

PRIME FACTORISATION

Lesson 2: Prime Dice

Australian Curriculum: Mathematics - Year 7

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

- Applying knowledge of factors to strategies for expressing whole numbers as products of powers of prime factors, such as repeated division by prime factors or creating factor trees.

Lesson abstract

Students play a game using dice labelled with prime numbers and learn how to use prime factorisations of a number to determine properties of a number. For example they learn how the prime factorisation shows if a number is even or odd, a perfect square or a perfect cube.

Mathematical purpose (for students)

To use the prime factorisation of a number to determine some of its properties.

Mathematical purpose (for teachers)

This lesson encourages students to determine properties of a number, such as whether it is a square, odd or even, from its prime factorisation without multiplying. This demonstrates why prime numbers are important.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- square number
- 4th power
- prime factorisation
- number properties

Lesson Materials

- Four prime dice per group of four students
- [Student Sheet 1 - Prime Dice Game](#) (1 per student)
- [Student Sheet 2 - Prime Dice Scorecard](#) (1 per student)



These prime number dice were made by writing the numbers onto blank foam cubes. Foam cubes don't make noise and are cheap and readily available from most education suppliers.

We value your feedback after this lesson via <http://tiny.cc/lesson-feedback>.



Prime Dice Game: How to Play

The object of the game is to roll the highest score with the prime factor dice. There are 6 rounds in the game. You need to roll the prime factorisation of six numbers with the properties listed in the table. The score for each category is also listed. You can only record one number when it is your turn. You can only score once in each category.

Categories	Score for the Categories
A number to the 4 th power	The product of the prime numbers showing on the dice
A number ending in 00	This roll scores 500 points
A number ending in 0	The product of the prime numbers showing on the dice
Square	The product of the prime numbers showing on the dice
Odd	The product of the prime numbers showing on the dice
Even	The product of the prime numbers showing on the dice

When it is your turn, you get the chance to roll the dice twice. After your first roll, you may choose to pick up and roll any or all of the dice again to try to get the highest combination for one of the six categories.

After you finish rolling, you must place a score or a zero in one of the categories on the scorecard.

The game ends when all the players have filled the six sections on the scorecard.

		Player's Names							
Categories	Numbers rolled	Score	Numbers rolled	Score	Numbers rolled	Score	Numbers rolled	Score	
A number to the 4 th power									
A number ending in 00 (500 pts)									
A number ending in 0									
Square									
Odd									
Even									
Total Score:									

Sample game

Amy's Turn:



Amy's first roll was 2, 3, 5 and 5.

She knows that this is an even number and ends in a zero.

She decides to re-roll the 3 to try and get a number ending in a double zero or a perfect square number.

Only re-rolling the 3 means that her new number will still be even and that it will still end in zero.



Amy rolls a 7.

This is not a number ending in a double zero. It is not a perfect square.

It is an even number and it is a number ending in zero.

She places it in the category '*A number ending in 0*' and scores 350 points.

Barry's Turn:



Barry's roll was 3, 5, 5 and 7.

He knows that this is an odd number.

He decides to reroll the 3 to try and get a perfect square number or a larger odd number.



He rolls a 5. It is not a perfect square number but it is a higher scoring odd number.

He scores 875 for his odd number. Great score Barry!

Begin the Lesson: Playing Prime Dice

Ask students if they have ever played the game *Yahtzee*. Explain that this game is similar; you get multiple rolls to try to score points for different categories. It might be helpful to show the sample game between Amy and Barry. Alternatively, you may show some sample rolls to illustrate the game.

Divide the class into groups of four. Each group of four will need four prime factor dice, and copies for each student of [Student Sheet 1 - Prime Dice Game](#) and [Student Sheet 2 - Prime Dice Scorecard](#).

Invite the students to play the game. Ask them to work out how the prime factorisation of a number reveals the properties of the different categories.

Enabling Prompt

- Ask the students to explore the prime factorisations for some even numbers and then odd numbers. What is different about the factorisations for odd and even numbers?
- Explore the prime factorisations for square numbers below 100. What is common to all these prime factorisations?

As students play the game, ask questions such as:

- *How are you working out where to place your score on the scorecard?*
- *What prime factors and/or combination of prime factors are needed for each category on the scorecard?*

<i>A number to the 4th power</i>	Four of a kind
<i>A number ending in 00</i>	There is only one option: 2, 2, 5, 5
<i>A number ending in 0</i>	You need prime factors of 2 and 5
<i>Square</i>	This can be four of a kind or 2 different pairs
<i>Odd</i>	Does not have 2 as a prime factor
<i>Even</i>	Must have 2 as a prime factor

Students will need to make decisions about which rolls are most likely, and place their scores strategically. This can form the basis of a productive class discussion. For example, if a student rolls a very small even number it may be a good strategy to put a zero in one of the less likely categories and hope for a higher scoring even number in a subsequent turn.

Extending Prompt

- What is the smallest number that is possible to roll for each category? What is the largest number that is possible to roll for each category?
- What is the largest possible score in a complete game? What is the smallest possible score if you complete every category?

The largest and smallest scores in each category are:

Category	Largest score	Smallest score (except for 0)
<i>A number to the 4th power</i>	$7^4 = 2401$	$2^4 = 16$
<i>A number ending in 00</i>	500	500
<i>A number ending in 0</i>	$7^2 \times 2 \times 5 = 490$	$3^2 \times 2 \times 5 = 90$
<i>Square</i>	$7^4 = 2401$	$2^4 = 16$
<i>Odd</i>	$7^4 = 2401$	$3^4 = 81$
<i>Even</i>	$7^3 \times 2 = 686$	$2^4 = 16$
<i>Total for the game</i>	8879	719

The game could be modified so that no combination can be used more than once.

Consolidation

Roll 6 prime factor dice. Using the prime factors, what can you tell me about your product?



My number is:

$$2 \times 3^2 \times 5^2 \times 7$$



My number is even because it has 2 as a prime factor.



My number ends in 0 (zero) as 5 and 2 are prime factors. If 2 was not a prime factor it would end in 5.

My number is greater than 1000. I know this because 7×5 is 35 and multiplying this by 3×3 will be greater than 100. I would then need to multiply this by $2 \times 5 = 10$.



My number is not square. For it to be square there would need to be an even number of each different prime factor. To make my number square I could multiply it by 2 and 7 or 14. I could also divide it by 14.



My number is not a perfect cube. For it to be a cube number there would need to be a multiple of three of each different prime factor. To make my number a perfect cube I could multiply it by $(2 \times 2) \times 3 \times 5 \times (7 \times 7)$, or something very large like $(2 \times 2) \times 3 \times 5 \times (7 \times 7) \times (31 \times 31 \times 31)$.

Objective

In this game you will be rolling prime dice and trying to get the biggest product possible in each of 6 categories.

Categories	Score for the Categories
A number to the 4 th power	The product of the prime numbers showing on the dice
A number ending in 00	This roll scores 500 points
A number ending in 0	The product of the prime numbers showing on the dice
Square	The product of the prime numbers showing on the dice
Odd	The product of the prime numbers showing on the dice
Even	The product of the prime numbers showing on the dice

How to Play

- When it is your turn, roll the dice. Think about the product of the numbers you have rolled.
 - What categories will the product be in?
 - Which category will give you the biggest score?
 - If you rolled one of the dice again could you get a different category or a bigger score?
- If you decide to try to improve your score, roll one of the dice again.
- Decide on a category and work out your score.
- The game ends when all the players have filled the six sections on the scorecard.

Rules

- If your product doesn't fit any of the remaining categories you will have to put a zero score instead.
- You can only record one number when it is your turn.
- You can only score once in each category.

Categories	Player's Names							
	Numbers rolled	Score						
A number to the 4 th power								
A number ending in 00 (500 pts)								
A number ending in 0								
Square								
Odd								
Even								
Total Score:								

Prime Dice Scorecard

Name: _____

Player's Names								
Categories	Numbers rolled	Score						
A number to the 4 th power								
A number ending in 00 (500 pts)								
A number ending in 0								
Square								
Odd								
Even								
Total Score:								

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