

# Introduction to Bar Models

## Lesson 8: Comparison Model with Multiplication

### Australian Curriculum: Mathematics (Years 4 and 5)

**ACMNA076:** Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder (Year 4).

**ACMNA082:** Solve word problems by using number sentences involving multiplication or division where there is no remainder (Year 4).

**ACMNA100:** Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies (Year 5).

### Lesson abstract

Through this lesson, students learn how the comparison bar model can display the mathematical relationships in problems involving multiplication and division. The calculations only involve whole numbers. Examples demonstrate bar models for problems solved by multiplying and problems of two visually different types solved by division. Students then complete tasks, individually or in groups, to consolidate and extend their learning.

### Mathematical purpose (for students)

The comparison bar model helps to organise information in problems involving multiplication and division.

### Mathematical purpose (for teachers)

In earlier lessons of this unit, the comparison bar model was used for ‘additive’ comparison. This lesson demonstrates its use with multiplicative comparison (which involves both multiplication and division). ‘Additive comparison’ compares absolute size e.g. an elephant is about *2m taller than* a cow. Multiplicative comparison compares relative size e.g. an elephant is about *3 times as tall as* a cow. There are several grammatical forms to express multiplicative comparison, and these are included across the example set. Division of the appropriate bar into equal sized ‘units’ is one of the key steps for students to master. Problems involving division present in two different ways. In situations of quotient division, the number of parts in which to divide a bar is unknown until the division is carried out, so the bar model is initially only a sketch of what might happen.

The first examples are deliberately straightforward, to describe how to build and use the comparison bar model. Later tasks are more challenging, involving more pieces of information, more complex relationships and varied language.

Lesson Length                      60 minutes approximately

#### Vocabulary Encountered

- times as many as
- times more than
- twice as many
- one third as much as

#### Lesson Materials

- Slideshow *ST4\_BarModelIntro\_8a\_CompMult.pptx*
- [Student Sheet 1 - Bar Model Examples 8A](#) (1 per student)
- [Student Sheet 2 - Bar Model Examples 8B](#) (1 per student)
- Calculators as required

We value your feedback after these lessons via <https://www.surveymonkey.com/r/G6VGPZ8>



# Introducing the Multiplicative Comparison Model

The following examples introduce the comparison model involving multiplication and division. Whether the operation required is multiplication or division depends only on what is known and unknown in the question, not on the underlying mathematical structure.

The examples are contained in the animated slideshow *ST4\_BarModelIntro\_8a\_CompMult.pptx*, which can be used during initial instruction and class discussion. The slideshow demonstrates how the models are built.

Hand out [Student Sheet 1 - Bar Model Examples 8A](#). Students should write the solutions to these examples, for future reference.

Example 1 shows a situation requiring multiplication, Examples 2 and 3 show different types of division situations.

## Example 1

Read Example 1 and discuss the meaning of the question - that it is a multiplicative comparison, not a comparison of the absolute numbers (which would just give the answer 204). Show to draw and label the bar model, using the slideshow.

A farmer harvested 200 plums. He then harvested 4 times as many apricots as plums. How many apricots did the farmer harvest?

### Sample Solution



$$200 \times 4 = 800$$

**The farmer harvested 800 apricots.**

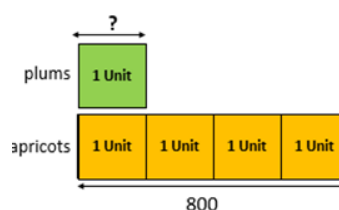
## Example 2

This example uses the same situation as Example 1, but finding the unknown requires division. The bar model looks the same. Labelling the unit helps to write the mathematical relationships.

A farmer harvested 800 apricots. He harvested 4 times as many apricots as plums. How many plums did the farmer harvest?

### Sample Solution

When students learn algebra, they will use a variable (e.g.  $x$ ) instead of "1 unit".



$$\begin{aligned} 4 \text{ units} &= 800 \text{ pieces of fruit} \\ 1 \text{ unit} &= 800 \div 4 \\ &= 200 \text{ pieces of fruit} \end{aligned}$$

**The farmer harvested 200 plums.**

### Example 3

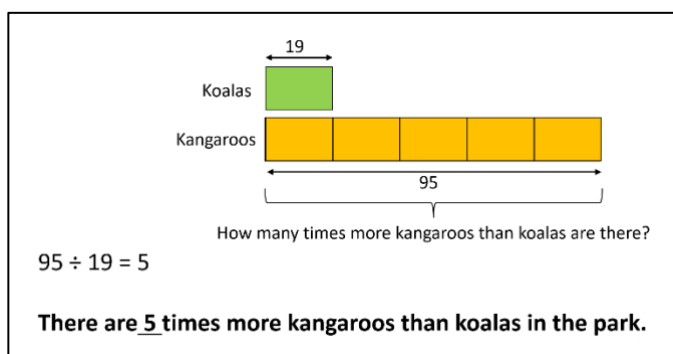
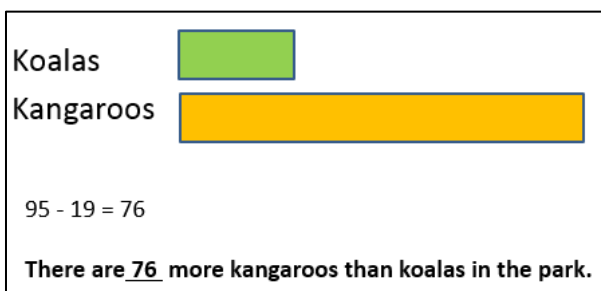
This example serves two functions. Parts a and b are included to contrast the additive comparison of “how many more” with the multiplicative comparison of “how many time more” or “how many times as many as” that is treated in this lesson.

The example also shows a different type of division to Example 2. In Example 2, the number of parts were known, so the bar could be divided into 4 and a good model drawn. In Example 3, the size of the part (19) is known, but the number of parts is to be found. This means that the “final” bar model cannot be drawn until the division has been done.

A park has 95 kangaroos and 19 koalas.

- How many more kangaroos than koalas are there in the park?
- How many times more kangaroos than koalas are there in the park?

#### Sample Solution



### Example 4

This example requires more bars in the model and uses new language to describe the comparison. Demonstrate to students how to check the answer: writing the numbers on the model, and checking against all parts of the question.

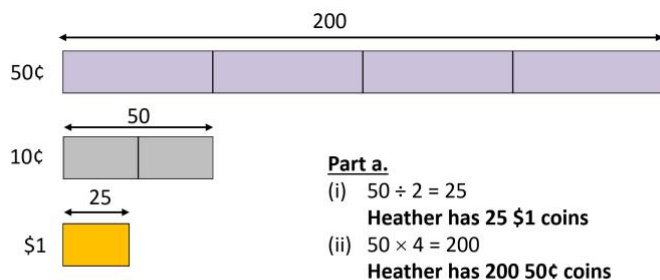
Heather collects coins. She has fifty 10¢ coins.

She has half as many \$1 coins as 10¢ coins.

She has four times as many 50¢ coins as 10¢ coins.

- How many coins of each denomination does Heather have?
- How much money does Heather have?

#### Sample Solution



#### **Part b.**

$$\text{Amount of money} = 0.50 \times 200 + 0.10 \times 50 + 1.00 \times 25$$

$$= \$130.00$$

**Heather has \$130 in coins**

# Consolidating and Concluding

## Further Practice

Students work through selected tasks from [Student Sheet 2 - Bar Model Examples 8B](#) either individually, in pairs or in groups.

Note that the problems in the tasks use larger numbers and have more layers of comparisons than those in the examples, which will require students to use more complex strategies.

Students who struggle with the computation should use calculators, so that they can pay attention to the structure of the bar models.

Discuss solutions as time permits. Worked solutions are provided in [Teacher Sheet - Bar Model Solutions 8B](#). Solutions to Tasks 1, 2 and 3 are included in the slide show *ST4\_BarModelIntro\_8a\_CompMult.pptx*

## Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The comparison model can be used to organise information in problems involving multiplication and division.
- Representing the parts which have equivalent values as “units” can be help us write mathematical sentences.
- Sometimes we have to calculate (by division) how many parts to divide a bar into.
- The bar model is only intended to be a sketch. The lengths do not have to be measured.

Draw bar models to represent the situations below and use them to solve the problems.

### Example 1

A farmer harvested 200 plums. He then harvested 4 times as many apricots as plums. How many apricots did the farmer harvest?

### Example 2

A farmer harvested 800 apricots. He harvested 4 times as many apricots as plums. How many plums did the farmer harvest?

### Example 3

A park has 95 kangaroos and 19 koalas.

- How many times more kangaroos than koalas are there in the park?
- How many times more kangaroos than koalas are there in the park?

## Task 1

Jolly's farm has 215 sheep.

Megan's farm has 3 times more sheep than Jolly's farm.

There are 4 times more cows than sheep on Jolly's Farm.

There are 2 times more cows than sheep on Megan's farm.

- How many cows are there on Jolly's farm?
- How many sheep are there on Megan's farm?
- How many cows are there on Megan's farm?

## Task 2

A gardener harvested 120 cherries.

She harvested 4 times more cherries than oranges.

The number of oranges harvested was 3 times more than the number of mangoes.

- How many oranges did the gardener harvest?
- How many mangoes did the gardener harvest?
- How many times more cherries than mangoes did the gardener harvest?

## Task 3

A supermarket sold 90 packets of chocolates. It sold:

- 3 times more packets of chocolates than cookies
  - 2 times more packets of cookies than candy sticks
  - 3 times more packets of candy sticks than nougats.
- How many packets of candy sticks were sold by the supermarket?
  - How many packets of nougats were sold by the supermarket?
  - How many times more packets of cookies than nougats were sold by the supermarket?
  - How many times more packets of chocolate than candy sticks were sold by the supermarket?

## Task 4

Jane's garden has 408 apples.

Molly's garden has twice as many apples as Jane's garden.

There are 4 times more kiwis than apples in Jane's garden.

There are 3 times more kiwis than apples in Molly's garden.

- How many kiwis are there in Jane's garden?
- How many apples are there in Molly's garden?
- How many kiwis are there in Molly's garden?

## Task 5

A supermarket sold 432 packets of chocolates. It sold

- twice as many packets of chocolates as cookies,
  - 3 times as many packets of cookies as lollipops
  - 3 times as many packets of lollipops as chewing gum.
- How many packets of lollipops were sold by the supermarket?
  - How many packets of chewing gum were sold?
  - How many times as many packets of cookies as chewing gum were sold?
  - How many times as many packets of chocolate as lollipops were sold?

## Task 1

Jolly's farm has 215 sheep.

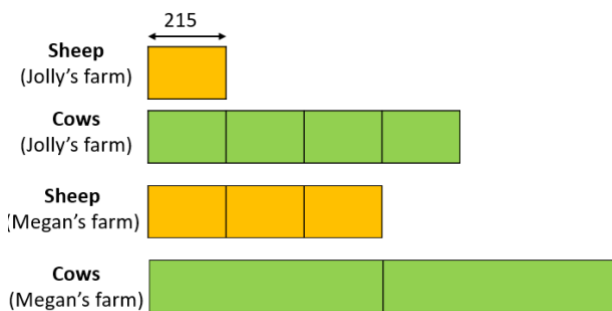
Megan's farm has 3 times more sheep than Jolly's farm.

There are 4 times more cows than sheep on Jolly's Farm.

There are 2 times more cows than sheep on Megan's farm.

- How many cows are there on Jolly's farm?
- How many sheep are there on Megan's farm?
- How many cows are there on Megan's farm?

## Solution



- a. How many cows are there on Jolly's farm?

$$215 \times 4 = 860$$

There are 860 cows on Jolly's farm.

- b. How many sheep are there on Megan's farm?

$$215 \times 3 = 645$$

There are 645 sheep on Megan's farm.

- c. How many cows are there on Megan's farm?

$$645 \times 2 = 1290$$

There are 1290 cows on Megan's farm.

## Task 2

A gardener harvested 120 cherries.

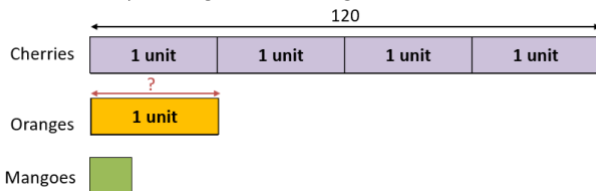
She harvested 4 times more cherries than oranges.

The number of oranges harvested was 3 times more than the number of mangoes.

- How many oranges did the gardener harvest?
- How many mangoes did the gardener harvest?
- How many times more cherries than mangoes did the gardener harvest?

### Solution

- a. How many oranges did the gardener harvest?

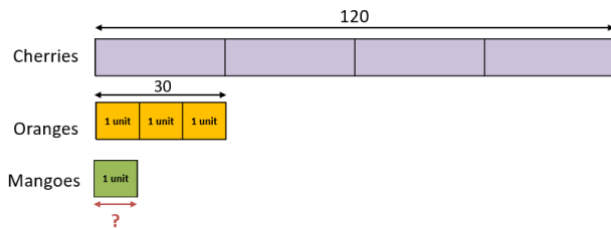


4 units = 120 pieces of fruit

1 unit =  $120 \div 4 = 30$  pieces of fruit

**The gardener harvested 30 oranges.**

- b. How many mangoes did the gardener harvest?

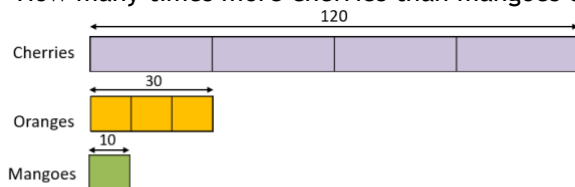


3 units = 30 pieces of fruit

1 unit =  $30 \div 3 = 10$  pieces of fruit

**The gardener harvested 10 Mangoes.**

- c. How many times more cherries than mangoes did the gardener harvest?



$120 \div 10 = 12$

**The gardener harvested 12 times more cherries than mangoes.**



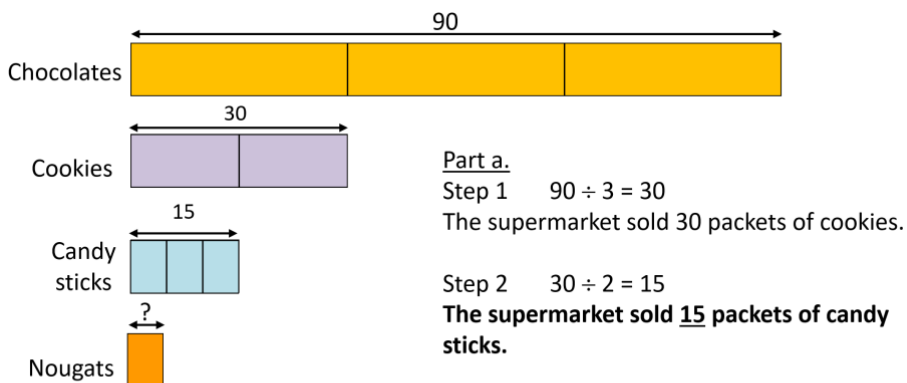
### Task 3

A supermarket sold 90 packets of chocolates. It sold:

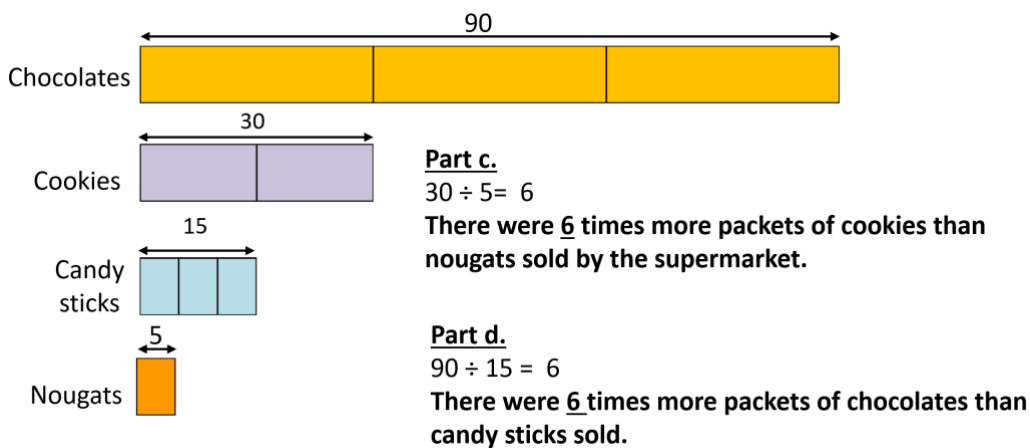
- 3 times more packets of chocolates than cookies
- 2 times more packets of cookies than candy sticks
- 3 times more packets of candy sticks than nougats.

- a. How many packets of candy sticks were sold by the supermarket?
- b. How many packets of nougats were sold by the supermarket?
- c. How many times more packets of cookies than nougats were sold by the supermarket?
- d. How many times more packets of chocolate than candy sticks were sold by the supermarket?

### Solution



**Part b.**  
 $15 \div 3 = 5$   
**The supermarket sold 5 packets of nougats.**



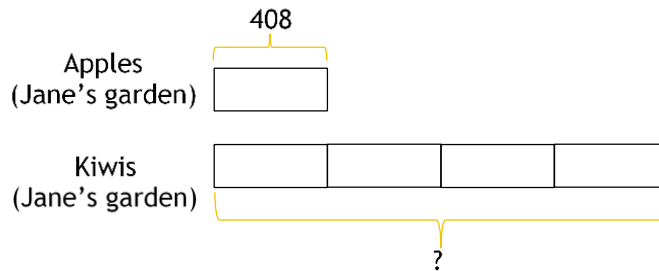
#### Task 4

Jane's garden has 408 apples. Molly's garden has twice as many apples as Jane's garden. There are 4 times more kiwis than apples in Jane's garden. There are 3 times more kiwis than apples in Molly's garden.

- How many kiwis are there in Jane's garden?
- How many apples are there in Molly's garden?
- How many kiwis are there in Molly's garden?

#### Solution

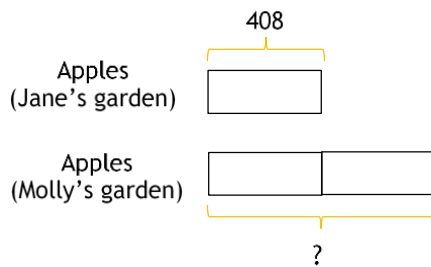
- How many kiwis are there in Jane's garden?



$$408 \times 4 = 1632$$

There are 1632 kiwis in Jane's garden.

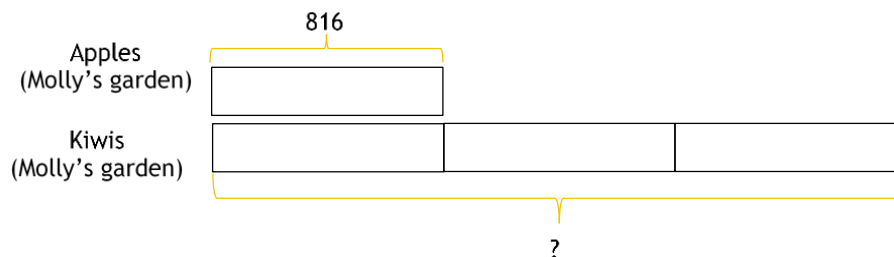
- How many apples are there in Molly's garden?



$$408 \times 2 = 816$$

There are 816 apples in Molly's garden.

- How many kiwis are there in Molly's garden?



$$816 \times 3 = 2448$$

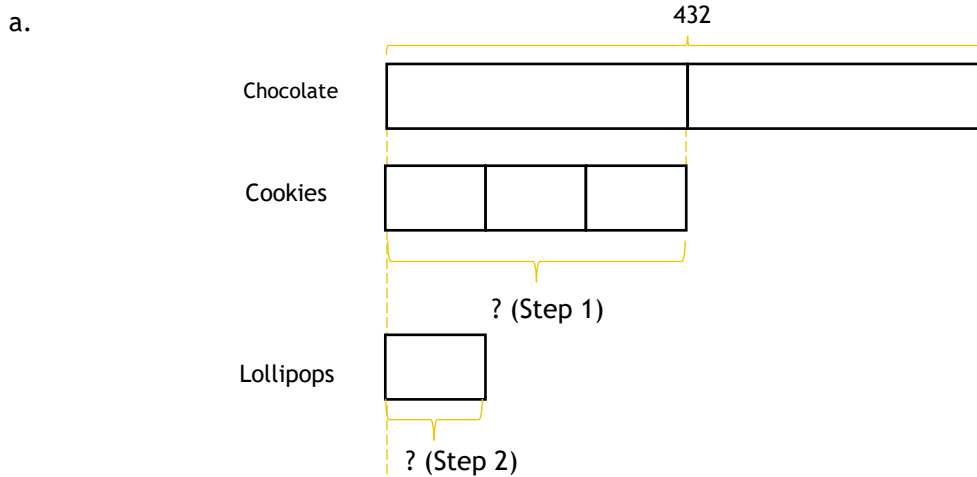
There are 2448 kiwis in Molly's garden.

## Task 5

A supermarket sold 432 packets of chocolates. It sold 2 times more packets of chocolates than cookies. 3 times more packets of cookies than lollipops were sold by the supermarket. It sold 3 times more packets of lollipops than chewing gum.

- How many packets of lollipops were sold by the supermarket?
- How many packets of chewing gum were sold by the supermarket?
- How many times more packets of cookies than chewing gum were sold by the supermarket?
- How many times more packets of chocolate than lollipops were sold by the supermarket?

### Solution

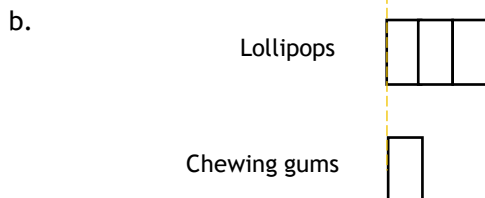


$$432 \div 2 = 216$$

The supermarket sold 216 packets of cookies.

$$216 \div 3 = 72$$

The supermarket sold 72 packets of lollipops.



$$72 \div 3 = 24$$

The supermarket sold 24 packets of chewing gum.

- $216 \div 24 = 9$   
There were 9 times as many cookies as chewing gum.

- $432 \div 72 = 6$   
There were 6 times as many chocolates as lollipops.