Year 4 Exemplar
Matchsticks

Australian Curriculum: Mathematics (Year 4)

ACMNA081: Explore and describe number patterns resulting from performing multiplication.

- Identifying examples of number patterns in everyday life.

Abstract

Students explore making rows of squares from matchsticks. The patterns can be described pictorially, numerically and symbolically. Students learn to work systematically and keep a record of results that assist them to develop and test conjectures. As the students describe and explain patterns, they will begin to move from additive to multiplicative reasoning. By changing from rows of squares, the task is easy to adapt to other year levels.

Mathematical purpose (for students)

To make and test conjectures about how many matchsticks are needed to construct a long row of squares.

Mathematical purpose (for teachers)

Teachers support and challenge students to:

- Notice numerical and spatial patterns (Analysing).
- Form and test conjectures about the pattern (Generalising).
- Communicate a conjecture about the pattern using words or symbols (Generalising).
- Use a rule to communicate the general case using words or algebraic symbols (Generalising).
- Explain the conjecture or rule using more than one example (Generalising).
- Justify the rule using a water tight argument (Justifying).

Time Needed

90 minutes approximately

Vocabulary Encountered

- explain why/because
- test/verify your idea
- if… then…
- rule/pattern
- generalise

Materials

- Matchsticks (approx. 25 per student)
- Student Sheet 1 – Matchsticks (1 per student)
- Student Sheet 2 - Matchsticks Table (1 per student)
- Reasoning Prompt Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- Assessment Sheet (1 per student)

Matchsticks: The Lesson

Reasoning Task

I used some matchsticks to make squares connected side by side as shown:

How many matchsticks would I need to make eight squares?
What if I wanted to make 100 squares? How many matchsticks would I need?

Introducing the Reasoning Task

To orient students' thinking, hand out about 20 matchsticks per student and ask students to make a shape. Ask, "How many matchsticks have you used in your shape?" Share responses and then draw attention to those who made squares. Show students how matchsticks can be used to make squares that are joined together in a row such as:

Hand out Student Sheet 1 – Matchsticks. It is useful to have students explore the task on their own to see what they do, such as using informal methods and/or formal methods to represent their thinking.

- Some students will naturally lean towards using addition to solve the problem (adding 3 matches to make the row one square longer). However, it is intended that students will move towards multiplication (e.g. multiplying length of row by 3 and adding 1).
- Later, encourage students to sort and organise their thinking into a table to assist them in working systematically and to notice and describe the patterns. See Student Sheet 2 - Matchsticks Table:

No. Squares	1	2	3	4	5	6	7	9	…
No. Matchsticks	4	7	10	13					

Reasoning Prompts

For more prompts in context, see this table.

- What stays the same and what changes? (Analysing)
- Sort or organise the following according to … (Analysing)
- What is the pattern here? (Generalising)
- What is the rule? (Generalising)
- Is that … (pattern) always going to work? (Generalising)
- If…then… (Justifying)

What stays the same and what changes?	Sort or organise the following according to …	What is the pattern here?	What is the rule?	Is that…(pattern) always going to work?	If…then…

Enabling Prompts

- Ensure students are using matchsticks to make the pattern for themselves. If necessary, suggest they make the pattern a couple of times, looking for patterns in how they construct it.
- Make a row of 5 squares. How many matchsticks are needed? Write down your findings in this table.
- As you add more squares, what stays the same, and what changes?
- Can you show your findings in an organised way using a table?

Extending Prompts

- Can you express your thinking as a rule?
- How do you know the rule works for any number of squares? Convince me. ?
- Can you find a different rule that is also correct?

Summary Phase

Invite students to share their solutions in order of complexity to develop a whole class mathematical discussion. The Formative Assessment Table shows the likely variation in responses. You might:

- Encourage students to explain each other's thinking.
- Ask
    - "What is one thing you know now about making and testing conjectures that you did not know before?"
    - "What have you learned about explaining your reasoning to others?"

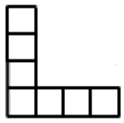
Further Activities

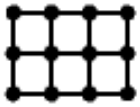
Follow Up Tasks

1. Change the polygons used in the row

Repeat the task by forming the matchsticks into a row of triangles or other polygons. Students can apply any of the strategies they have previously used and heard to the follow-up task.

2. Use other shapes https://nrich.maths.org/8111





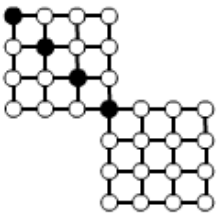
L shapes with arms of variable length.
Rectangles with a fixed height (e.g. 2) and variable length

Extension Tasks

3. Chicken Boxes https://www.resolve.edu.au/chicken-boxes-year-56?special_topic=80

This reSolve unit of four lessons progresses from simple to more complex patterns. Each lesson gives a strong emphasis to justifying why the conjectured patterns are true, to expressing the patterns using multiplication. Beginning with the context of cages at a poultry show, students explore the how the number of walls used increases as the number of cages increases, making tables and graphs and writing rules. They discuss the equivalence of different rules. Later, they design some different cages, observing the changes in the relationships between variables. The move from 1 dimension to 2 dimensions to 3 dimensions brings more complexity into the patterns.

4. Two Squares https://nrich.maths.org/8111



This pattern with two squares has four black dots.

Work out the number of white dots and the number of lines needed to draw a pattern with:

- 25 black dots
- 100 black dots
- n black dots

Formative Assessment

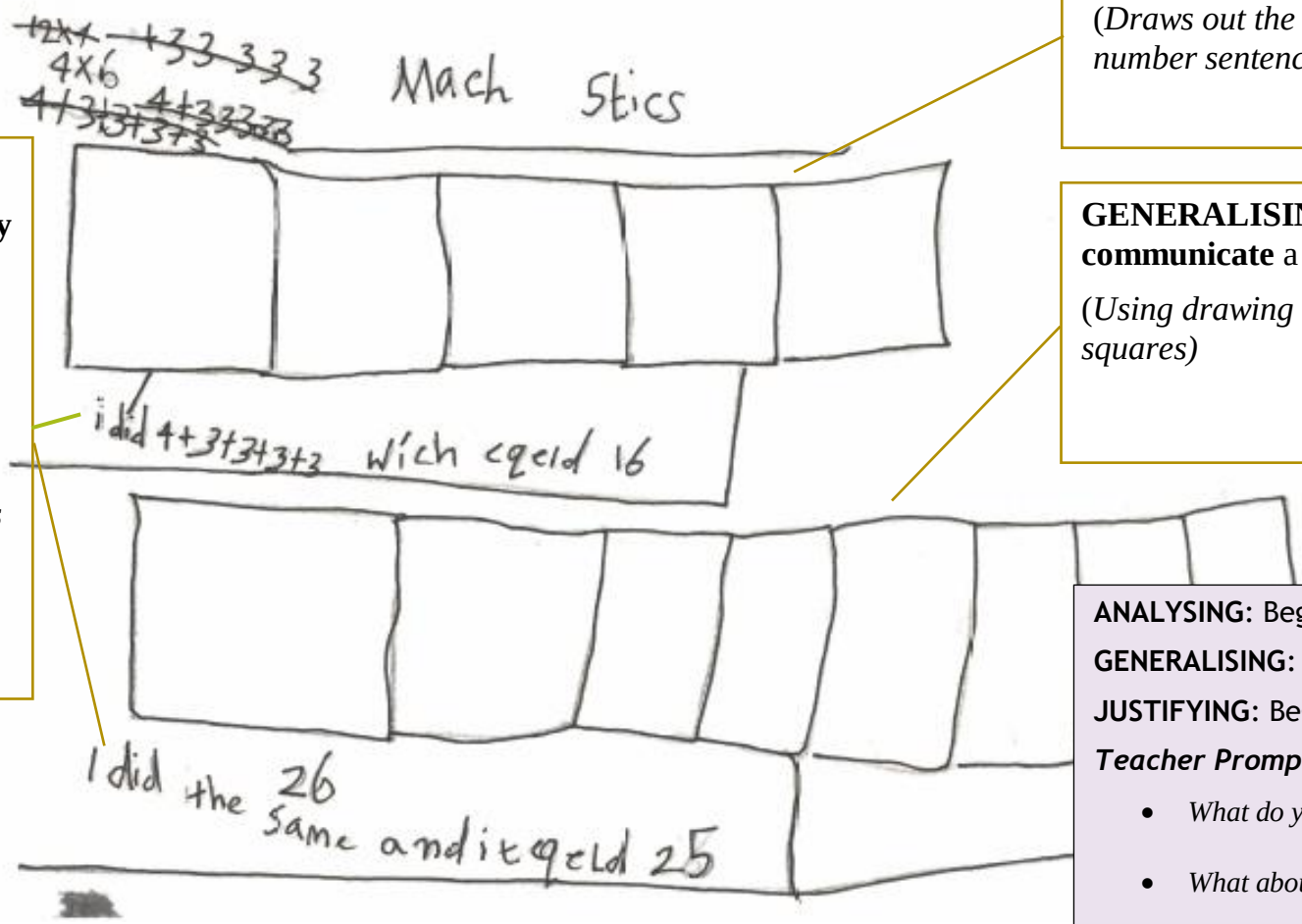
The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students' reasoning during the lesson. Suitable prompts are suggested to support or extend such students' reasoning.

Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers' assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student's reasoning.

ANALYSING Possible Student Response	Level	Suggested Prompts
You would need a lot more matchsticks.	Not Evident	Offer matchsticks for the student to physically make the pattern.
The student draws out the squares and counts all. **(See Annotated Work Sample 1)**	Beginning	Is there another way you can work it out? (Steer away from 'count all') What do you notice? Is there a pattern? How can you change this to make a row that is one square longer?
The student has drawn out the pattern and labelled each step. E.g. For 8 squares, you need 25 matchsticks. I worked this out by counting by 3s seven times and then I added four more on the end because that's the first square in the pattern. Therefore you would need 28 matchsticks for 9 squares and 31 matchsticks for 10 squares. You just keep adding 3 on each time. **(See Annotated Work Sample 2)**	Developing	How might you organise your thinking? (Encourage use of a table).
The student notices the significance of the '1' and the '3'. **(See Annotated Work Sample 3)**	Consolidating	Explain your thinking.
Notices the patterns in each square having 3 matchsticks and the 1 left over in the initial square. The students wrote "That matchstick brings it [the square] together." **(See Annotated Work Sample 4)**	Extending	Explain the pattern.
GENERALISING **Possible Student Response**	**Level**	**Suggested Prompts**
The student draws attention to repeated components of a pattern by drawing different amounts of squares e.g. "I did 4+3+3+3+3." **(See Annotated Work Sample 1)**	Beginning	What do you notice? What stays the same? What changes?
The student applies their rule to the example of 100 squares. E.g. For 8 squares you need 25 matchsticks. I did this by working out 4 + (7 x 3) = 25. In other words, you add the 4 matchsticks used in the first square to the additional lots of 3 matchsticks used to make additional squares. **(See Annotated Work Sample 2)**	Developing	Is there a pattern? How can you describe it?
The student identifies and describes the significance of the '1' and the '3'. E.g. "I noticed that what is the same is that each square has 3 matchsticks, so you could times 3 by the amount of squares and then	Consolidating	Convince me that this will work for any given number of squares.

add 1 to make up the fourth matchstick that is in the first square." (33x3)+1; (8x3)+1 **(See Annotated Work Sample 2)**		
The student applies their rule to further examples. e.g. "For 8 squares you need 25 matchsticks. I did this by working out (8 x 3) + 1 = 25. For 10 squares it is (10x3)+1=31, for 100 squares it is (100x3)=1=301, for 1000 squares it is 3001 **(See Annotated Work Sample 3)**	Consolidating	Does that rule work for any number of squares? Even those that do not end with a 0? Convince me.
The student applies the rule for any number of squares. e.g. There is a pattern with the 0s: (3x100)+1=301 (3x10000)+1 =30001 It does not always have to have a 0 (3x5321)+1 =15964 **(See Annotated Work Sample 4)**	Extending	So, what is the rule? Can you use a letter to represent the number of squares?
JUSTIFYING **Possible Student Response**	**Level**	**Suggested Prompts**
Explains how he added to find 5 squares and then repeated the process to find 8 squares. There is no mention of patterns. **(See Annotated Work Sample 1)**	Beginning	What about 10 or 12 squares?
The student verifies the total of matchsticks required for a certain amount of squares using number sentences and/or diagrams. **(See Annotated Work Sample 2)**	Developing	What about 100 squares? What about 150 squares?
Uses a number sentence and drawing to check it works for each case. **(See Annotated Work Sample 4)**	Consolidating	Put your rule into words. Why does your rule work?
The rule to find the number of matchsticks for any given number of squares could be written as 3 x number +1.	Extending	Create your own pattern of matchsticks for a friend to solve.

Annotated Work Sample 1



ANALYSING: **Repeats patterns**

(*Draws out the squares and writes number sentence*).

GENERALISING: Attempts to communicate a common property

(*Using drawing tries 5 squares and then 8 squares)*

JUSTIFYING:

Describes what they did and recognises what is correct or incorrect.

(*Explains how he added to find 5 squares and then repeated the process to find 8 squares. There is no mention of patterns.)*

ANALYSING: Beginning

GENERALISING: Beginning

JUSTIFYING: Beginning

Teacher Prompt:

- *What do you notice? Is there a pattern?*
- *What about 10 or 12 squares?*
- *How might you organise your thinking?*

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Work Sample 1 Rubric

Student Name: Work Sample 1 Reasoning Task: MATCHSICKS Date:

Observation of student's reasoning:

-Drew out squares & tried different amount e.g 5 then 8 squares but couldn't/didn't describe any patterns

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct. *
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Next step is to notice & describe pattern i.e the '1' matchstick + 3 more.

Annotated Work Sample 2

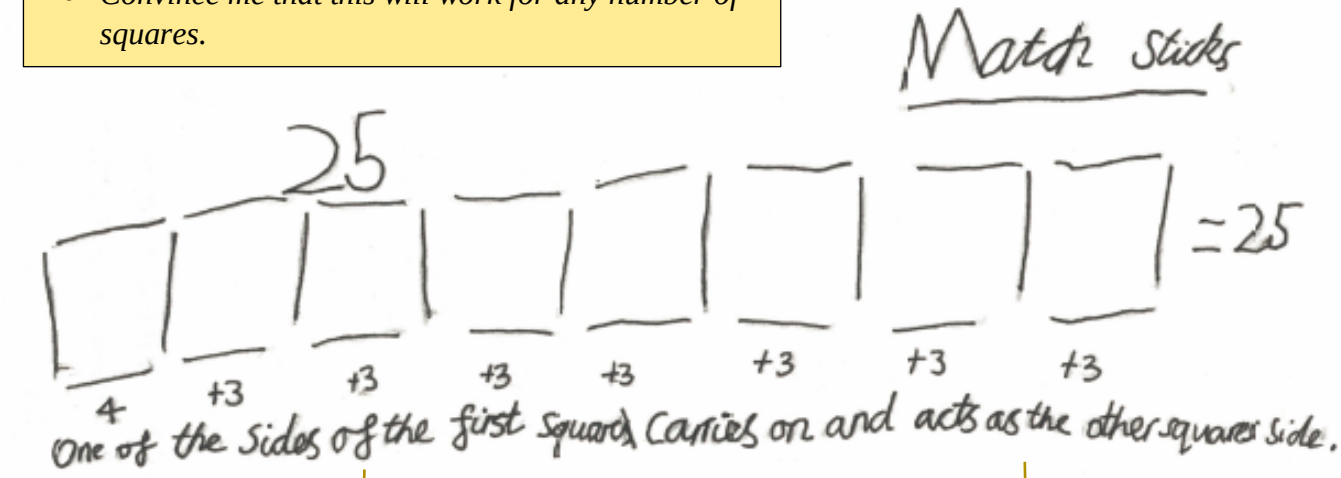
ANALYSING: Developing

GENERALISING: Developing - Consolidating

JUSTIFYING: Consolidating

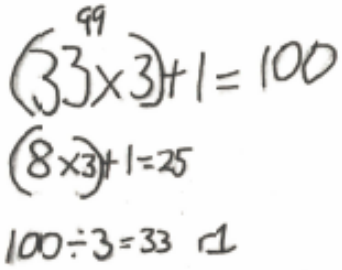
Teacher Prompt:

- *Convince me that this will work for any number of squares.*



JUSTIFYING: Verifies truth of statements by confirming all cases. Uses a correct **logical argument**

Student uses another example of 33 squares to find number of matchsticks



ANALYSING:

Repeats and extends patterns

The student has drawn out the pattern, labelled each step and described the pattern in words

GENERALISING: Communicates a rule using mathematical terms

The student identifies and describes the significance of the '1' and the '3'.

GENERALISING:

Explains how the rule works using examples

The student can apply their rule to the example of 33 squares to get 100 matchsticks but did not apply rule to get 100 squares.

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Work Sample 2 Rubric

Student Name: WORK SAMPLE 2 Reasoning Task: MATCHSTICKS Date:

Observation of student's reasoning:

- Drew pattern & labelled each step.
- Identified the '1' & '3'
- Applied rule to find 100 but needs to develop argument

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Look at general rule for finding any number of squares.

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Annotated Work Sample 3

ANALYSING:
Consolidating

GENERALISING:
Consolidating

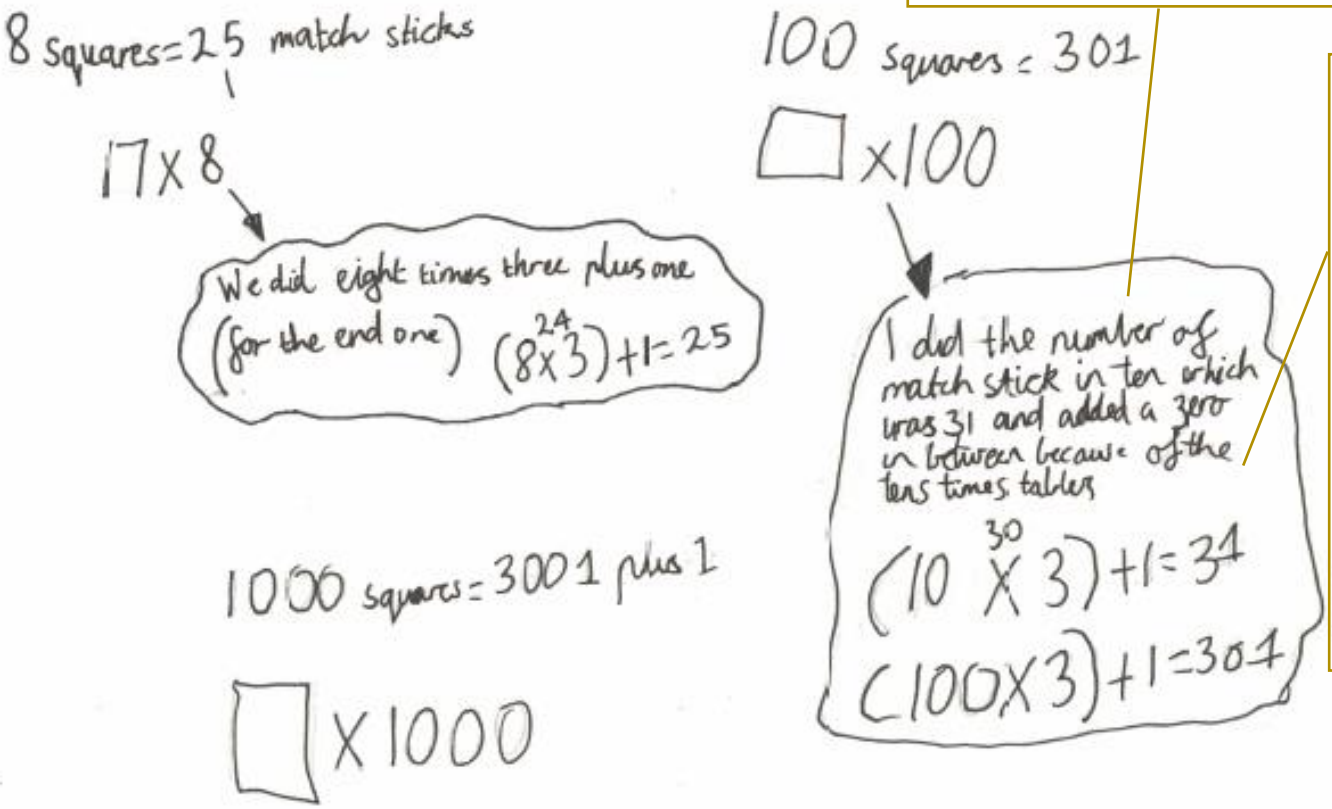
JUSTIFYING:
Consolidating

Teacher Prompt:

- *How could we describe the pattern?*
- *What is the rule?*

JUSTIFYING: Verifies truth of statements by confirming all cases. Uses a correct **logical argument.**

The next step for the student is to develop a watertight argument and use algebraic notation to describe the rule.



GENERALISING:
communicates a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.

The student creates more examples with multiples of 10 to test the rule.

ANALYSING:
Systematically searches for examples, extends pattern or analyses structure to form a conjecture.
Makes **predictions** about other cases.

The student explains the significance of the '1' and the '3'. Predicts for 1000 squares

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Work Sample 3 Rubric

Student Name: Work Sample 3 Reasoning Task: Matchsticks Date:

Observation of student's reasoning:

Notices the '1' & '3' in the pattern & creates more examples to test it.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Look at rule for finding any number.

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Annotated Work Sample 4

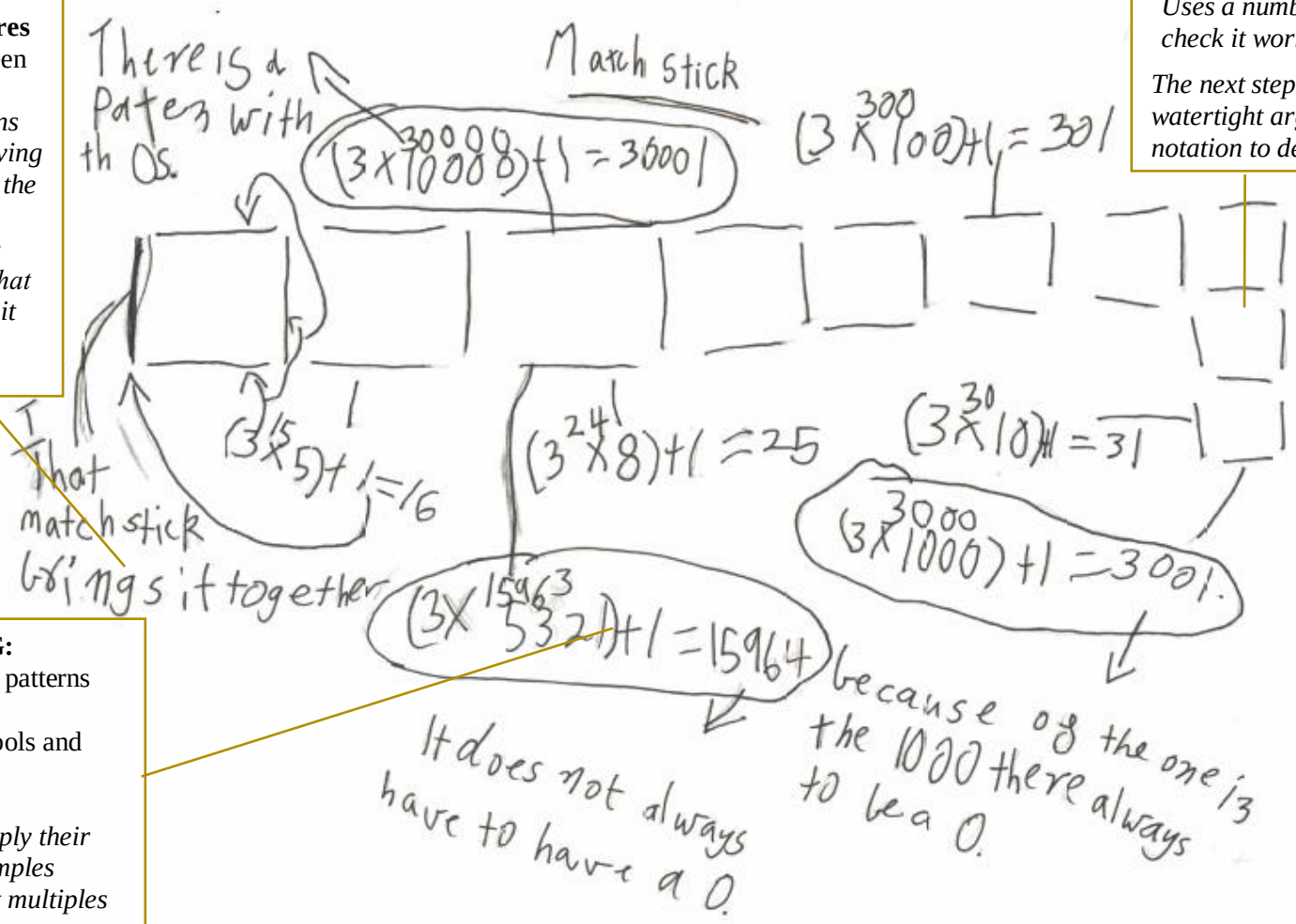
ANALYSING:
Notices and explores relationships between properties.

Notices the patterns in each square having 3 matchsticks and the 1 left over in the initial square. The students wrote "That matchstick brings it [the square] together."

JUSTIFYING: Verifies truth of statements by confirming all cases

Uses a number sentence and drawing to check it works for each case.

The next step for the student is to develop a watertight argument and use algebraic notation to describe the rule.



GENERALISING:
Generalises cases, patterns or properties using mathematical symbols and **applies** the rule.

The student can apply their rule to further examples including those not multiples of 10.

ANALYSING:
Extending

GENERALISING:
Extending

JUSTIFYING:
Consolidating

Teacher Prompt:

- *What is the rule?*

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Work Sample 4 Rubric

Student Name: WORK SAMPLE 4 Reasoning Task: MATCHSTICKS Date:

Observation of student's reasoning:

Notices '1' & '3'.
Applies rule but needs watertight argument

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	✱ • Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Look at rule for finding any number of squares.

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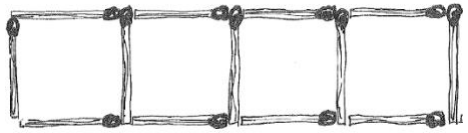
Matchsticks

Name:_____

I used some matchsticks to make squares connected side by side as shown:



How many matchsticks would I need to make 8 squares?
What if I wanted to make 100 squares? How many matchsticks would I need?

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Matchsticks Table

Name:_____

Using a table can help mathematicians work systematically to notice and describe patterns.
Complete the following table.

Number of Squares									
Number of Matchsticks									

What do you notice?

How can this table help you to find out how many matchsticks you would need to make 100 squares?

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Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • A**rgument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

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