

# Introduction to Bar Models

## Lesson 6: Comparison Model for Addition

### Australian Curriculum: Mathematics (Years 2 to 6)

**ACMNA077:** Investigate equivalent fractions used in contexts (Year 4).

**ACMNA103:** Investigate strategies to solve problems involving addition of fractions with the same denominator.

**ACMNA291:** Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (Year 5).

**ACMNA127:** Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (Year 6).

### Lesson abstract

Students are introduced to a new type of bar model, the comparison model, through two very simple examples. Problems, for individual or group solving, involve quantities with known differences (e.g. one quantity is 10 more or less than another; one quantity is one fifth larger than another). Drawing the bar model supports an intuitive approach to the fraction calculations involved.

### Mathematical purpose (for students)

Bar models can represent relationships between quantities that are being compared.

### Mathematical purpose (for teachers)

This lesson introduces students to the comparison bar model, that is drawn by vertically aligning the bars for two quantities to highlight the difference between them. After a very easy start, later examples are more challenging by having more information in the problems, more relationships and larger numbers. Some problems are only with whole numbers, other involve fractions (for example one quantity being a fifth less than another). Students are required to find fractional parts of the whole as a solution step in some of the problems. This is supported by the bar models. In this way, the lesson can assist in strengthening the development of fractional concepts and skills. The connections made between real situations, diagrams, and symbolic statements strengthen mathematical understanding.

Lesson Length      60 minutes approximately

#### Vocabulary Encountered

- Comparison Model

#### Lesson Materials

- Slide show *ST4\_BarModelIntro\_6a\_CompAdd.pptx*
- [Student Sheet 1 - Bar Model Examples 6A](#) (1 per student)
- [Student Sheet 2 - Bar Model Examples 6B](#) (1 per student)
- Calculators as required

We value your feedback after these lessons via <<https://www.surveymonkey.com/r/G6VGPZ8>>



# Introducing the Comparison Model

This lesson introduces the comparison bar model in simple contexts involving addition of whole numbers. Throughout the examples, teachers should emphasise how the comparison bar model helps students to visualise the differences and relationships between quantities.

The simplest comparison bar model is shown in Example 1: two bars, vertically aligned at one end, to highlight the difference between their lengths. The first two examples are deliberately very simple, to focus attention on the comparison model. Later examples and problems provide more challenge.

The five examples and three of the consolidation problems are contained in the slide show *ST4\_BarModelIntro\_6a\_CompAdd.pptx* which can be used during initial instruction and class discussion.

Hand out [Student Sheet 1 - Bar Model Examples 6A](#). Students should write the solutions to these examples, for future reference.

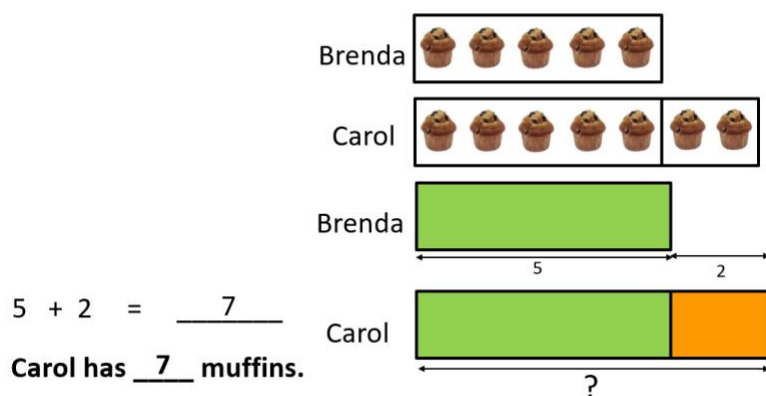
## Example 1

Read Example 1 with the class and discuss how to draw and label the model. This is a quick, simple introduction to the comparison model.

Use the slide show to demonstrate the model being built.

Brenda has 5 muffins.  
Carol has 2 muffins more than Brenda.  
How many muffins does Carol have?

### Sample Solution



## Example 2

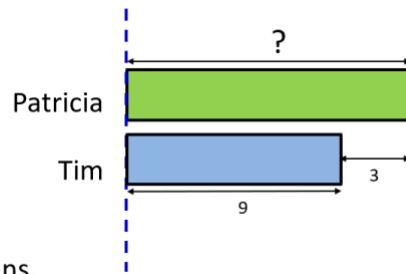
Students could try to draw the relevant bar model for themselves before the teacher discusses the solution as a group.

Tim has 9 muffins.  
Tim has 3 muffins fewer than Patricia.  
How many muffins does Patricia have?

## Sample Solution

$$9 + 3 = \underline{12}$$

Patricia has 12 muffins.



## Example 3

This example is more challenging because it involves larger numbers and the comparison is expressed as a fraction. Using the slide show as a guide.

Read Example 3 with the class and discuss how to draw and label the bar model.

There are 225 Poll Dorset sheep on a farm.

The number of Merino sheep on the farm is one fifth more than the number of Poll Dorset sheep.

a. How many Merino sheep are there on the farm?

b. What is the total number of Poll Dorset and Merino sheep on the farm?

## Sample Solution

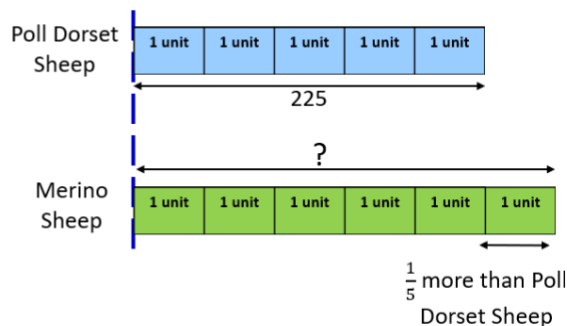
Part a.

$$5 \text{ units} = 225$$

$$1 \text{ unit} = 225 \div 5 = 45$$

$$6 \text{ units} = 6 \times 45 \text{ or } 225 + 45 = 270$$

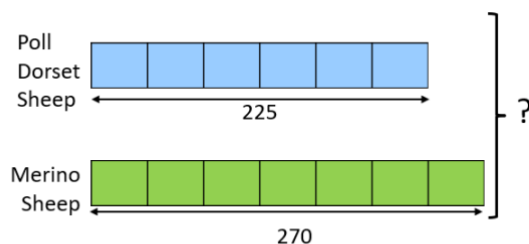
**There are 270 Merino sheep in the farm.**



Part b.

$$225 + 270 = 495$$

**There are 495 Poll Dorset and Merino sheep in the farm.**



## Enabling Prompts

Ask students to consider the following points:

- How many types of sheep are mentioned in the problem? (ANS: 2 types).
- Do we know whether there is more of one type of sheep than the other? (ANS: yes, there are more Merino sheep than Poll Dorset sheep on the farm).
- How would this information translate to the comparison bar model?
  - draw the first bar for the number of Poll Dorset sheep because we know the exact number
  - sub-divide the bar for Poll Dorset sheep into 5 equal units, because we are looking at one fifth extra of Merino sheep
  - define 1 unit as being one fifth of the Poll Dorset sheep
  - the next bar for Merino sheep should be 6 units long because it contains 1 more unit than the first bar for Poll Dorset sheep
  - make sure the bars are aligned at the left
  - mark out the difference between the two bars as 1 unit
- How can we find the number of Merino sheep? (Ans: 5 units are known, so 1 unit is known, so 6 units are known.)

## Extending Prompts

- Could we use the comparison model on fractions as well? (ANS: yes, but the difference must be marked out in terms of fractional parts)
- Can we combine the use of comparison and part-whole models of earlier lessons in a problem? (ANS: yes. We have done that in this problem)

## Example 4

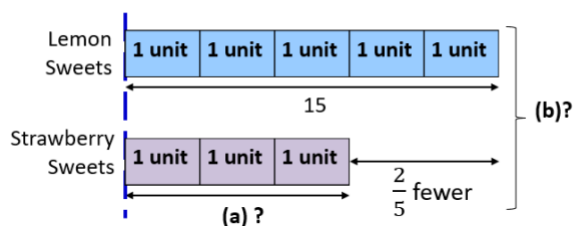
Read Example 4 with the class and discuss how to draw and label the bar model.

Jennifer has 15 lemon sweets.

She has  $\frac{2}{5}$  fewer strawberry sweets than lemon sweets.

- How many strawberry sweets does Jennifer have?
- How many lemon sweets and strawberry sweets does Jennifer have in total?

## Sample Solution



$$5 \text{ units} = 15$$

$$1 \text{ unit} = 15 \div 5 = 3$$

$$3 \text{ units} = 3 \times 3 = 9$$

**a. Jennifer has 9 strawberry sweets.**

$$15 + 9 = 24$$

**b. Jennifer has 24 lemon sweets and strawberry sweets in total.**

## Enabling Prompts

Ask students to consider the following:

- What sweets are mentioned in the problem, and which is more? (ANS: lemon & strawberry; more lemon)
- Mark all the information you know on the two bars that you draw. Align them to show the difference easily.

# Consolidating and Concluding

## Further Practice

Hand out [Student Sheet 2 - Bar Model Examples 6B](#). Students work through selected tasks either individually, in pairs or in groups.

Note that some of the problems in the tasks use larger numbers or have a greater number of values to compare than those in the examples. Calculators might help keep the focus on the bar models.

Discuss solutions as time permits. Worked solutions are provided in [Teacher Sheet - Bar Model Solutions 6B](#). Solutions to Tasks 1-3 are included in the slide show.

## Conclusion

Summarise the learning points for the lesson:

- The comparison model is used to highlight the difference between quantities.
- At least two quantities are compared when we use the comparison model.
- Sometimes we use a comparison model 'nested' into a part-whole model, drawn in a 'vertical' arrangement.
- When drawing a comparison model, be careful to align the bars, often drawing a dotted line to show equal amounts on different bars.
- Sometimes it is helpful to define equal parts in a divided bar as 'units', as this helps to organise the calculations.

**Example 1**

Brenda has 5 muffins.

Carol has 2 muffins more than Brenda.

How many muffins does Carol have?

**Example 2**

Tim has 9 muffins.

Tim has 3 muffins fewer than Patricia.

How many muffins does Patricia have?

**Example 3**

There are 225 Poll Dorset sheep on a farm.

The number of Merino sheep on the farm is one fifth more than the number of Poll Dorset sheep.

- a. How many Merino sheep are there on the farm?
- b. What is the total number of Poll Dorset and Merino sheep on the farm?

**Example 4**

Jennifer has 15 lemon sweets.

She has  $\frac{2}{5}$  fewer strawberry sweets than lemon sweets.

- a. How many strawberry sweets does Jennifer have?
- b. How many lemon sweets and strawberry sweets does Jennifer have in total?

Draw bar models to represent the situations below and use them to solve the problems.

### Task 1

Brenda, Carol and Abby are preparing to give a party.  
Brenda baked 12 muffins.  
Carol bought 8 muffins more than Brenda had baked.  
Abby made 4 muffins more than Carol bought.

- How many muffins did Carol bring?
- How many muffins did Abby bring?
- How many muffins do Brenda, Carol and Abby have in total for the party?

### Task 2

Tim, Patricia and Robin were three artists who held a joint exhibition in an art gallery.  
Tim contributed 10 paintings to the exhibition.  
Tim contributed 4 paintings fewer than Patricia.  
Robin contributed 6 more paintings than Patricia.

- How many paintings did Patricia contribute?
- How many paintings did Robin contribute?
- How many paintings did the three artists contribute in total?

### Task 3

A farm has Poll Dorset, Merino and Suffolk sheep.  
There are 300 Poll Dorset sheep on the farm.  
There are  $\frac{3}{10}$  fewer Suffolk sheep than Poll Dorset sheep on the farm.  
There are  $\frac{1}{3}$  more Merino sheep than Suffolk sheep on the farm.

- How many Suffolk sheep are there on the farm?
- How many Merino sheep are there on the farm?
- What is the total number of Poll Dorset, Merino and Suffolk sheep on the farm?

#### Task 4

Ben, Carl and Albert are preparing for a picnic.

Ben bought 10 buns.

Carl bought 4 buns more than Ben.

Albert bought 8 buns more than Carl.

- a. How many buns did Carl buy?
- b. How many buns did Albert buy?
- c. How many buns did Ben, Carl and Albert buy in total?

#### Task 5

There are 2968 chickens on Farm A.

There are  $\frac{1}{2}$  as many **more** chickens on Farm B than Farm A.

- a. How many chickens are there on Farm B?
- b. What is the total number of chickens on the two farms?



## Task 1

Brenda, Carol and Abby are preparing to give a party.

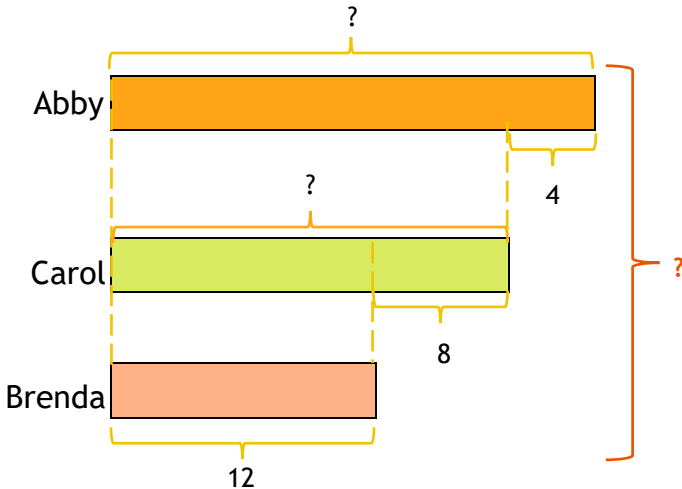
Brenda baked 12 muffins.

Carol bought 8 muffins more than Brenda had baked.

Abby made 4 muffins more than Carol bought.

- How many muffins did Carol bring?
- How many muffins did Abby bring?
- How many muffins do Brenda, Carol and Abby have in total for the party?

- An animated solution for this task is included in the slide show *ST4\_BarModelIntro\_6a\_CompAdd.pptx*

<p>(a)</p> $12 + 8 = 20$ <p>Carol brought <u>20</u> muffins.</p>	
<p>(b)</p> $20 + 4 = 24$ <p>Abby brought <u>24</u> muffins.</p>	
<p>(c)</p> $12 + 20 + 24 = 56$ <p>Brenda, Carol and Abby have <u>56</u> muffins in total.</p>	

Note that the 3 bars are aligned and roughly proportionally drawn.

It is important to mark the difference in quantity (i.e. 4 and 8) in the bars.

This an example of a comparison model nested within a part-whole model (vertical arrangement).

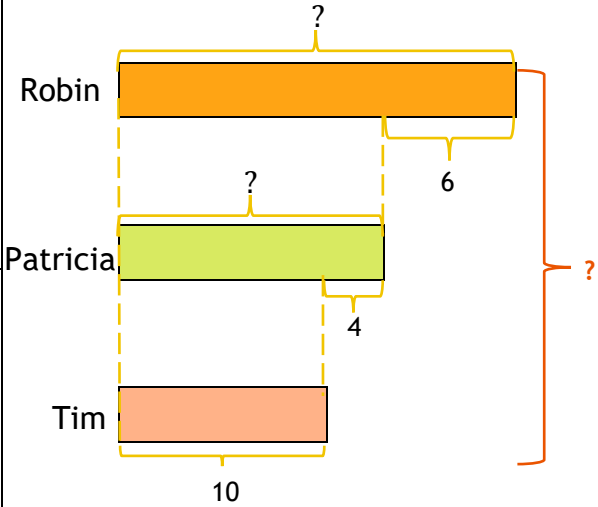
Students can label the bar model with the answers for parts a and b before they work on part c.

## Task 2

Tim, Patricia and Robin were three artists who held a joint exhibition in an art gallery.  
 Tim contributed 10 paintings to the exhibition.  
 Tim contributed 4 paintings fewer than Patricia.  
 Robin contributed 6 more paintings than Patricia.

- How many paintings did Patricia contribute?
- How many paintings did Robin contribute?
- How many paintings did the three artists contribute in total?

An animated solution for this task is included in the Slide show *ST4\_BarModelIntro\_6a\_CompAdd.pptx*

<p>(a)</p> $10 + 4 = 14$ <p>Patricia contributed <u>14</u> paintings to the exhibition.</p>	
<p>(b)</p> $14 + 6 = 20$ <p>Robin contributed <u>20</u> paintings to the exhibition.</p>	
<p>(c)</p> $10 + 14 + 20 = 44$ <p>The three artists contributed <u>44</u> paintings to the exhibition in total.</p>	

This is another example of a comparison model nested within part-whole model (vertical arrangement).

Here, the arrangements of the bars are important.

The students have to identify which artist contributed the most paintings and then arrange the bars representing the paintings from each artist in ascending or descending order for ease of comparison.

The differences between bars be labelled.

Students can label the bar model with the answers for parts a. and b. before they work on part c.

### Task 3

A farm has Poll Dorset, Merino and Suffolk sheep.

There are 300 Poll Dorset sheep on the farm.

There are  $\frac{3}{10}$  fewer Suffolk sheep than Poll Dorset sheep on the farm.

There are  $\frac{1}{3}$  more Merino sheep than Suffolk sheep on the farm.

- How many Suffolk sheep are there on the farm?
- How many Merino sheep are there on the farm?
- What is the total number of Poll Dorset, Merino and Suffolk sheep on the farm?

An animated solution for this task is included in the slide show *ST4\_BarModelIntro\_6a\_CompAdd.pptx*

<p>(a)</p> <p>10 units = 300</p> <p>1 unit = 30</p> <p>7 units = 210</p> <p>There are <u>210</u> Suffolk sheep in the farm.</p> <p>(b)</p> <p>3 units = 210</p> <p>1 unit = 70</p> <p>4 units = 280</p> <p>(or <math>210 + 70 = 280</math>)</p> <p>There are <u>280</u> Merino sheep in the farm.</p> <p>(c)</p> <p><math>300 + 210 + 280</math></p> <p><math>= 790</math></p> <p>There are <u>790</u> Poll Dorset, Merino and Suffolk sheep in the farm.</p>	<p>The diagram shows four horizontal bars representing different sheep breeds. The top bar, labeled 'Poll Dorset Sheep', is yellow and divided into 10 equal units, with a bracket below it labeled '300'. The second bar, labeled 'Suffolk Sheep', is green and divided into 7 equal units, with a bracket below it labeled '?'. A bracket to the right of this bar indicates it is '<math>\frac{3}{10}</math> fewer than Poll Dorset Sheep'. The third bar, also labeled 'Suffolk Sheep', is orange and divided into 3 equal units, with a bracket below it labeled '?'. A bracket to the right of this bar indicates it is '<math>\frac{1}{3}</math> more than Suffolk Sheep'. The bottom bar, labeled 'Merino Sheep', is purple and divided into 4 equal units, with a bracket below it labeled '?'. A large bracket on the right side of the diagram groups the Suffolk and Merino bars together, with a question mark '?' next to it.</p> <p>Students need to first identify that there are more Poll Dorset sheep than the other breeds, and there are fewer Suffolk sheep than any of the other breeds. Next, students draw the bar representing the number of Poll Dorset sheep, divide into 10 equal “units”, then draw the bar representing the number of Suffolk sheep.</p> <p>Then the ratio of the lengths of bars for Suffolk sheep to Merino sheep is 3 : 4.</p> <p>When they draw the bar for Merino sheep students will probably be uncertain whether it is longer or shorter than the bar for Poll Dorset sheep. They should go ahead with sketching the bar model anyway. Just make a decision - longer or shorter does not matter.</p> <p>Bar models in these lesson plans have been drawn in proportion, after we had solved the problems. But in live problem solving this is not the case. Occasionally, a bar model may have to be redrawn in the middle of a solution to better reflect the real situation.</p>
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## Task 4

Ben, Carl and Albert are preparing for a picnic.

Ben bought 10 buns.

Carl bought 4 buns more than Ben.

Albert bought 8 buns more than Carl.

- How many buns did Carl buy?
- How many buns did Albert buy?
- How many buns did Ben, Carl and Albert buy in all?

<p>(a)</p> $10 + 4 = 14$ <p>Carl bought <u>14</u> buns.</p>	
<p>(b)</p> $14 + 8 = 22$ <p>Albert bought <u>22</u> buns.</p>	
<p>(c)</p> $10 + 14 + 22 = 46$ <p>Ben, Carl and Albert bought <u>46</u> buns in total.</p>	

## Task 5

There are 2968 chicken on Farm A.

There are  $\frac{1}{2}$  as many **more** chickens on Farm B than Farm A.

- How many chickens are there on Farm B?
- What is the total number of chickens on the two farms?

<p>(a) 2 units = 2968 1 unit = 1484 3 units = 4452 There are <u>4452</u> chickens on Farm B.</p>	
<p>(b) <math>2968 + 4452 = 7420</math> There are <u>7420</u> chickens on the two farms.</p>	