Introduction to Bar Models
Lesson 4: Addition of Fractions

Australian Curriculum: Mathematics (Years 4 -6)

ACMNA077: Investigate equivalent fractions used in contexts (Year 4).
ACMNA103: Investigate strategies to solve problems involving addition of fractions with the same denominator (Year 5).
ACMNA126: Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6).

Lesson abstract

In this lesson, students learn how to use bar model as a tool to represent a variety of worded addition problems involving fractions, by studying worked examples and practising with further tasks. They encounter situations where the bars that make up a bar model may simultaneously represent an absolute number (e.g. $50) and a fraction of a specified quantity (quarter of a cost).

Mathematical purpose (for students)

Bar models can help us to visualise parts of a whole when adding fractions.

Mathematical purpose (for teachers)

This lesson introduces students to using bar models with fractions. In some situations, the parts of the bar model are only known as fractions of a whole (e.g. finding what fraction of a cake is eaten by 2 people) and in other situations the parts of the bar model represent the fractions (a fifth of a cake) as well as an absolute number (e.g. 150 gm).

Basic fraction calculations (e.g. finding the quantity that is a given fraction of a whole, making a common denominator) are well scaffolded by bar models, so that this lesson can strengthen developing fraction skills. The lesson provides several different addition contexts involving the use of fractions. The connection between fraction word problems and diagrammatic interpretations builds mathematical understanding of addition of fractions. Links are shown to symbols addition statements involving fractions derived from model drawing.

Lesson Length 60 minutes approximately

Vocabulary Encountered

Lesson Materials

- Slide show *ST4_BarModelIntro_4a_AddFract.pptx*
- Student Sheet 1 – Bar Model Examples 4A (1 per student)
- Student Sheet 1 – Bar Model Examples 4B (1 per student)
- Calculators as required

We value your feedback after these lessons via https://www.surveymonkey.com/r/G6VGPZ8

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Representing Fractions on Bar Models

This lesson introduces bar models for contexts involving the addition of fractions. Emphasis is placed on how the part-whole bar model helps students in visualising additive relationships between fractional quantities.

The examples are contained in the animated slide show *ST4_BarModelIntro_4a_AddFract.pptx*, which can be used during initial instruction and class discussion.

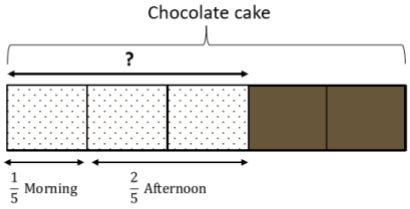
Hand out Student Sheet 1 – Bar Model Examples 4A. Students should write and draw the solutions to these examples, for future reference.

Example 1

Read Example 1 with the class and discuss how to draw and label the model. The slide show can be used as a prompt.

> Abby ate $\frac{1}{5}$ of a chocolate cake in the morning.
>
> She ate another $\frac{2}{5}$ of the same chocolate cake in the afternoon.
>
> What fraction of the chocolate cake did Abby eat in total?

Sample Solution



$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

Abby ate $\frac{3}{5}$ of the chocolate cake.

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

Abby ate $\frac{3}{5}$ of the chocolate cake.

Enabling Prompts

Ask students to consider:

- Did Abby eat the whole chocolate cake? (ANS: no)
- What fraction of the chocolate cake did Abby eat in the morning? (ANS: $\frac{1}{5}$ of a chocolate cake)
- Did Abby eat more chocolate cake later? (ANS: yes, she ate another $\frac{2}{5}$ of the same chocolate cake)
- What do we need to find? (ANS: the fraction of the chocolate cake did Abby eat in total)
- How can we draw the bar model to show the fractional parts? (ANS: the whole bar is divided in to 5 units; shade 1 unit to represent what Abby ate in the morning; shade another 2 units to show what she ate in the afternoon)
- Which are the "parts" to form the "whole" in the bar model? (ANS: the "whole" is the whole cake)

Note: All the shaded units in the bar have been shown concurrently in making this model. Highlight how the bar model can help students come up with the mathematical statements.

Extending Prompts

Ask students to consider:

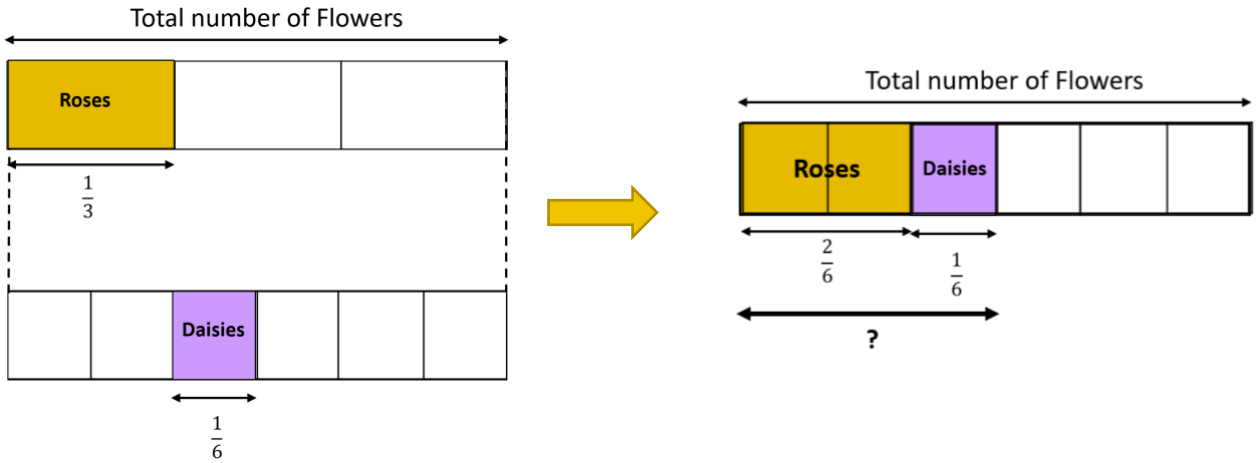
- What if Abby ate more of the same chocolate cake? How do we show this on the bar model? (ANS: shade more parts)
- What if Abby ate more than 1 whole chocolate cake? How do we show this on the bar model? (ANS: If the next chocolate cake is of the same size, we can extend the bar, but clearly mark the length that represents one whole chocolate cake. If the next cake is of a different size, we cannot do it simply.)
- What if the fraction of the chocolate cake that Abby ate subsequently is not in fifths? How do we show this on the bar? (ANS: we sub-divide the bars to show the required fractional parts – the common denominator is really helpful; see next example and the Tasks)

Example 2

Students could try this example for themselves before the teacher discusses the solution as a group. Use the slide show to see the models being built and to highlight how the bar model can help students come up with the mathematical statements.

> $\frac{1}{3}$ of the flowers in Brenda's shop are roses.
>
> Another $\frac{1}{6}$ of the flowers in her shop are daisies.
>
> What fraction of the flowers in Brenda's shop are roses and daisies combined?

Sample Solution



$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$

$= \frac{3}{6}$ or $\frac{1}{2}$

$\frac{1}{2}$ of the flowers at Brenda's shop are roses and daisies combined.

Enabling Prompts

Ask students to consider:

- What flowers are mentioned in the problem? (ANS: roses and daisies, but Brenda has others too)
- What fraction of the flowers in Brenda's shop are roses? (ANS: $\frac{1}{3}$)
- What fraction of the flowers in Brenda's shop are daisies? (ANS: $\frac{1}{6}$)
- What do we need to find? (ANS: the fraction of flowers in Brenda's shop that are roses and daisies combined)
- How do we draw the bar model to show the fractional parts? (ANS: the whole bar represents all Brenda's flowers)

- How do we know how many equal units the divide the bar into? (ANS: based on $\frac{1}{3}$ and $\frac{1}{6}$; it is good to divide the bar into 6 equal units, but other numbers e.g. 12 would do)
- How do we represent $\frac{1}{3}$ and $\frac{1}{6}$ on the same bar if the bar is now divided in to 6 equal units now? (ANS: $\frac{1}{3} = \frac{2}{6}$)

Extending Prompts

- What if we have to show a third type of flower in the bar model? How do we do this? (ANS: if the third fraction is halves, thirds or sixths, this bar will work, but otherwise we will need to divide the bar up further)

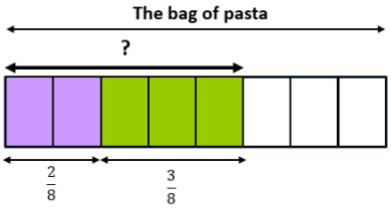
Example 3

The solution for this example in the animated slideshow demonstrates a slightly different way of drawing the bar model, where the various parts are being added on dynamically, as the word problem progress. This contrasts with the more static approach in the earlier two solutions.

Read Example 3 to the class and discusses how to draw and label it.

> Vanessa first cooked $\frac{2}{8}$ of a bag of pasta for her own lunch.
>
> She had to cook another $\frac{3}{8}$ of the same bag of pasta because her best friend joined her for lunch.
>
> What fraction of the bag of pasta did Vanessa cook?

Sample Solution



$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

Vanessa cooked $\frac{5}{8}$ of the bag of pasta.

Enabling Prompts

Ask the students to consider:

- Did Vanessa cook the whole bag of pasta? (ANS: no, she cooked $\frac{2}{8}$ bag of pasta)
- Why did Vanessa have to cook more paste? (ANS: her best friend joined her for lunch)
- How much more of the same bag of pasta did Vanessa cook for her friend? (ANS: another $\frac{3}{8}$ bag from the same bag)
- How do we represent this on the bar model? (ANS: we draw one bar to represent one whole bag of pasta, then sub-divide the bar into 8 equal units because the fraction is in eighths, then shade parts)
- What do we need to find? (ANS: fraction of the bag of pasta Vanessa cooked)

Extending Prompts

- What if more friends joined Vanessa for lunch and Vanessa had to cook more pasta from the same bag? (ANS: we shade more units in the bar; but if the fraction of the bag of pasta for more friends is not in eighths, we have to consider the units of the bar again)
- What if Vanessa cooks more than one bag?

Consolidating and Concluding

Further Practice

Hand out Student Sheet 2 – Bar Model Examples 4B. Students work through selected tasks either individually, in pairs or in groups. Discuss solutions as time permits.

When attempting the tasks, please note the following points:

- Remind children to draw part-whole bar models to accompany their working steps.
- Task 2 (a) is similar to Task 1 (c) but students must first understand that the bar representing the total sum of money has 8 equal parts, with 5 parts representing notes and the remaining 3 parts representing coins.
- Task 2 (b) is similar to Example 2 where the bar is transformed from fourths to eighths.
- Task 3 (c) requires another bar division from eighths to sixteenths.

Worked solutions are provided in Teacher Sheet - Bar Model Solutions 4B. Animated solutions to Tasks 1 and 2 are included in the slideshow (*ST4_BarModelIntro_4a_AddFract.pptx*).

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The part-whole model can involve the use of many parts (more than two parts) within the whole; some of the parts can be equal in quantity. This is important when they are fractional parts.
- To represent fractions, the bar is divided into a convenient number of units. The common denominator of fractions tells us how many.
- In the examples discussed, we used addition to find the unknown, because we could see the additive relationship between the various parts making up the whole.

Bar Model Examples 4A

Name: _____

Example 1

Abby ate $\frac{1}{5}$ of a chocolate cake in the morning.
She ate another $\frac{2}{5}$ of the same chocolate cake in the afternoon.
What fraction of the chocolate cake did Abby eat in total?

Example 2

$\frac{1}{3}$ of the flowers in Brenda's shop are roses.
Another $\frac{1}{6}$ of the flowers in her shop are daisies.
What fraction of the flowers in Brenda's shop are roses and daisies combined?

Example 3

Vanessa first cooked $\frac{2}{8}$ bag of pasta for her own lunch.

She had to cook another $\frac{3}{8}$ bag of the same pasta because her best friend joined her for lunch.

What fraction of the bag of pasta did Vanessa cook?

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Bar Model Examples 4B

Name: _____

Draw bar models to represent the problem situations below and use them to help you solve the problems.

Task 1

$\frac{1}{2}$ of the books in library are detective stories.

Another $\frac{1}{4}$ of the books in the library are comics.

(a) What fraction of the books in the library are detective stories and/or comics?

(b) If there are 33 848 books altogether in the library, what is the total number of books that are detective stories and/or comics?

Task 2

The ticket machines at Flinders Street Train Station collected a total amount of $33 760 on Monday.

$\frac{5}{8}$ of the total amount of money collected by the machines was in notes. The rest was in coins.

(a) How much money collected by the machines was in notes?

(b) A quarter of the total amount collected by the machines was in one-dollar coins. What fraction of the total amount was made up of notes and one-dollar coins?

(c) The station master converted $2110 worth of coins into notes. What fraction of the total amount of money collected was made up of notes in the end?

Task 3

The vending machine which sells cold and hot drinks at Melbourne Central Train Station collected a total sum of $ 24 000 in a month.

One third of the total sum of money collected by the machine came from the sales of cold drinks. The rest was from the sales of hot drinks.

(a) How much money collected by the machines was from the sales of cold drinks?

(b) A quarter of the total sum was from the sales of hot coffee. What fraction of the total sum was collected from the sales of cold drinks and hot coffee?

(c) For $2000 worth of hot drinks, customers opted to have them served cold (by machine adding ice). What fraction of the money was spent on cold drinks?

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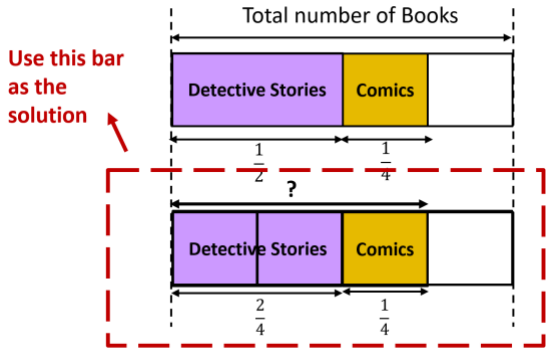
Bar Model Solutions 4B

Task 1

$\frac{1}{2}$ of the books in library are detective stories.

Another $\frac{1}{4}$ of the books in the library are comics.

(a) What fraction of the books in the library are detective stories and/or comics?

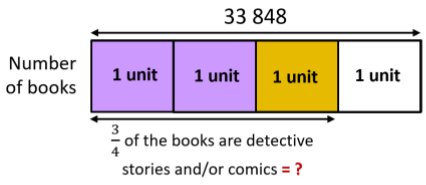


$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4}$ of the books in the library are detective stories and comics.

[Note: The Part-Whole Model can be used in an additive way. Fill in the other known information in the bar in preparation for part (b). The bar becomes the one below. There isn't a need to draw two different bars for parts (a) and (b). Just fill in additional information on the same bar as the problem solution progresses.]

(b) If there are 33 848 books altogether in the library, what is the total number of books that are detective stories and/or comics?



Call each ¼ of the total number of books 1 unit.
So…
4 units = 33 848
1 unit = 33 848 ÷ 4
= 8462
3 units = 3 x 8462
= 25 386

There are 25 386 detective stories and/or comics in the library.

[Note: The Part-Whole Model is used in a mutliplicative way for this problem]

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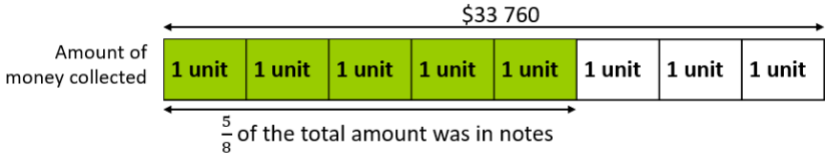
Task 2

The ticket machines at Flinders Street Train Station collected a total amount of $33 760 on Monday. $\frac{5}{8}$ of the total amount of money collected by the machines was in notes. The rest was in coins.

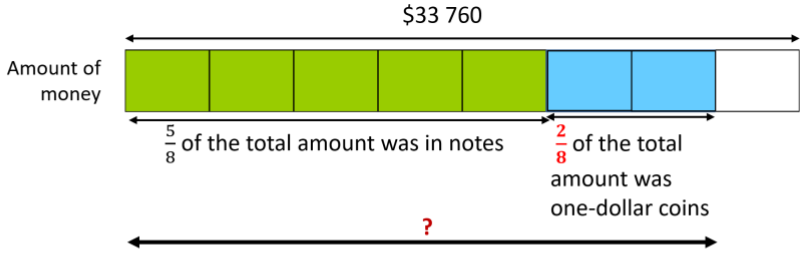
(a) How much money collected by the machines was in notes?



Call $\frac{1}{8}$ of the total amount of money collected '1 unit'.

8 units = $33 760
1 unit = $33 760 ÷ 8
= $4220
5 units = 5 x 4220
= $21 110
$21 110 was in notes.

(b) A quarter of the total amount collected by the machines was in one-dollar coins. What fraction of the total amount was made up of notes and one-dollar coins?



$\frac{5}{8} + \frac{1}{4} = \frac{5}{8} + \frac{2}{8}$

$= \frac{7}{8}$

$\frac{7}{8}$ of the total amount of money was made up of notes and one-dollar coins.

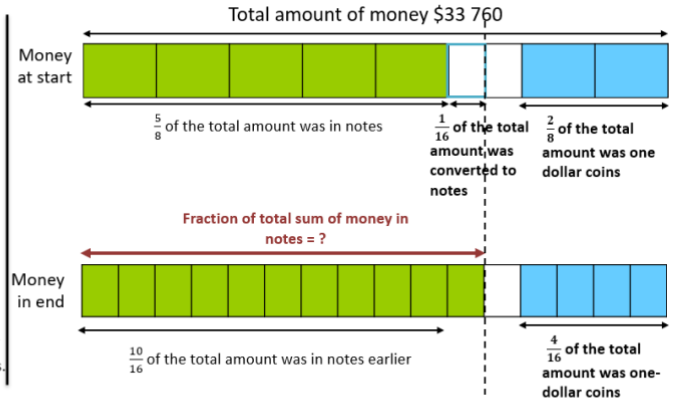
(c) The station master converted $2110 worth of coins into notes. What fraction of the total amount of money collected was made up of notes in the end?

$\frac{2110}{33\,760} = \frac{1}{16}$ of coins were converted to notes

$\frac{5}{8} + \frac{1}{16} = \frac{10}{16} + \frac{1}{16}$

$= \frac{11}{16}$

$\frac{11}{16}$ of the total amount of money was in notes.



$\frac{11}{16}$ of the total amount of money was in notes.

Alternatively,

$21 110 + $2110 = $23 220

$\frac{23220}{33760} = \frac{11}{16}$

This can be verified by the bar model.

Discuss with students how the bar model can be easily drawn because one sixteenth is half of one eighth.

Task 3

The vending machine which sells cold and hot drinks at Melbourne Central Train Station collected a total sum of $ 24 000 in a month.
One third of the total sum of money collected by the machine came from the sales of cold drinks. The rest was from the sales of hot drinks.

(a) How much money collected by the machines was from the sales of cold drinks?
(b) A quarter of the total sum was from the sales of hot coffee. What fraction of the total sum was collected from the sales of cold drinks and hot coffee?
(c) For $2000 worth of hot drinks, customers opted to have them served cold (by machine adding ice). What fraction of the money was spent on cold drinks?

(a)

3 units = $24 000
1 unit = $ 8000

$8000 was collected from the sales of cold drinks.

(b)

$$\frac{1}{3} + \frac{1}{4}$$

$$= \frac{4}{12} + \frac{3}{12}$$

$$= \frac{7}{12}$$

$\frac{7}{12}$ of the total sum was collected from the sales of cold drinks and hot coffee.

(c)

$$\frac{2000}{24\,000} = \frac{1}{12}$$

$$\frac{1}{3} + \frac{1}{12}$$

$$= \frac{4}{12} + \frac{1}{12}$$

$$= \frac{5}{12}$$

$\frac{5}{12}$ of the total sum was collected from the sales of cold drinks.