Year 4 Exemplar
Shapeshifter

Australian Curriculum: Mathematics (Year 4)

ACMMG088: Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies.

Abstract

Shapeshifter is based on a picture story book. Students find a rule for using a straight line to dissect a polygon to make a polygon with one more side. The content foci are naming shapes and their components. The reasoning foci are especially Analysing and Justifying – seeing and explaining when and why cutting off a corner increases the number of sides.

Mathematical purpose (for students)

To find a way of splitting a shape to make a different type of shape, and to explain why it works.

Mathematical purpose (for teachers)

Teachers support and challenge students to:

- Compare and contrast outcomes of splitting for different triangles and other polygons (Analysing).
- Form conjectures about splitting shapes (Justifying).
- Express the conjecture verbally, using diagrams and in a written statement using words including geometric terms (Generalising).
- Test their conjecture to show that it works for all triangles and other 2D shapes and explain why their rule works (Justifying).

Time Needed 120 minutes approximately

Vocabulary Encountered

- Polygon
- names of 2D shapes
- vertex
- if… then…

Materials

- *The Greedy Triangle* by Marilyn Burns or online version e.g. https://www.youtube.com/watch?v=kPuI4XyyZUE
- Student Sheet 1 – Shapeshifter (1 per pair, optional)
- Scissors, poster paper, glue
- Coloured paper triangles (at least 10 per pair; optional template here)
- *Geoboard* app; *Geoboard* whiteboard interactive (optional)
- Reasoning Prompt Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- Assessment Sheet (1 per student)

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

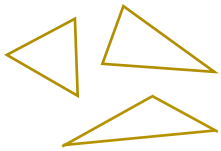
aamt

Shapeshifter: The Lesson

Introducing "The Greedy Triangle"

- Read or listen to "The Greedy Triangle" by Marilyn Burns, Syd Hoff and Gordon Silveria (1995 Scholastic Press). There are several readings of the book on the internet e.g. https://www.youtube.com/watch?v=kPuI4XyyZUE
- This lesson is best conducted by students working in pairs but it can also be done in small groups.
- Record the names of the two-dimensional shapes and discuss the properties of these two-dimensional shapes.

Reasoning Task



Imagine you are the Shapeshifter in "The Greedy Triangle". You have only one tool that you can use. It cuts one straight line only. Work out **a rule** for cutting shapes that the Shapeshifter could use to change the triangle to a quadrilateral and then to a pentagon and so on. Create a poster to show that your rule works and explain why your rule works.

- Hand out Student Sheet 1 – Shapeshifter and multiple triangles for students to use for their trials. Triangles can be cut from coloured paper (use a variety of shapes), or printed in greyscale or colour from Triangles for Printing.
- Encourage students to look at all their shapes and compare and contrast the shapes they have created by cutting in different orientations.
- Students can cut triangles, or fold, or just draw lines with a ruler. They need to keep the pieces of any split triangles together in order to compare and record the outcomes. Sometimes students end up with a pile of shapes and do not know which two started off as joined together. One way to get around this problem is to encourage students to explore by folding or drawing and then when they have found the rule to use the scissors to cut and then paste onto their poster.
- When roaming the classroom prompt students to tell you what they think the rule is and explain it. This will help students to verbalise the rule to make a quadrilateral (e.g. "To make a quadrilateral, cut a corner off the triangle, but do not cut through a corner.")
- This investigation could also be conducted using digital tools with students investigating outcomes of cutting triangles and preparing a digital poster or report.

Reasoning Prompts

For more prompts in context, see this table.

- What is the same about the shapes you make with this cut? (Analysing)
- What are all the different ways in which the Shapeshifter can cut a triangle? (Analysing)
- How can you describe the cut you need to use? (Generalising)
- Is [a cut that fits this description] always going to work? (Generalising and Justifying)
- What happens in general (for cutting shapes beyond triangles)? (Generalising)
- What is the rule? (Generalising)
- Why does your rule work? (Justifying)
- How do you know [the rule] always works? (Justifying)

What is the same and different about …?	Alter an aspect of something to see an effect. If we change this what will happen?	How could you explain the rule to someone else?	Is that… (pattern) always going to work?	What happens in general?	What is the rule?	Explain – why does this work?	How do you know?

Enabling Prompts

- Suggest that students make folds rather than cuts to examine and record effect of different cuts/folds.
- Where do I start a cut to make a quadrilateral? Where do I finish the cut?
- How can you use diagrams to show how your rule works?

Extending Prompts

- Does your rule work for all types of triangles? How do you know?
- How does the angle of the cut affect the properties of the shape created?
- Does your rule work for making pentagons work for all types of quadrilaterals? How do you know?

Summary Phase

Invite students to share their solutions in order of complexity to develop a whole class mathematical discussion. The Formative Assessment Table shows the likely variation in responses. You might:

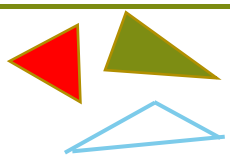
- Encourage students to explain each other's thinking.
- Ask "What is one thing you know now about making and testing conjectures that you did not know before?".
- Ask students to write a reflection on their learning: "What helped you to justify your rule?"

Further Activities

Follow Up Tasks

1. Cutting to get *fewer* sides: starting from a quadrilateral or a pentagon or a polygon with more sides.

2. Shapeshifter Welder



Suppose the Shapeshifter had a welding machine. What shape would the Shapeshifter need to weld onto The Greedy Triangle to change it to a quadrilateral? Where on the triangle would Shapeshifter do the weld? Does your rule work for other shapes? Create models (or a poster) to show your rule and explain why it works.

Alternate Tasks

3. **Shapeshifter Extension**

   Colleen says that the Shapeshifter can use one cut to turn a triangle into all the different types of quadrilaterals? Verify or refute this claim by showing examples. If the claim is true, explain why.

For this task students will need to choose one triangle and then find out whether they can make a square, rectangle, parallelogram, rhombus, trapezium and an irregular quadrilateral from this triangle. They will need explain changes made to the rule or to the type of triangle needed to create other special quadrilaterals and to explain why not where it is not possible. For this task students will need to consider angle size and parallel lines.

4. **Shapeshifter Attacks Concave Polygons**

These shapes are polygons. Not all polygons are convex. Some are concave with reflex angles. Investigate the Shapeshifter task considering concave polygons.

References and Resources

"The Greedy Triangle" by Marilyn Burns, Syd Hoff and Gordon Silveria (1995 Scholastic Press)

Geoboard https://apps.mathlearningcenter.org/geoboard/

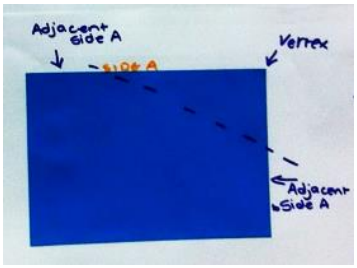
Geoboard app https://itunes.apple.com/us/app/geoboard-by-the-math-learning-center/id519896952?mt=8

Formative Assessment

The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students' reasoning during the lesson. Suitable prompts are suggested to support or extend such students' reasoning.

Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers' assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student's reasoning.

ANALYSING **Possible Student Response**	**Level**	**Suggested Prompts**
Cuts up shapes to create lots of pieces but can't put them back together.	Not Evident	Let's fold them first to see what works and what doesn't.
Cuts up shapes and puts them back together. These are randomly placed on the poster.	Beginning	How could we sort these cases? How could we organise them on your poster?
Sorts cases to find and present those that worked. **(See Annotated Work Sample 1)**	Developing	Where was the "corner that you cut off" to make these shapes?
Orders their experiments starting with triangles and then other shapes. Sorts cases into those that worked and didn't work. **(See Annotated Work Sample 4)**	Consolidating	What other ways of cutting don't work?
Explores all the possible cuts including angle of cut to notice the properties of the shape created.	Extending	How can you include these ideas in your rule? (Generalise cases)
GENERALISING **Possible Student Response**	**Level**	**Suggested Prompts**
The shapes created after the cut are pasted on poster.	Not evident	Show us how you created these shapes? What was the same about how you cut the first shape?
Pointing at the triangle - "You cut from here to here" (Using gesture).	Beginning	How can we describe "here" (pointing) and "here" (pointing)?
"Cut off a corner."	Developing	What do you mean by "corner"? How does it work?
"Cut off a corner with one cut making sure that the off cut is a triangle." **(See Annotated Work Sample 3)**	Consolidating	If you do this, will it always result in one extra side?
"Cut between two adjacent sides to make a triangle off cut." Diagrams illustrate the different cases to show how the rule works and when it does not work.	Extending	Why does your rule work? (Justify using logical argument)

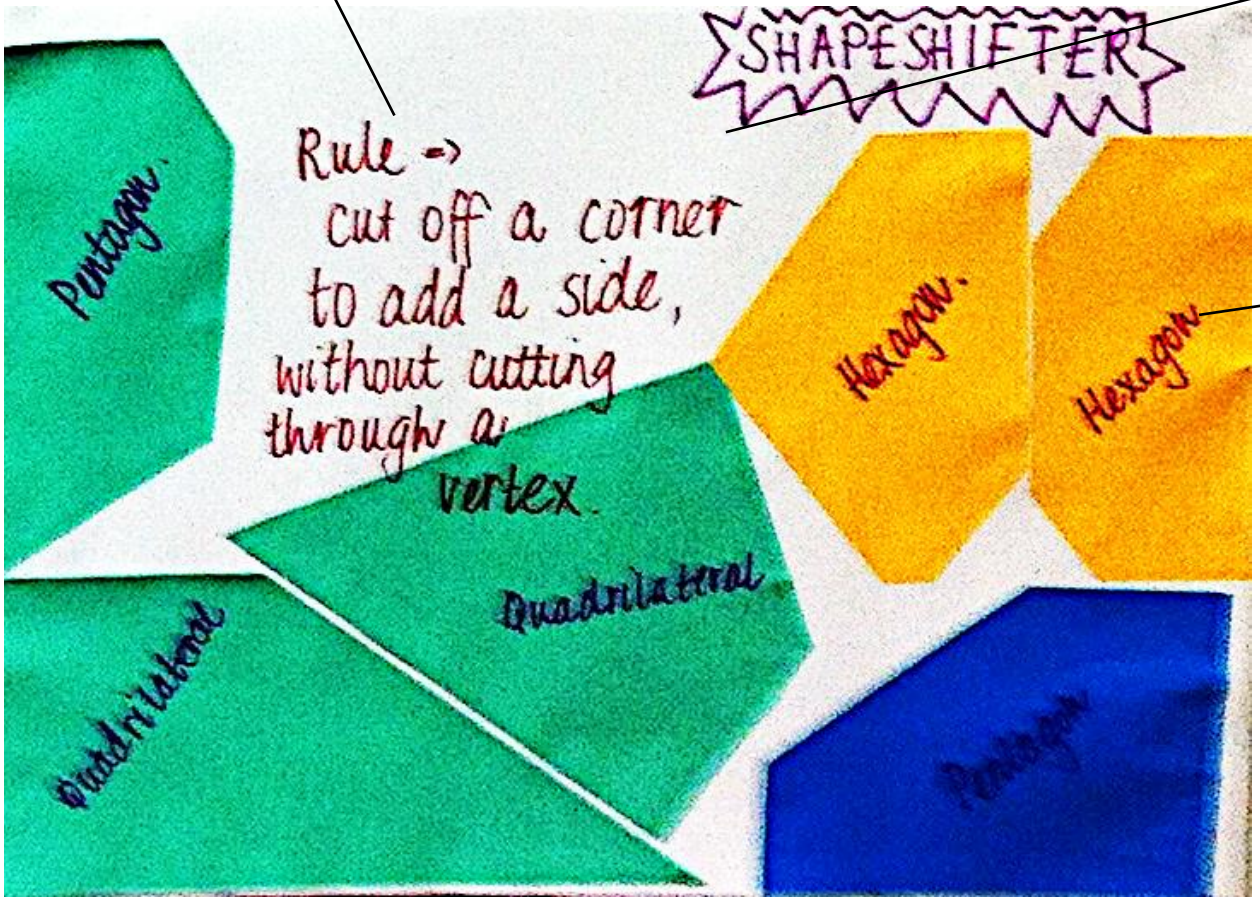
JUSTIFYING Possible Student Response	Level	Suggested Prompts
The shapes created after the cut are pasted on poster. **(See Annotated Work Sample 1)**	Not evident	Explain how these shapes [on your poster] show that your rule works.
Includes a diagram of a rule that works (e.g. cut off a corner) and one that does not work.	Beginning	Does this cutting procedure work for all types of triangles and other two-dimensional shapes? How do you know?
Includes diagrams of multiple cases that work (e.g. cut off a corner), but these are not ordered or labelled.	Developing	How could you display your cases to verify that your rule works?
Diagrams verifying that the rule works are organised systematically and labelled to show the applied rule and outcome shapes. **(See Annotated Work Sample 2)**	Consolidating	Why does you rule work? Convince us. (Develop a logical argument.)
"If you cut between two adjacent sides then you cut off one angle or corner only. The cut line creates an extra side, and there are two corners at the meeting of the cut line and the starting side and end side of the cut. This means there is one extra angle." 	Extending	Extending task: How does the angle of the cut affect the properties of the shape created?

Annotated Work Sample 1

ANALYSING: **Sorts** cases.

Rule indicates they found cases that work for the rule and those that do not.

GENERALISING: **Communicates** a rule (conjecture) using mathematical terms, and records other cases.
Rule explains what not to do when cutting but examples do not show how the rule works.



JUSTIFYING: Does not justify

ANALYSING: Developing

GENERALISING: Developing

JUSTIFYING: Not evident

Teacher Prompt

What happens if you cut through a vertex (corner)? How is it different from cutting off a vertex (corner)?

How does your rule work?

Convince me that your rule works?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

re(Solve) MATHS BY INQUIRY

Work Sample 1 Rubric

Student Name: WORKSAMPLE 1 Reasoning Task: Shapeshifter Date:

Observation of student's reasoning:

Produced examples six different shapes
Poster doesn't show how their rule works.—
only the outcome of the cut.
Identify a critical element of the rule - not cutting through a corner.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Need to show the piece that was cut off to explain the rule.
What happens when you cut through a vertex?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

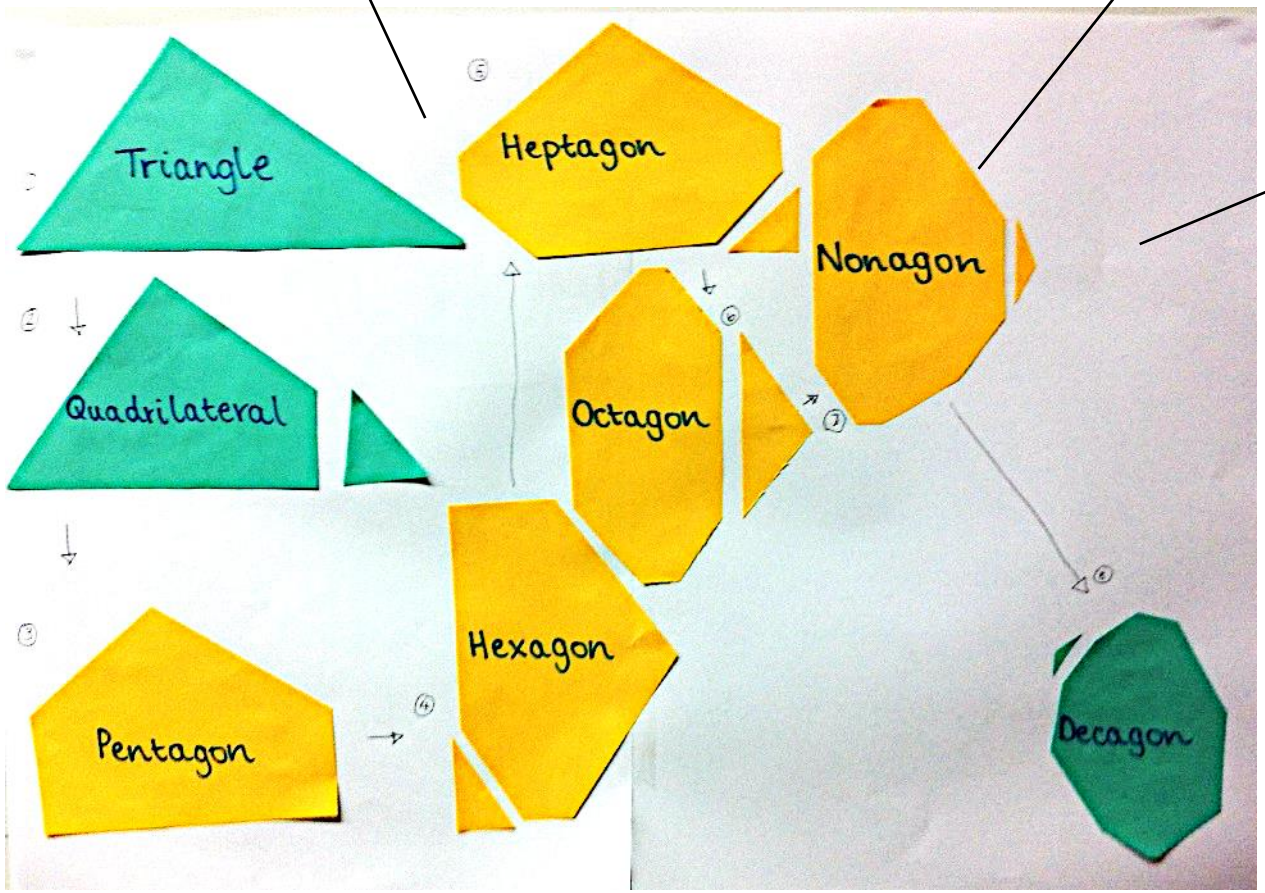
Annotated Work Sample 2

ANALYSING: **Systematically searches** for a common property.

Models illustrate that there has been testing of the conjecture to find out whether it works for other cases. The cases are ordered.

GENERALISING: **Communicates** a rule using diagrams.

The diagrams show what the rule is and how it works.



JUSTIFYING: **Verifies** that rule works by confirming for a number of cases.

Interestingly, the same triangle has been used to make all these shapes by cutting off a corner of each new shape created.

ANALYSING: Consolidating

GENERALISING: Developing

JUSTIFYING: Consolidating

Teacher Prompt:

Tell us, what is your rule?

Tell us, how does your rule work?

Why does your rule work?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

re(Solve) MATHS BY INQUIRY

Work Sample 2 Rubric

Student Name: WorkSample2 Reasoning Task: Shapeshifter Date:

Observation of student's reasoning:

Experimented with different shapes and different cuts. Then started again to recreate the Greedy Triangle's story of changing shape from a triangle to decagon

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples. diagrams	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

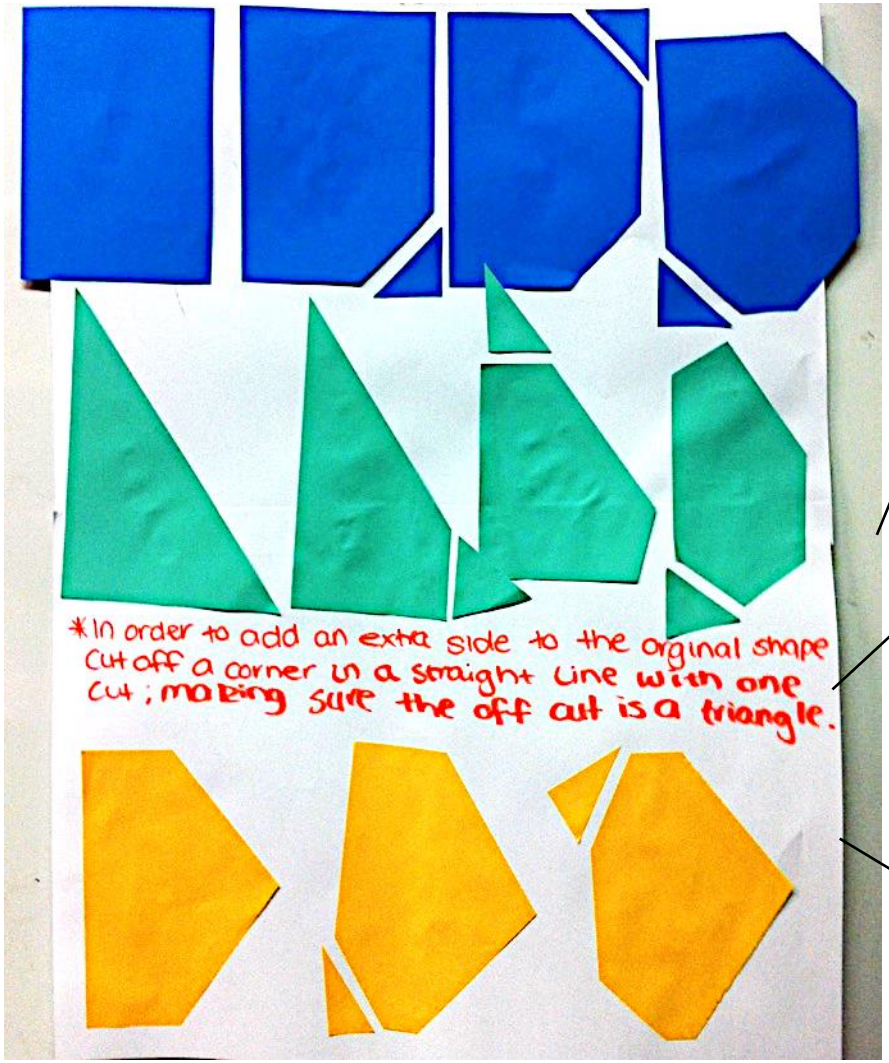
Diagrams show the rule. Prompt, need to explain the rule to others. And to justify. Why does this cut work? When doesn't it work?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Annotated Work Sample 3



ANALYSING: **Systematically searches** for a common property.

Models illustrate there has been testing of the conjecture to find out whether it works when starting with different shapes.

GENERALISING: **Communicates** a rule using mathematical terms and diagrams and explains how the rule works using other cases.

Rule and examples explains what to do, but not how to be sure that the "off cut is a triangle".

JUSTIFYING: **Verifies** the rule works by confirming for a number of cases.

Tests the rule for different starting shapes to verify that it works for these cases.

ANALYSING:
Consolidating

GENERALISING:
Consolidating

JUSTIFYING:
Consolidating

Teacher Prompt

How do you make sure that the off cut is a triangle?

What happens if you cut through a corner [vertex]?

Why does your rule work?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Work Sample 3 Rubric

Student Name: WORK SAMPLE 3 Reasoning Task: Shape shifter Date:

Observation of student's reasoning:

Started by experimenting with different cuts on the triangle. When they found what worked with triangle they tested it with a rectangle and pentagon.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using ~~terms~~ mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples. + diagram	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

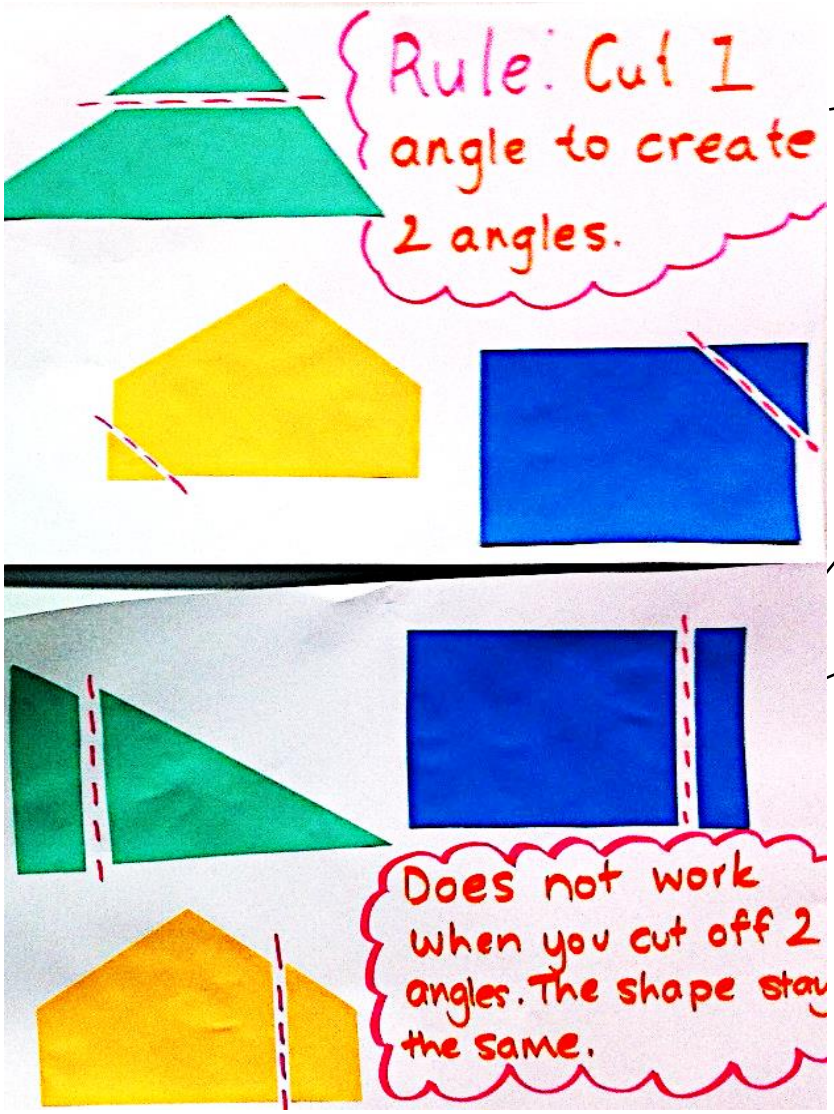
Could make the rule more specific. What happens when you cut through a corner ~~to~~ to cut off a triangle? Why does your rule work?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Annotated Work Sample 4



GENERALISING: **communicates a rule** using mathematical terms and diagrams and explains what the rule means and how it works.

ANALYSING: **Systematically searches** for a common property.

Models illustrate that there has been testing of different placement of cuts to form a conjecture. They've also tested the conjecture to find out whether it works when starting with different shapes.

JUSTIFYING: **Verifies** that rule works by confirming for a number of cases.

Tests the rule for different starting shapes to verify that it works for these cases.

ANALYSING: Consolidating

GENERALISING: Consolidating

JUSTIFYING: Consolidating

Teacher Prompt

Why does your rule work?

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Work Sample 4 Rubric

Student Name: Work Sample 4 Reasoning Task: Shapeshifter Date:

Observation of student's reasoning:

Experimented with different cuts on different shapes. Sorted into cuts that worked and cuts that didn't. Noticed the difference for re-entrant angles. Used "angle" in their rule.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using diagrams mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming **all cases** or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Need prompts to develop a logical argument. If.... then..., because.....

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

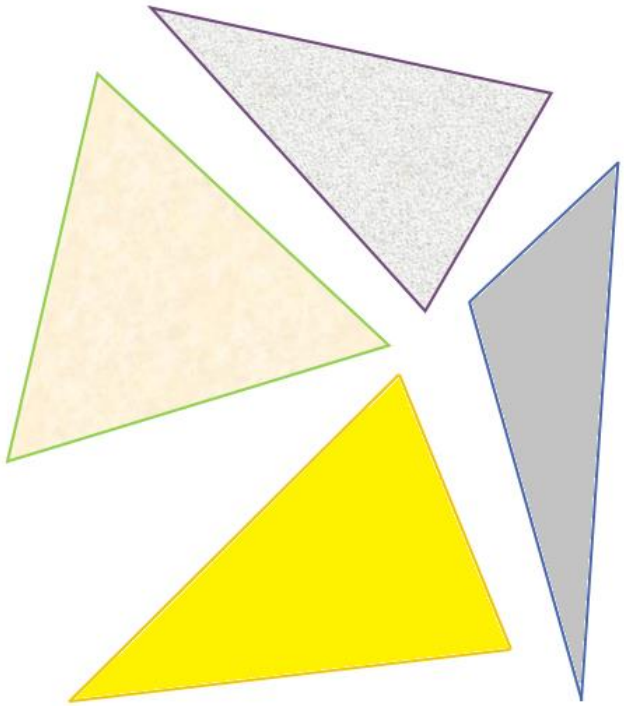
Shapeshifter

Name:_____

Imagine you are the Shapeshifter in "The Greedy Triangle". You have only one tool that you can use. It cuts one straight line only.

*Work out **a rule** for cutting shapes that the Shapeshifter could use to change the triangle to a quadrilateral and then to a pentagon and so on.*

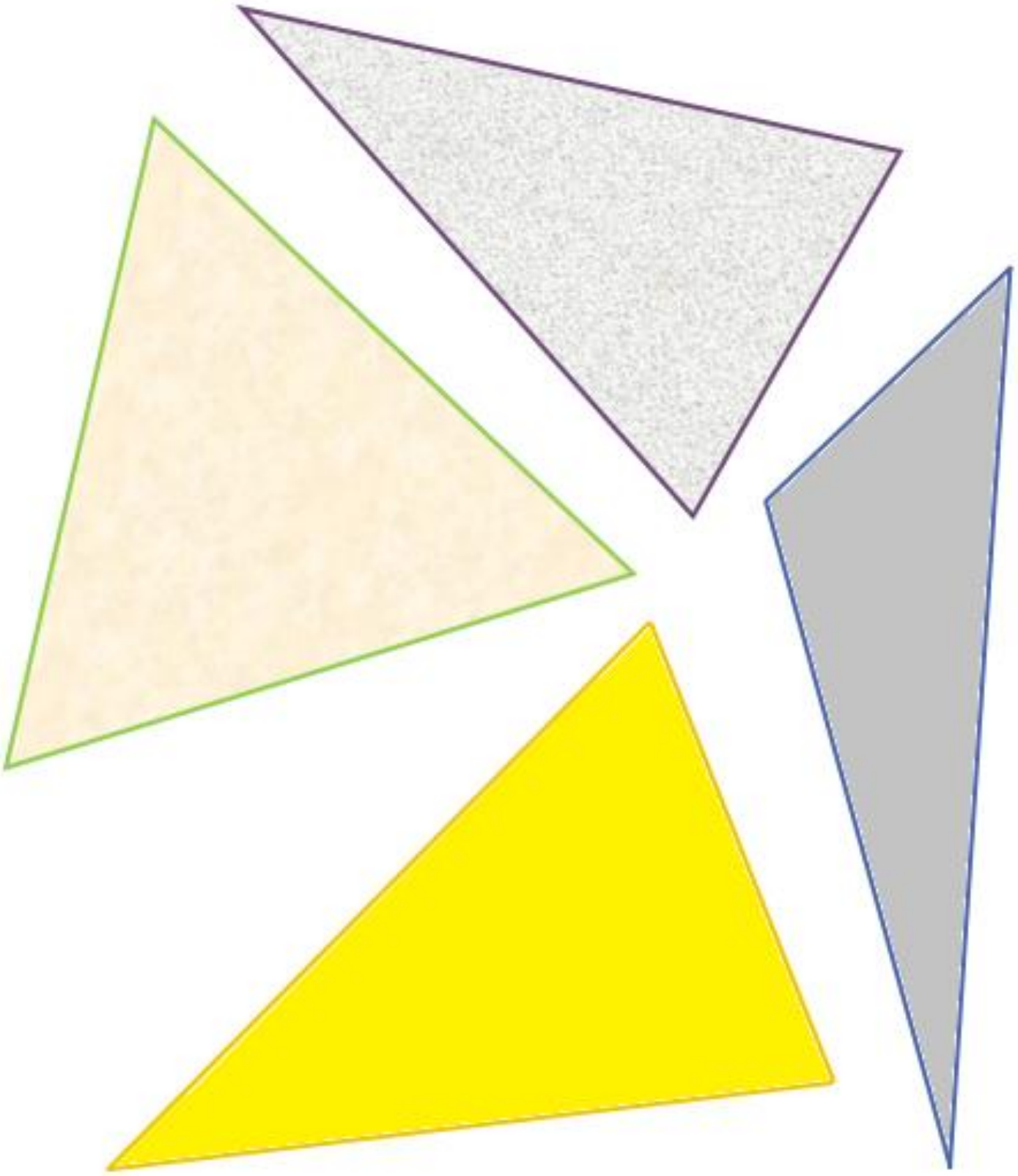
Create a poster to show that your rule works and explain why your rule works.



Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt



Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • A**rgument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt