

Introduction to Bar Models

Lesson 3: Multiplication & Division

Australian Curriculum: Mathematics (Years 3 to 5)

ACMNA057: Representing and solving problems involving multiplication using efficient mental and written strategies and appropriate digital technologies (Year 3).

ACMNA076: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder (Year 4).

ACMNA082: Solve word problems by using number sentences involving multiplication or division where there is no remainder (Year 4).

ACMNA100: Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies (Year 5).

Lesson abstract

Students learn how the bar model can help represent multiplication and division contexts by studying examples and practicing with further tasks. The tasks encountered involve equal groups multiplication and partition and quotient variations of division problems. The examples use very simple numbers. Consolidation tasks have larger numbers and contain more mathematical information to sort through.

Mathematical purpose (for students)

Bar models help our thinking with multiplication and division word problems.

Mathematical purpose (for teachers)

In this lesson, students encounter three different problem solving situations involving the use of multiplicative thinking with whole numbers. There are problems of equal groups multiplication and the two simplest meanings for division. These are partition (dividing a known quantity into a known number of parts of unknown size) and quotient (dividing a known quantity into an unknown number of parts of a known size). The diagrammatic representations of these situations promotes a greater depth of mathematical understanding of multiplication and division.

Some of the examples also introduce the pre-algebra concept of partitioning the bars into 'units' of equivalent quantity, with students beginning to consider this general term to represent an unknown number.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- Part-whole Model
- Bar Model
- Unit

Lesson Materials

- Slide show *ST4_BarModelIntro_3a_MultDiv.pptx*
- [Student Sheet 1 - Bar Model Examples 3A](#) (1 per student)
- [Student Sheet 1 - Bar Model Examples 3B](#) (1 per student)
- Calculators as required

We value your feedback after these lessons via <https://www.surveymonkey.com/r/G6VGPZ8>



Bar Model For Multiplication

This section of the lesson introduces bar models for equal groups (repeated addition) multiplication. These are the simplest situations for multiplication. The examples use simple numbers to emphasise the bar model.

The examples are contained in the slide show *ST4_BarModelIntro_3a_MultDiv.pptx*, which can be used during initial instruction and class discussion.

Hand out [Student Sheet 1 - Bar Model Examples 3A](#). Students should write the solutions to these examples, for future reference.

Example 1

Read Example 1 and discusses how to draw and label the bar model. The slide show *ST4_BarModelIntro_3a_MultDiv.pptx* can be used as a prompt.

There are 3 trays of cookies. Each tray has 2 cookies. How many cookies are there?

Sample Solution

$$3 \times 2 = \underline{\quad 6 \quad}$$

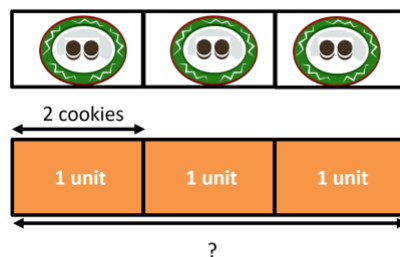
There are 6 cookies.

Using the bar model:

1 unit = 2 cookies

3 units = 3×2

= 6 cookies



There are 6 cookies in total

Prompts

Prompt students to analyse the problem:

- How do you figure out the number of cookies on the 3 trays?

Draw and label a bar model to represent this situation with the students, discussing each step as you proceed. The animated slideshow shows these steps.

- Draw a rectangle to represent 1 tray. Label "2 cookies" for each rectangle.
- Draw 3 rectangles to represent 3 trays.
- Optional step: Label each rectangle as "1 unit". Discuss the concept of partitioning the bar into "units", where there are multiple units representing the same value (in this instance, the number of cookies on each tray).
- From the bar model, how can you figure out the number of cookies on the 3 trays?
- Introduce the concept of a 'unit' to represent the number of cookies on one tray.

Check the answer:

- How do you know that you are correct? From the bar model, $2 + 2 + 2 = 6$

Bar Models For Division

This section introduces how to use a bar model for problems involving division concepts. There are two basic types of division problem Partition and Quotition. Partition problems involve giving out equal parts (e.g. sharing 12 items among 4 people and finding how much each person gets). Quotition problems involve distributing a quota of a given size (e.g. having 15 biscuits and serving some people 3 biscuits each and finding how many people can be served). The bar models look very different for these two types of division situations.

Example 2 (Partition)

Read Example 2 and discuss how to draw and label the model.

6 cookies are shared equally among 3 children. How many cookies does each child get?

Sample Solution

$$6 \div 3 = \underline{2}$$

Each child gets 2 cookies.

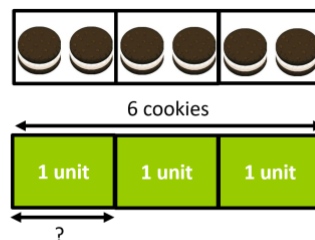
Using the bar model:

3 units = 6 cookies

1 unit = $6 \div 3$

= 2 cookies

Each child gets 2 cookies



Prompts

Prompt students to analyse the problem:

- How might you figure out the number of cookies each child gets?

Draw and label a bar model to represent this situation with the students, discussing each step as you proceed:

- Draw a rectangle to represent the number of cookies 1 child has.
- Draw 3 rectangles to represent the number of cookies each of the 3 children have. Label “6” for the 3 rectangles.
- Optional step: Label each rectangle as “1 unit”. Discuss with students the concept of partitioning the bar into “units”, where there are multiple units representing the same value (in this instance, the number of cookies each child receives).
- Ask students: From the bar model, how can you figure out the number of cookies each child gets?
- Highlight that the 3 rectangles represents the “whole” (i.e. the total number of cookies) and 1 rectangle is 1 part (the number of cookies 1 child has).

Check the answer:

- Each rectangle represents 2 cookies. From the bar model, 3 rectangles represents $2 \times 3 = 6$ cookies.

Example 3 (Quotition)

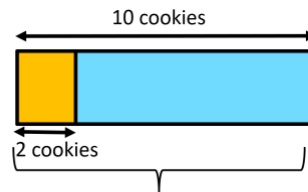
Students could try this example themselves, before discussing as a group.

There are 10 cookies. Catherine puts 2 cookies on each plate. How many plates of cookies are there?

Sample Solution

$$10 \div 2 = \underline{5}$$

There are 5 plates of cookies.



How many groups of 2
are there?

Prompts

Prompt students to analyse the problem:

- How do you figure out the number of plates of cookies?
- Let's use the bar model method.

Draw and label a bar model with the students, discussing each step with them as you proceed:

- Draw a rectangle to represent 10 cookies. Label it as "10 cookies".
- Draw a smaller bar within the bar representing all cookies, to show 2 cookies.

Ask students:

- How many groups of "2" are there in 10?
- How would you represent this in the bar model?
- From the bar model, how many plates of cookies are there?

Check the answer:

- There are 5 groups of 2. This gives $5 \times 2 = 10$ cookies.

Consolidating and Concluding

Further Practice

Students work through tasks from [Student Sheet 2 - Bar Model Examples 3B](#) either individually, in pairs or in groups.

Note that the problems in the tasks use larger numbers than those in the examples and the problems contain more pieces of information to sort through. Students can use calculators if arithmetic distracts from learning about the bar models.

Discuss solutions as time permits. Worked solutions to all problems are provided in [Teacher Sheet - Bar Model Solutions 3B](#). Solutions to Tasks 1-3 are included in the slide show *ST4_BarModelIntro_3a_MultDiv.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- Bar models can help in organising the information in a word problem involving multiplication or division.
- Labelling equal parts in the bar model as 'units' can be helpful.

Example 1

There are 3 trays of cookies. Each tray has 2 cookies. How many cookies are there?

Example 2: Sharing

6 cookies are shared equally among 3 children. How many cookies does each child get?

Example 3: Distributing Equal Amounts

There are 10 cookies. Catherine puts 2 cookies on each plate. How many plates of cookies are there?

Task 1

Keith was in charge of wrapping presents for a Christmas event. He wrapped some presents and filled 3 bags on Monday. He wrapped some more presents and filled 2 bags on Tuesday, 4 more bags on Wednesday and 2 more bags on Thursday. Each bag contained 35 presents.

- How many bags did Keith use?
- How many presents did Keith wrap?
- Keith was asked to stick 2 Christmas stickers on each present. How many stickers did Keith need?

Task 2

At a company lunch, \$3645 was spent on fish and chips. Each plate of fish and chips cost \$15.

- How many plates of fish and chips were ordered at the company lunch?
- The amount spent on fish and chips could be used to sponsor 3 people on a weekend cruise. What is the cost of the weekend cruise for each person?

Task 3

A gardener sold 2760 tulips and 2944 roses to 8 florists.

Each florist bought the same number of tulips and the same number of roses.

One of the florists, Joanne, arranged some tulips and some roses in each vase.

There were 15 tulips in each vase. Each vase also had an equal number of roses.

- How many tulips did each florist buy?
- How many vases were there?
- How many roses did Joanne put in each vase?

Task 4

A driver delivered 6000 chocolate bars and 2520 candies to 6 retail outlets. Each outlet received the same number of chocolate bars and the same number of candies.

At each retail outlet, the sales assistant packed some chocolate bars and candies into some bags. There were 50 chocolate bars in each bag. Each bag also had an equal number of candies.

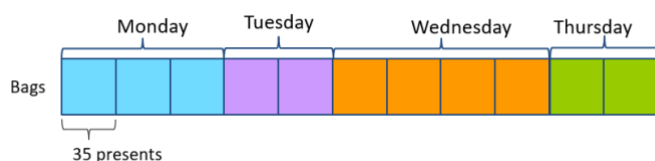
How many candies did the sales assistant pack into each bag?

Task 1

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- How many bags did Keith use?
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Solution (Equal groups multiplication)



- $3 + 2 + 4 + 2 = 11$
 Keith used 11 bags.
- $1 \text{ bag} = 35 \text{ presents}$
 $11 \text{ bags} = 35 \times 11 = 385 \text{ presents}$
 Keith wrapped 385 presents.
- $385 \times 2 = 770$
 Keith needed 770 stickers.

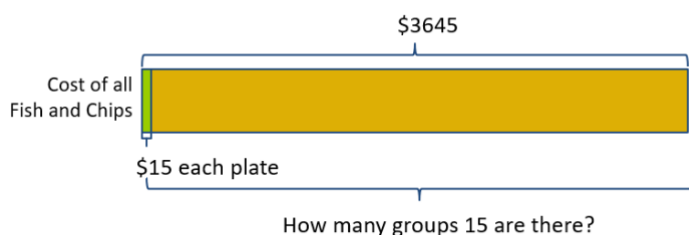
Task 2 (Quotition division first, then partition)

At a company lunch, \$3645 was spent on fish and chips. Each plate of fish and chips cost \$15.

- How many plates of fish and chips were ordered at the company lunch?
- The amount spent on fish and chips could be used to sponsor 3 people on a weekend cruise. What is the cost of the weekend cruise for each person?

Solution

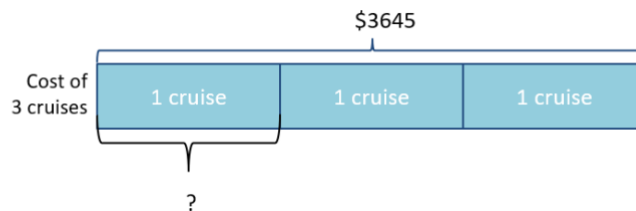
a.



$$\$3645 \div 15 = 243$$

243 plates of fish and chips were ordered at the company lunch.

(b)



Cost of fish & chips = 3 cruises

$$3 \text{ cruises} = \$3645$$

$$1 \text{ cruise} = \$3645 \div 3 \\ = \$1215$$

The cost of a weekend cruise for one person is \$1215

Task 3

A gardener sold 2760 tulips and 2944 roses to 8 florists.

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There were 15 tulips in each vase. Each vase also had an equal number of roses.

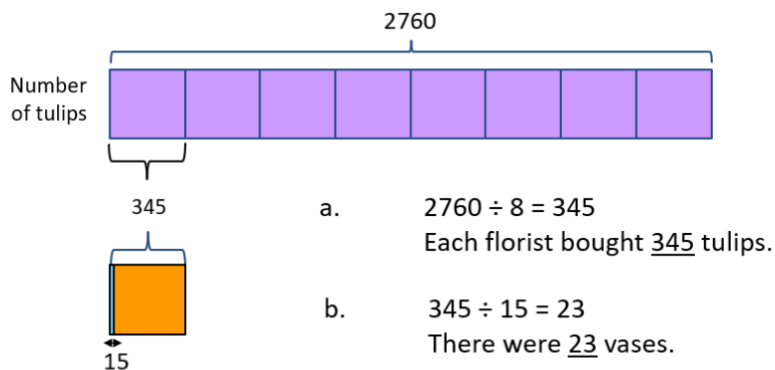
(a) How many tulips did each florist buy?

(b) How many vases were there?

(c) How many roses did Joanne put in each vase?

Solution

a. & b.

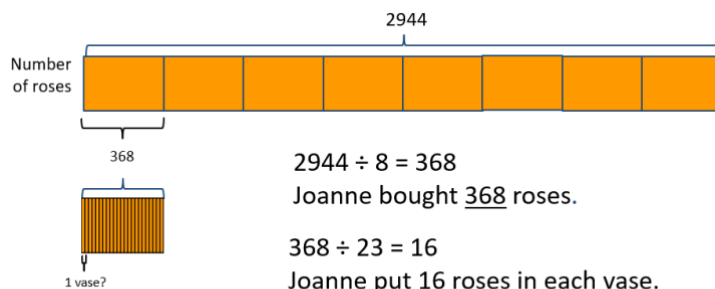


a. $2760 \div 8 = 345$
Each florist bought 345 tulips.

b. $345 \div 15 = 23$
There were 23 vases.

How many groups of 15 are there?

c.



$$2944 \div 8 = 368$$

Joanne bought 368 roses.

$$368 \div 23 = 16$$

Joanne put 16 roses in each vase.

(From part b. we know there were 23 vases)

Task 4

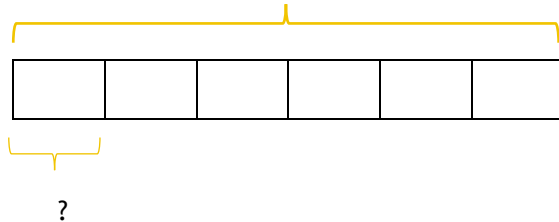
A driver delivered 6000 chocolate bars and 2520 candies to 6 retail outlets. Each outlet received the same number of chocolate bars and the same number of candies.

At each retail outlet, the sales assistant packed some chocolate bars and candies into some bags. There were 50 chocolate bars in each bag. Each bag also had an equal number of candies.

How many candies did the sales assistant pack into each bag?

Solution

6000 chocolate bars

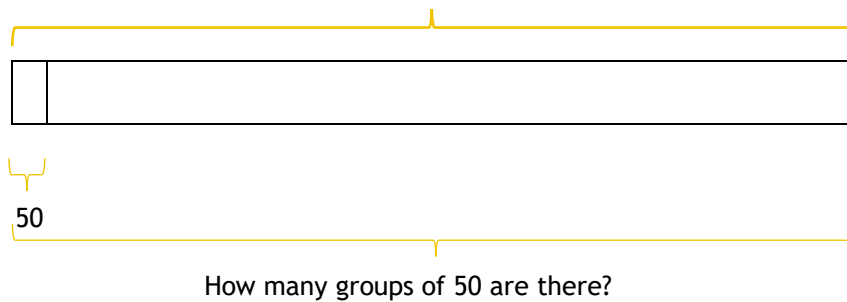


$$6000 \div 6 = 1000$$

Each outlet received 1000 chocolate bars.

Enlarged version from Step 1

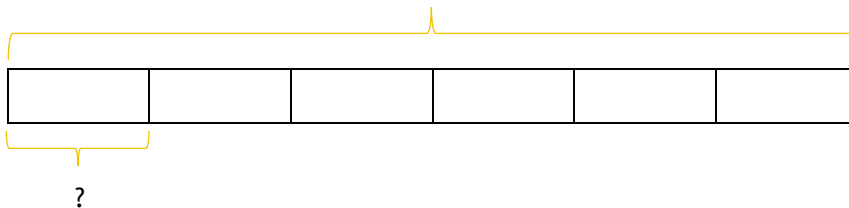
1000 chocolate bars



$$1000 \div 50 = 20$$

There were 20 bags.

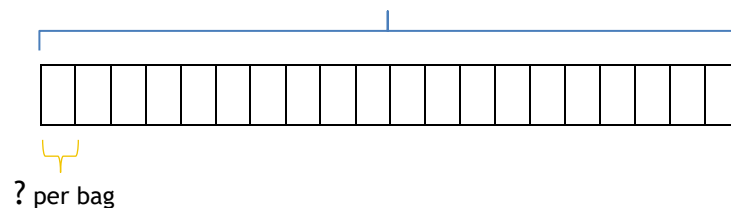
2520 candies



$$2520 \div 6 = 420$$

Each outlet received 420 candies.

420 candies



$$420 \div 20 = 21$$

The sales assistant packed 21 candies into each bag.