

CIRCLE AREA

Lesson 2: Rounding out Area

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Lesson abstract

The lesson consists of three stand-alone activities that, in different ways, consolidate students' knowledge of the formula for the area of a circle. The first activity provides pictures of circular objects in the outdoors for students to estimate the areas. The second activity enriches students' class textbook-based practice with the circle area formula by illustrating the variation that is required for comprehensive understanding. The third activity, **Precious Pendants**, provides challenging multi-step area calculations and an opportunity for creativity.

Mathematical purpose (for students)

The area of a circle can be estimated or calculated exactly using the formula, for many practical applications.

Mathematical purpose (for teachers)

This material builds students' skills with computation and estimations involving circle area in many variations. Together these ideas present a comprehensive view of the use of the formula for the area of a circle. Students calculate area from the radius or diameter and vice versa. The areas of compound shapes are calculated and some complex problems are solved. Students calculate with measurements given in a range of forms - whole numbers, fractions and decimals with differing numbers of decimal places. Sensible limitations of accuracy of answers can be discussed.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- formula
- substitute
- accuracy
- decimal places
- compound shape

Lesson Materials

- scientific calculator
- slide show [2a Area Calculations Intro powerpoint](#)
- [Student Sheet 1 - Circle Area Fluency](#) (1 per student, optional)
- [Teacher Sheet 1 - Circle Area Fluency](#)
- [Student Sheet 2 - Precious Pendants](#) (1 per student)
- [Teacher Sheet 2 - Precious Pendants](#)
- [Student Sheet 3 - Open Brief](#) (1 per student)

We value your feedback after this lesson via <http://tiny.cc/lesson-feedback>



Getting Started

In this introduction, students should view the slide show [2a Area Calculations Intro powerpoint](#) that presents a series of photos of five real world circular objects and asks a question related to the area of each.

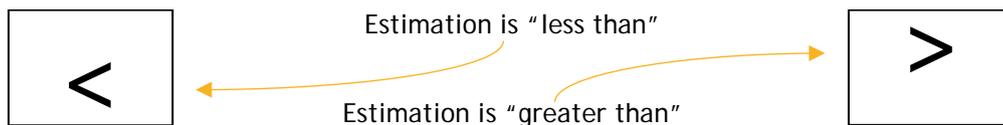
Suggested strategy

Move fairly quickly through the slides. In each set:

- Slide 1 describes a scenario.
- Slide 2 provides limited visual information.
- Slide 3 gives measurements. Continue to encourage estimation rather than precise calculations. For example the manhole cover (first problem) has a diameter of 60cm, which is not much more than half a metre. It would fit inside a square of side 60 cm, so the area is considerably less than a square metre.



Students should draw a large 'greater than' symbol in their notebook. When inverted it becomes 'less than'. They respond to the first slide by holding up the symbol in the direction that reflects their estimation. When new information becomes available on subsequent slides they can make increasingly refined estimations and if desired can change their chosen symbol. Alternatively a show of hands can be used.

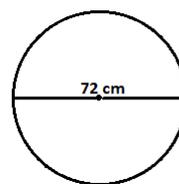


Fluency Task

[Student Sheet 1 - Circle Area Fluency](#) provides a variety of scenarios that are all part of fluency with area calculations. Because of the variety of items, this sheet is most suitable for review. Students should present their calculations so they can be easily followed by a classmate.

Solutions should include:

- A diagram showing measurements given.
- The appropriate formula.
- The value to be substituted.
- The result of the calculation.
- The answer to question with units.



$$A = \pi r^2$$

$$\text{When } r = 36 \text{ cm}$$

$$A = \pi \times 36^2$$

$$\approx 4\,071.51 \text{ cm}^2$$

Students should be encouraged to check the reasonableness of their answers. For the example above, the area should be about half way between these lower and upper bounds:

- Area is less than $3 \times 40^2 \rightarrow$ or $3 \times 4^2 \times 10^2 \rightarrow$ about 300×16 , say $4\,800 \text{ cm}^2$
- Area is more than $3 \times 30^2 \rightarrow$ or $3 \times 3^2 \times 10^2 \rightarrow$ about 300×9 , say $2\,700 \text{ cm}^2$

Students should be encouraged to carefully monitor the correctness of the answers to build self-assessment skills.

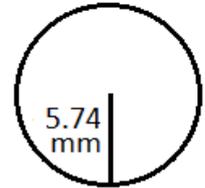
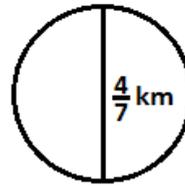
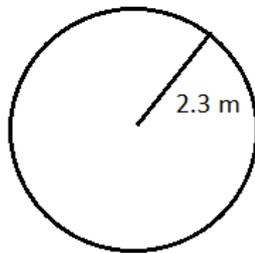
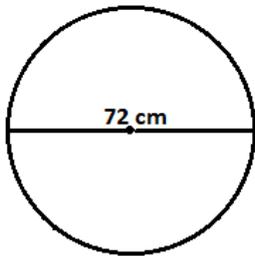
Application and Possible Assessment

In [Student Sheet 2 - Precious Pendants](#) students calculate the amount of material used in items of jewellery to provide a reasonable quote for the cost of materials. These are multi-step problems requiring complex calculations in an attractive context.

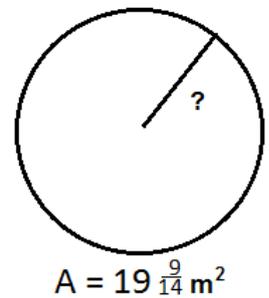
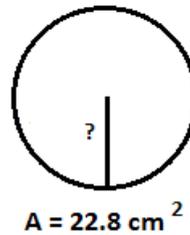
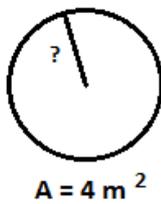
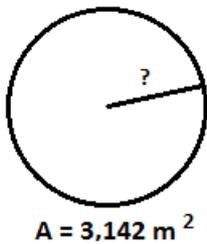
[Student Sheet 3 - Open Brief](#) provides open questions and allows students to use their creativity while meeting multiple criteria for the design.

Circle Area Fluency

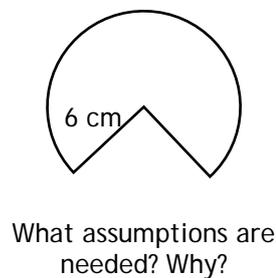
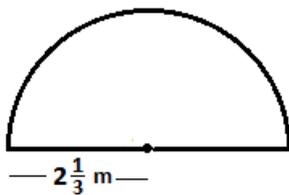
1. Find the area of each of these circles. *These items vary number type and radius/diameter.*



2. Find the radius of each of these circles. *These items vary number type.*



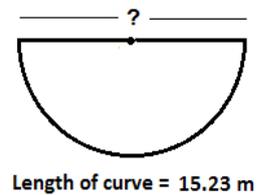
3. Find the area of these shapes that are sectors (fractions) of circles. *These are multi-step problems.*



What assumptions are needed? Why?



What assumptions are needed? Why?



Teacher Sheet 1 - Circle Area Fluency

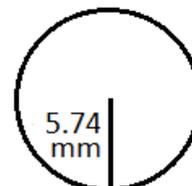
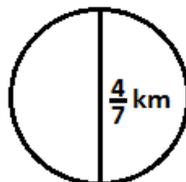
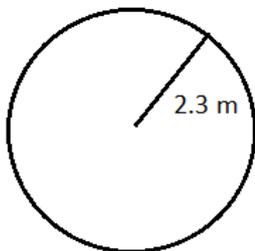
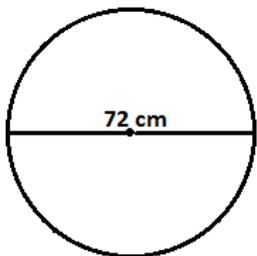
1. Find the area of each of these circles. *These items vary number type and radius/diameter*

$A = 4\,071.50\text{ cm}^2$

$A = 16.62\text{ m}^2$

$A = 0.26\text{ km}^2$

$A = 103.51\text{ mm}^2$



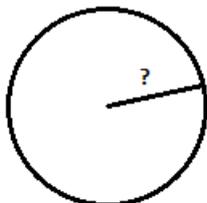
2. Find the radius of each of these circles. *These items vary number type.*

$r = 1.00\text{ m}$

$r = 1.13\text{ m}$

$r = 2.69\text{ cm}$

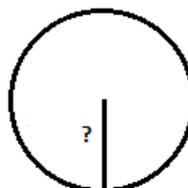
$r = 2\frac{1}{2}\text{ m}$



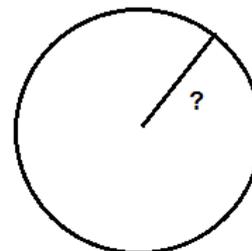
$A = 3,142\text{ m}^2$



$A = 4\text{ m}^2$



$A = 22.8\text{ cm}^2$



$A = 19\frac{9}{14}\text{ m}^2$

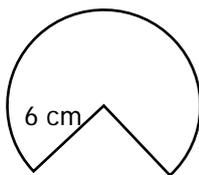
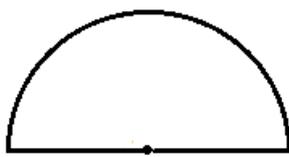
3. Find the area of these shapes that are sectors (fractions) of circles. *These are multi-step problems.*

$A = 8.55\text{ m}^2$

$A = 84.82\text{ cm}^2$

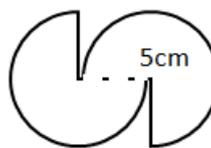
$A = 117.81\text{ cm}^2$

$A = 36.92\text{ m}^2$

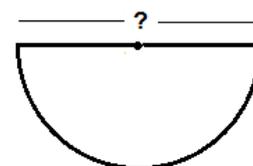


A sector with internal angle of 270°

i.e. $\frac{3}{4}$ of a circle



Two $\frac{3}{4}$ circles placed together



Length of curve = 15.23 m

Teacher Notes

- In Question 2, the fraction answer has been obtained using $3\frac{1}{7}$ for pi. It is reasonably accurate and was widely used before the advent of calculators.
- Fractions are rarely used as measurements nowadays. The two items with fraction measurements provide experience of dealing with fractions and decimals together (best to convert early!).
- Another important variation not on this worksheet is using pronumerals rather than number measurements.
- Answers on this sheet are given to two decimal places. To get this accuracy, more than two decimal places must be used for pi. We have also assumed that the given measurements are highly accurate (e.g. 72 cm is really 72.000000 cm and not just measured to the nearest centimetre as would probably really happen).

Precious Pendants

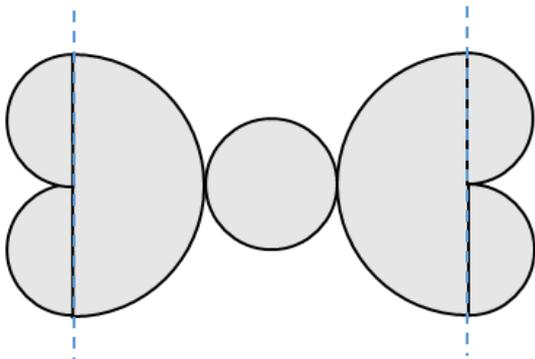
Jamos the Jeweller charges clients based on the amount of metal used in the solid pendant jewellery designs. A recent client wants some custom pieces made and Jamos is in a bit of jam regarding these intricate circular designs.

Can you help determine how much of the metal is in each design?
What would the metal cost?

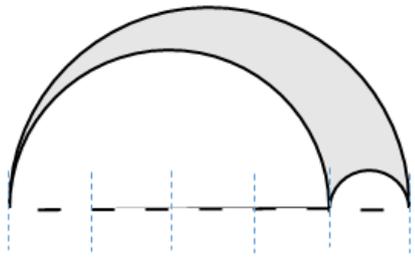
Note: Jamos charges clients \$45 per square centimetre of the metal used.

Please present your calculations as clearly as possible so that Jamos may check your figures.

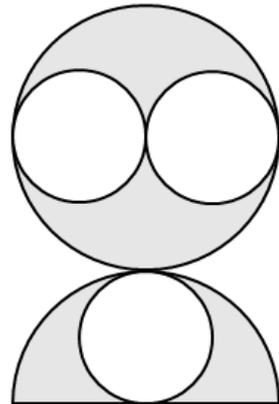
Note that if one circle looks to have double or one-quarter of the diameter of another it does.



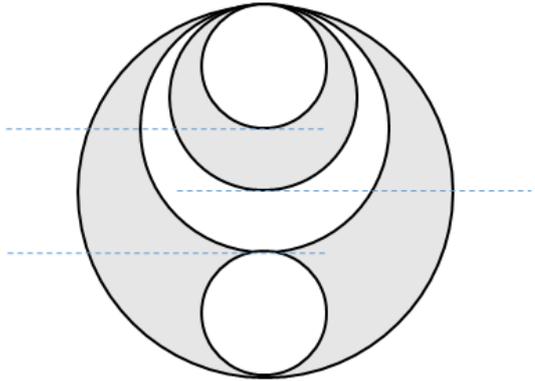
The full circle in the centre has a diameter of 2.8cm; it is the same size as each semi-circle on the sides



An arbelos with the smallest semi-circle diameter 11mm



The semi-circle has a diameter of 5.6cm



The small circle has a radius of 20mm

Recalculate the areas of each pendant, letting the radius of the smallest circle in each design be r cm. What do you notice about the areas of the component circles?

Teacher Sheet 2 - Precious Pendants

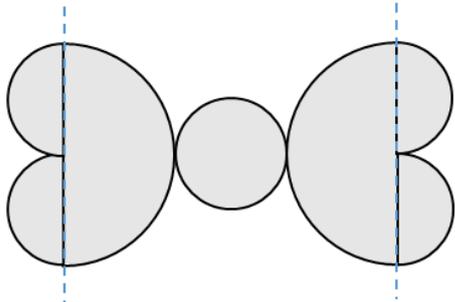
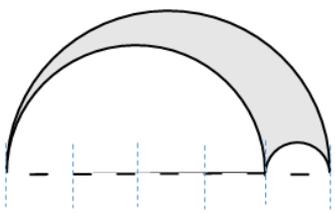
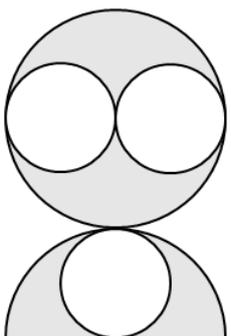
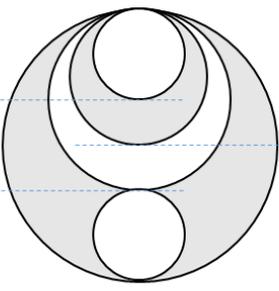
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Note that if one circle looks to have double or one-quarter of the diameter of another it does.

 <p>The full circle in the centre has a diameter of 2.8cm; it is the same size as each semi-circle on the sides</p> <p>Centre circle: $d = 2.8 \text{ cm}$ ($r = 1.4 \text{ cm}$) 4 semicircles $r = 1.4 \text{ cm}$; 2 semicircles $r = 2.8 \text{ cm}$ Total area = 43.1 cm^2 → Cost of metal = \$1939.60</p>	 <p>An arbelos with the smallest semi-circle diameter 11mm</p> <p>1 large semicircle $r = 27.5\text{mm}$ Cutting out: 1 semicircle $r = 5.5\text{mm}$; 1 semicircle $r = 22\text{mm}$. Total area = 380.13 mm^2 → Cost of metal = \$171.06 (Be sure to convert mm to cm!)</p>
 <p>3 semicircles $r = 2.8 \text{ cm}$ Cut outs: 3 circles $r = 1.4 \text{ cm}$ Total Area = 18.47 cm^2 → Cost of metal = \$831.27</p> <p>The semi-circle has a diameter of 5.6cm</p>	 <p>Large circle $r = 60 \text{ mm}$ Smaller circle $r = 30 \text{ mm}$ Cut outs: 2 circles $r = 20\text{mm}$ 1 circle $r = 40 \text{ mm}$ Total Area = 65.97 cm^2 → Cost of metal = \$2968.80</p> <p>The small circle has a radius of 20mm</p>
<p>Recalculate the areas of each pendant, letting the radius of the smallest circle in each design be $r \text{ cm}$. What do you notice about the areas of the component circles?</p> <p>SAMPLE FINDINGS: In the first diagram, the total area is 7 times the area of the central circle. The area of the arbelos is 10 times the area of the small cut-out semicircle. In general, if the small semicircle has diameter $1/n$th of the largest circle, the area of the arbelos is $2n$ times the area of the small cut out.</p>	

Open Brief

Name: _____

Design briefs have been provided by a customer who allows Jamos to create new designs. Can you help?

Draw each of your designs. Clearly mark the dimensions. Show the calculation of total area of metal and the cost.

Note: Jamos charges clients \$45 per square centimetre of the metal used.

<p>A circular pendant with a semi-circular cut-out and an overall cost for the metal of between \$250 and \$300.</p>	<p>A "snowman" consisting of three different sized circles, with the smallest circumference measuring exactly 1cm, and the total cost of metal is \$1025.</p>
<p>A large circular pendant with at least two circular cut outs and the cost of metal in the pendant equal to the cost of the metal in one or more of the cut outs. (opportunity for some algebra here)</p>	<p>A pendant consisting of seven different sized parts: one full circle; two semi-circles; and four quarter-circles. The part circles can either be joined to the main circle or cut out of it. The total cost of metal is between \$3500 and \$3800.</p>