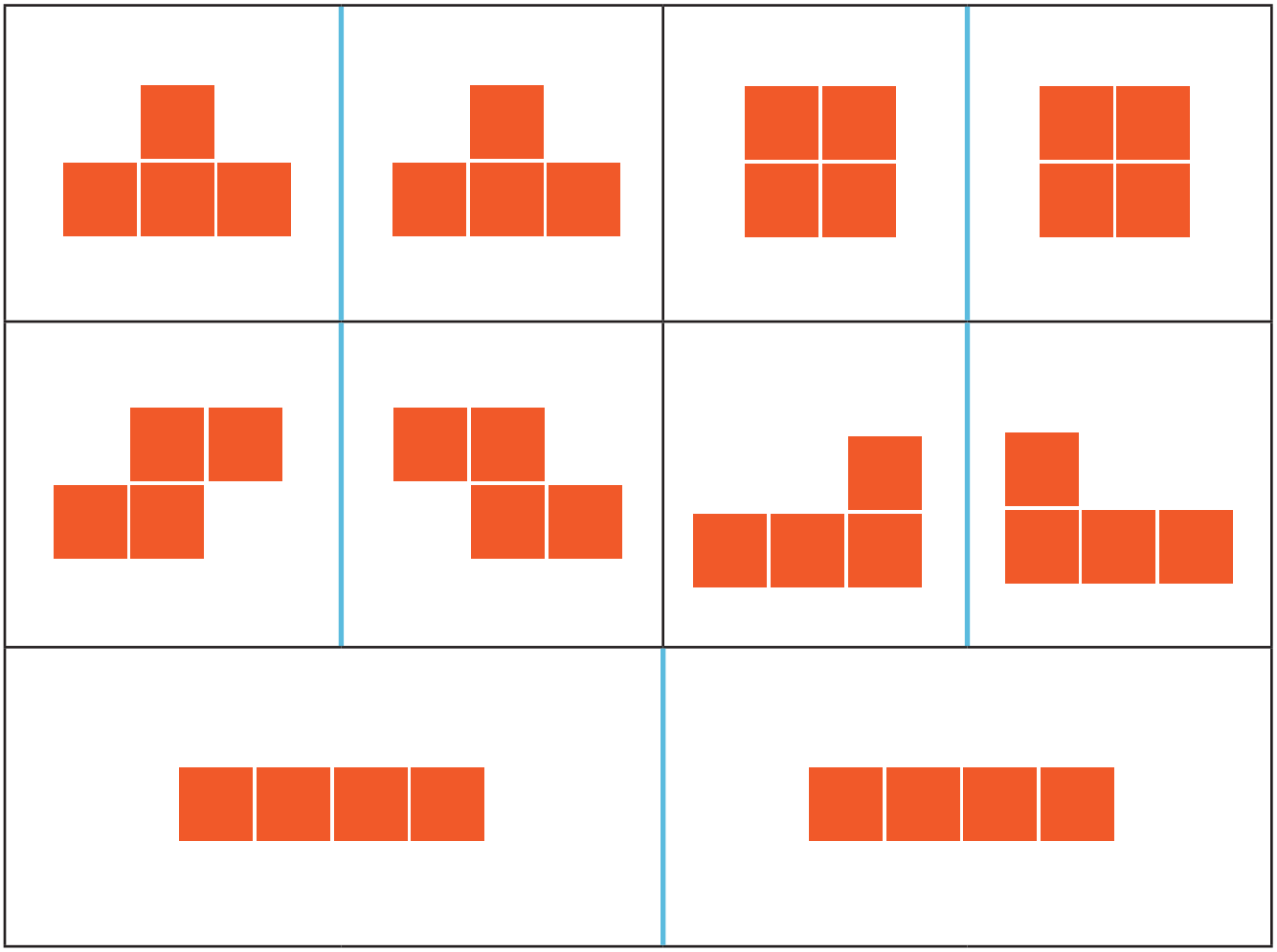
Reflection

Look at the different shapes that have been found.

Pose the question: *Which tetrominoes look different when flipped vertically?*

Teacher notes:

- It may assist students to draw a vertical line on a page and to flip each shape over this line, to see which shapes look different and which still look the same.



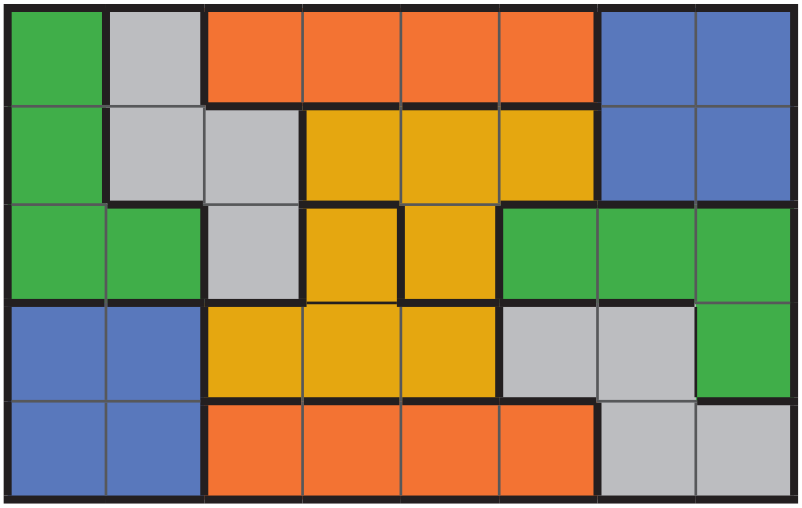
Further activities

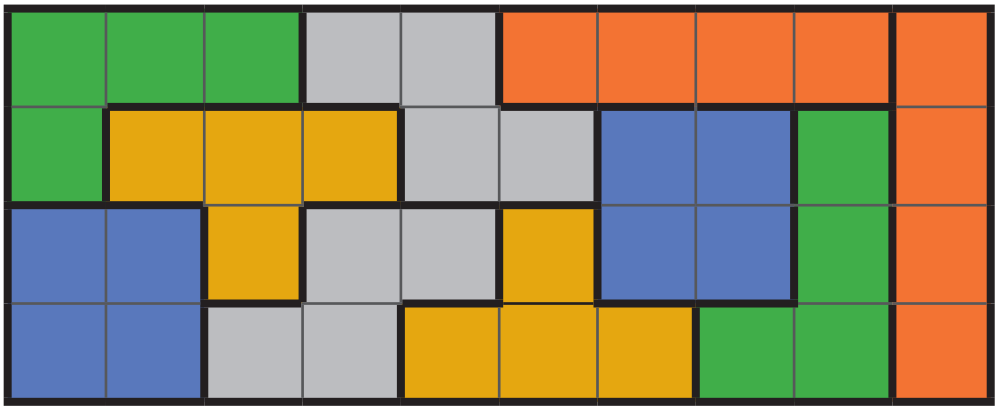
Activity 1

Tetrominoes are used in the game Tetris, where they must be rotated to fit into a space. Cut out a set of the previous 10 tetrominoes (shown in Teacher notes) and allow the students to make some different shapes. It is possible to make two different rectangles using these 10 tetrominoes. Cutting out the shapes this way will require students only to rotate the pieces to fit them together — no flips will be required.

Pose the challenge: *Can you make a rectangle using your tetrominoes?*

There are many ways to construct 5×8 and 4×10 rectangles. Two possibilities are illustrated below.





Enabling prompt:

- Give the students the dimensions of a finished rectangle and see if they can use this as a guide to put together their own rectangle.

Extending prompt:

- Once students have found one rectangle, **pose the challenge:** *Can you find a different rectangle using your tetrominoes?*

Activity 2

Students create a simple design using combinations of pattern blocks. They must recreate their design to show what it would look like when turned 180° or flipped horizontally or vertically.

Activity 3

Students use pattern blocks to find all the possible ways to make a regular hexagon. They might start by trying to make the yellow hexagon using different pattern blocks. They can then see if they can make larger regular hexagons.