re(Solve) MATHS BY INQUIRY

Reasoning with 3D Objects

Y3

Summary of learning goals

- This sequence explores symmetry and the composition of three-dimensional (3D) objects. This will require students to interpret and represent 3D objects in two dimensions.
- Students will apply their knowledge of transformations to show congruency or difference between objects. Applying transformations of these objects develops students' mental rotation skills.
- Students' abilities to work systematically are developed as they find all possible solutions in each task.

Australian Curriculum: Mathematics (Year 3)

ACMMG063: Make models of three-dimensional objects and describe key features.

ACMMG066: Identify symmetry in the environment.

Summary of lessons

Who is this sequence for?

- This sequence is for Year 3 students who have had prior experiences with identifying and creating symmetry in objects. They will need an understanding of transformations, particularly flips (reflections) and turns (rotations), and how these can be used to show congruency.
- Prior experience with 2D representations of 3D objects will also assist students with tasks.

Lesson 1: Building Symmetry

Students are presented with a non-symmetrical 3D model made of five interlocking cubes. They are asked to make the model symmetrical by changing the position of one block or by adding an extra block to the model. Students are asked to find all possible solutions to the challenge. As the students explore the different possibilities, the language of symmetry and transformations is explored.

Lesson 2: Four Cubes

This resource consists of two sections. The first section asks students to build objects using four cubes and following simple rules. After some exploration, they are encouraged to work systematically to find different solutions. Using transformations, the students reason and justify that all the constructions they have made are different and that they have found all possible solutions. In the second section, students then draw the 2D representations of these 3D objects.

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Reflection on this sequence

Rationale

Students' spatial reasoning skills are a significant indicator of their future performance in mathematics. These skills can be developed through practise and application. This sequence builds students' skills in key aspects of spatial reasoning, with particular focus on students' visualisation skills and their ability to mentally rotate an object, use transformations and identify symmetry.

It is common for symmetry and transformations to be explored with 2D objects in primary mathematics, whereas this sequence is built around 3D objects. Students' visualisation skills are built as they work with concrete models in different contexts and then construct 2D representations of 3D objects. Throughout the sequence they are also required to mentally rotate objects to assist in solving problems.

reSolve mathematics is purposeful

- Both tasks in this sequence invite students to construct and manipulate 3D objects made with interlocking cubes. The tasks' activities focus on building correct language use and students' abilities in the mathematical proficiencies of problem-solving and reasoning.
- The tasks are structured so that students move from hands-on exploration with the objects constructed, through to visualising the results of movements and rotations prior to physically manipulating the objects.

reSolve tasks are inclusive and challenging

- Both tasks in this sequence have a low floor and high ceiling, providing access and challenge for a wide range of abilities.
- The low floor of the task is demonstrated as students use trial and error with the concrete materials to find different solutions to the problems presented.
- The high ceiling of the task is demonstrated as students use spatial visualisation to find possible solutions without actually interacting with or manipulating the concrete materials.

reSolve classrooms have a knowledge-building culture

- There is a strong focus on building understanding through collaborative investigations. In the first task, students work as a class to find different ways that symmetry can be created by moving or adding blocks to a 3D object.
- The second task encourages the teacher to set up a class display of different possibilities for the way four cubes are joined together. Using this display, the class must reach consensus that they have found all possibilities and that each construction displayed is unique.

LESSON 1

re(Solve) MATHS BY INQUIRY

Reasoning with 3D Objects

Building Symmetry

Y3

About this lesson

Students are presented with a non-symmetrical three-dimensional (3D) model made of five interlocking cubes. They are asked to make the model symmetrical by changing the position of one block or by adding an extra block to the model. Students are asked to find all possible solutions to the challenge. As the students explore the different possibilities, the language of symmetry and transformations is explored.

Australian Curriculum: Mathematics (Year 3)

ACMMG063: Make models of three-dimensional objects and describe key features.

ACMMG066: Identify symmetry in the environment.

Mathematical purpose

- Students will build a deeper understanding of symmetry in 3D objects. They will look at different ways of creating symmetry when starting with a non-symmetrical model and build associated vocabulary to describe symmetry and transformations in space.

Learning intention

- To look at different ways of creating symmetrical 3D objects.

Time

A lesson of approximately 1 hour.

Vocabulary

- reflection
- rotation
- symmetry
- transformation
- translation

Resources

- interlocking cube blocks
- reSolve PowerPoint *1a Building Symmetry Images*
- mirrors to check symmetry (optional)

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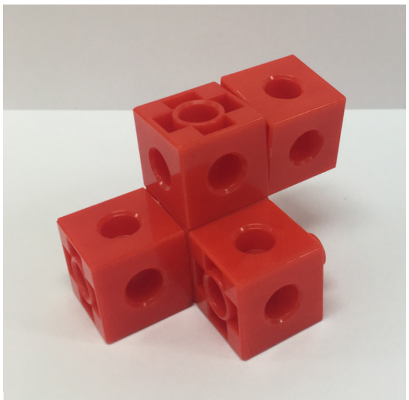
Making symmetrical shapes

Resources: Briefly show the class the first slide from reSolve PowerPoint *1a Building Symmetry Images*:



Hide the image and ask the students to recreate it from memory using five blocks of the same colour — this removes colour as an attribute when considering the symmetry of designs. Show the image again for students to check their solution.

Exploring symmetry

Pose the question: *How many symmetrical objects can you make by moving one block?*

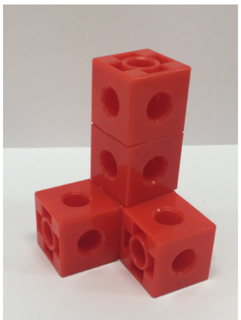
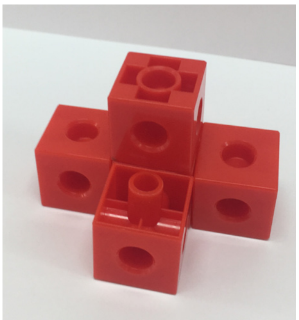
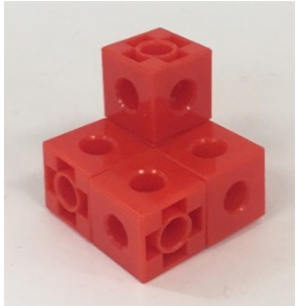
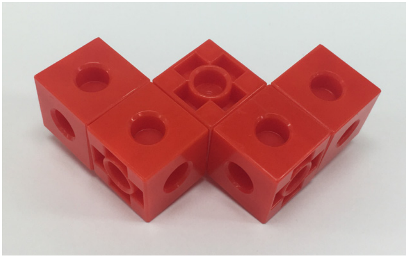
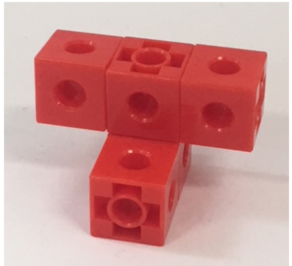
Students are looking for reflection (or mirror) symmetry. Allow the students to experiment with moving one block to create a symmetrical model.

Resources: A mirror can be used to check for symmetry.

Ask the students to keep any symmetrical models that they have created. As students create each symmetrical model, ask them to decide if it is a new design or one that they already have in their collection. Show slide 3 so that students can check their solutions.

Enabling prompt:

- *Can you remove one block to make this model symmetrical?*
  - ◊ There are a number of ways that this can be done. Students can keep the different solutions that they find. A mirror can be used to check for symmetry.
- *Look at the symmetrical objects that you have made. Can you add a block to your models and still keep the object symmetrical?*
  - ◊ By doing this, students should make all of the structures, thus answering the first question posed to the whole class; that is, can you change the original model into a symmetrical object by moving one block?

There are at least five different possible models:		
		
		

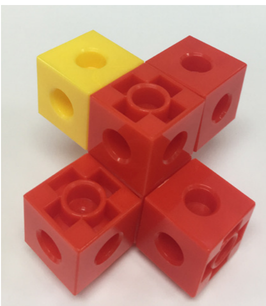
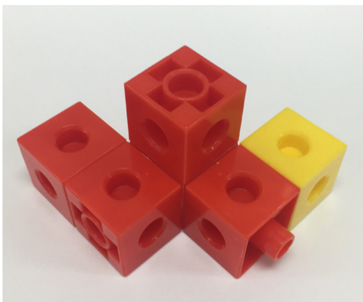
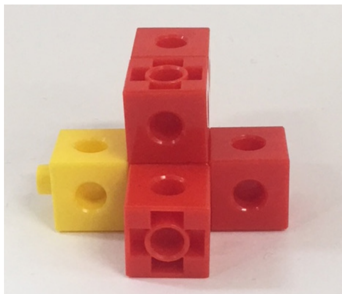
Teacher notes:

- This investigation provides an opportunity to explore some important language when working spatially. Students will see that symmetry is a reflection (or mirror) image.
- As the students compare the objects to see if they are the same or different, they will be using transformations to manipulate them. Students should be encouraged to use the language of transformations.
- Earlier in the curriculum, the terms *flip*, *slide* and *turn* have been used. At this point these words can still be used or the more formal terms *rotation*, *translation* and *reflection* may be introduced.

As students believe they have found all the different options, **pose the question:** *How many symmetrical objects can be created by adding one block to the original structure?*

- Make sure they are adding blocks to the original model, not the models they have just made.
- The block to be added should be a different colour, to make it easier to keep track of changes made.

Show slide 4 so that students can check their solutions.

There are at least three unique models that can be made from the original:		
		

Extending prompt:

- *What other symmetrical objects can you make by adding two or three blocks to the model?*
  - ◊ Students can explore the effects of adding more blocks to create symmetry.
  - ◊ Ask students: *Does adding two or three blocks instead of one make it easier or harder to create symmetry?* They may observe that the task becomes easier the more blocks you are allowed to add.

Reflection

This reflection provides an opportunity to explicitly model the language of symmetry and transformations.

- Ask one student to share a symmetrical model that they found by moving one block. Have them explain why it is symmetrical and where the line(s) of symmetry can be seen.
- Ask whether anyone found a different model. Have them show in what ways it is different and where the line of symmetry is.
- Create a class collection of the three models.

In the same way, share the different symmetrical models made by adding a block.

Further activities

Activity 1

Students make a symmetrical model using four to eight blocks. Have them move one block to make their model non-symmetrical, then pass the model to a friend. The friend should move one block to make the model symmetrical again. Did they move the same block back to recreate the original object, or did they move a different block and create a new symmetrical object?

Activity 2

Create symmetrical 3D objects using different building materials. Describe the symmetry that can be found in each object.

Where to next?

Lesson 2: Four Cubes asks students to explore all the ways that four cubes can be combined and to demonstrate with transformations that they have found all unique possibilities.

LESSON 2

re(Solve) MATHS BY INQUIRY

Reasoning with 3D Objects

Four Cubes

Y3

About this lesson

This resource consists of two sections. The first section asks students to build objects using four cubes and following simple rules. After some exploration, they are encouraged to work systematically to find different solutions. Using transformations, the students reason and justify that all the constructions they have made are different and that they have found all possible solutions. In the second section, students then draw the two-dimensional (2D) representations of these three-dimensional (3D) objects.

Australian Curriculum: Mathematics (Year 3)

ACMMG063: Make models of three-dimensional objects and describe key features.

Mathematical purpose

- Students make simple block constructions using four interlocking cubes. They work systematically to find all the ways that four cubes can be combined and apply transformations to show they have found all unique possibilities.
- Students then draw the various solutions from different perspectives.

Learning intention

- To build, compare, and draw 3D cube constructions.

Time

This task is best presented over two 45–60 minute lessons.

Resources

- interlocking cube blocks (at least 20 per student)
- isometric graph paper for drawing cube models

Vocabulary

- transformations

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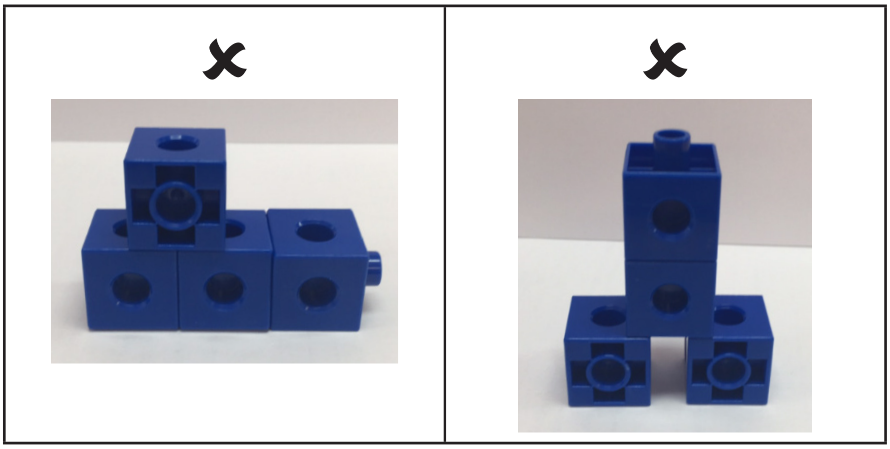
Construction with cubes

Explain to the students that they will be creating different 3D models using four interlocking cubes. The cubes must be joined by matching full faces of the cubes.



Pose the challenge: *Can you find all of the different ways four cubes can be joined together?*

Building

Students work in pairs to find some of the different possible constructions that can be made using four cubes.

Resources: Each group will need at least 20 cubes to make all the different combinations.

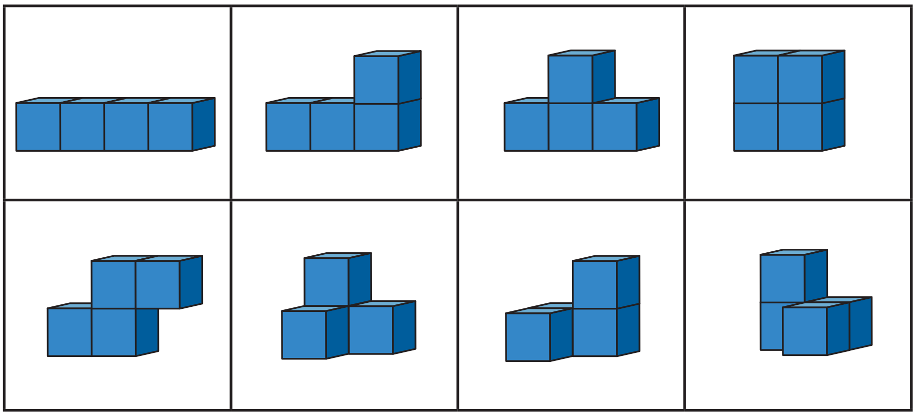
After the pairs have started collecting some different examples, ask students to consider whether some of the options that have been made are different or the same. The constructions are considered the same if they can be rotated or reflected to match the orientation of another four-cube construction.

As students work through the challenge, collect examples of the different constructions in a common area (e.g. on a table). At different times during the lesson, gather the students around to look at the number of constructions discovered and investigate whether any can be rotated or reflected to match another construction.

Comparing and drawing

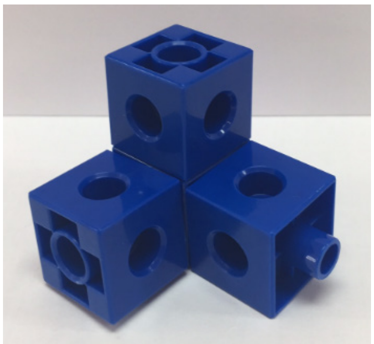
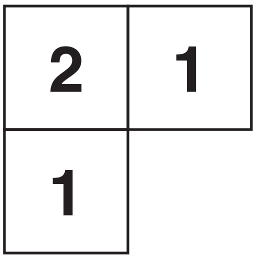
Gather students around the area where the different constructions have been collected. As a class, come to a consensus that each design is unique. Any duplicates can be eliminated.

There are eight unique designs in all. The final two designs in the illustration are the most difficult to recognise as different. They cannot be rotated to create identical shapes.

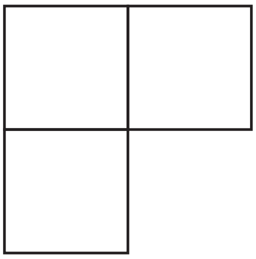
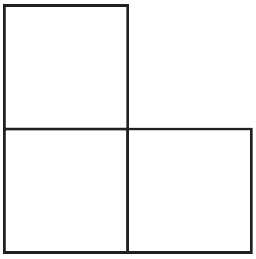
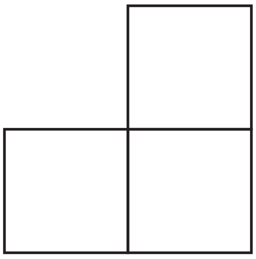


Ask the students to draw 2D representations for all the different four-cube designs. There are three ways they can record the designs:

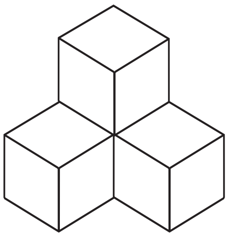
1. Record the top view of each model and the number of blocks in each level of the construction.

	The top view of this design is: 

2. The top, side and front views of each model can be drawn as squares.

	The top view of this design is: 
The front view of this design is: 	The side view of this design is: 

3. The front view of each model can be drawn on isometric grid paper or dot paper.

	The front view of this design is: 

Computer software can also be used for drawing plans:

- A simple cube can be made in Microsoft Word. Multiple cubes can be joined together to draw diagrams of the models.
- SketchUp is an excellent resource for 3D modelling.

Further activities

Activity 1

Have students create a non-symmetrical model using five or more cubes. Ask them to create a vertical reflection of their model. They must then show that the two models are different and that one model cannot be rotated to perfectly match the other model.

Activity 2

Create a set of cards showing the drawings of the different perspectives of the eight constructions. Each card should have just one image on it. Shuffle the cards and have the students match the cards to the different models.