WORKING WITH ALGEBRA
Lesson 2: Substitution

Australian Curriculum: Mathematics – Year 8

ACMNA190: Extend and apply the distributive law to the expansion of algebraic expressions.
- Applying the distributive law to the expansion of algebraic expressions using strategies such as the area model.

ACMNA191: Factorise algebraic expressions by identifying numerical factors.
- Recognising the relationship between factorising and expanding.

ACMNA192: Simplify algebraic expressions involving the four operations.
- Understanding that the laws used with numbers can also be used with algebra.

Lesson abstract

This resource contains a collection of tasks focussing on substitution. In Expressing Relationships, students determine a range of values that make a two-variable equation true. In What Can It Be?, students explore options that will make an incomplete identity true. In Temperature Conversion, students find values satisfying the relationship between temperatures expressed in degrees Fahrenheit and temperatures expressed in degrees Celsius.

Mathematical purpose (for students)

Practise substitution in algebraic expressions.

Mathematical purpose (for teachers)

These tasks develop fluency with substitution into algebraic expressions. They enable teachers to identify and address student misunderstandings about the use of algebraic notation. The tasks offer students opportunities to create and communicate a variety of solutions. Teachers can create tasks like this involving algebraic expressions of any desired complexity.

Lesson Length 20 to 30 minutes for each task

Vocabulary Encountered
- Coefficient
- Equation
- Expression
- Identity
- Square
- Substitute
- Term
- Trial and error
- Variable

Lesson Materials
- Student Sheet 1 – Expressing Relationships (1 per 10 students)
- Student Sheet 2 – What Can It Be? (1 per 6 students)
- Student Sheet 3 – Temperature Conversion (1 per 5 students)
- Teacher Sheet 1 – Expressing Relationships
- Teacher Sheet 2 – What Can It Be?
- Teacher Sheet 3 – Temperature Conversion

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Expressing Relationships

I know $\frac{2a}{3}+\frac{3b}{4}=\mathbf{120}$.
What might be the values of *a* and *b*?

(Give a range of possible answers)

Getting Started

- Provide students with the task card from Student Sheet 1 – Expressing Relationships and pose the task problem.
- Before beginning individual work, conduct a brief class discussion about the task, encouraging use of mathematical terminology such as variable, term, expression, substitute, solve, satisfy,
- Key discussion points might include:
  - Do the values of *a* and *b* need to be whole numbers or positive? (ANS: no)
  - How many answers are there likely to be?

Enabling Prompts

- Try a simpler problem first: I know $a + 2b = 60$. What might be the values of *a* and *b*? (Give a variety of possible answers.)
- Try simple special cases for the given problem. If $a = 0$, what is *b*? If $b = 0$, what is *a*?
- Choose values for *a* that are divisible by 3, to make the arithmetic easier.

Extending Prompts

- What if $a = b$? Solve this problem using logical reasoning, then using guess-check-improve, and then using algebraic equation solving.
- Represent all the solutions graphically.

Summarising

- Encourage students to share different strategies for finding values of *a* and *b* in groups and with the class.

Solutions

- There are many solutions. Some solutions and strategies are shown in Teacher Sheet 1 – Expressing Relationships.

What Can It Be?

The underlined box means that something is missing. What might be the letters or numbers missing in the following?

$$\mathbf{3}(\boldsymbol{a}+\underline{\square})-\underline{\square}=\underline{\square}\boldsymbol{a}+\underline{\square}$$

Getting Started

- Provide students with the task card from Student Sheet 2 – What Can It Be? or write it on the board.
- Initiate brief class discussion about the task, encouraging use of mathematical terminology such as variable, coefficient, expression, equation and identity.
- Clarify that this task is about finding different ways of making the expression on the left equal to the expression on the right for all values of *a* (that is, creating an identity). Note the difference with an equation.

Enabling Prompts

- Try an easier problem: Fill the boxes to make $2(a-\_\_)=2a-\_\_$ for all values of *a*?
- Check your answers numerically by substitution and also algebraically by expanding the left hand side of your identity to check that it is the same as the right hand side.

Extending Prompts

- Find another way to solve this problem.
- Explain why the missing symbols cannot all be the same as each other.
- Fill some of the boxes with numerical values that are not positive integers.
- Fill some of the boxes with variables.

Summarising

- Encourage students to share different strategies and solutions in groups and with the class. Ask them how they know that they have created an equation that is true for all values of *a*.

Solutions

- There are many solutions. Some solutions are available in Teacher Sheet 2 – What Can It Be?.

Temperature Conversion

There is a rule for converting Fahrenheit temperature (used in the US) to Celsius temperature (used almost everywhere else:

$$\mathbf{F} = \frac{9}{5}\mathbf{C} + \mathbf{32}.$$

What might be the values for F and for C?

(Give a range of possible answers)

Getting Started

- Provide students with the task card from Student Sheet 3 – Temperature Conversion or write it on the board.
- Before starting individual work, initiate brief class discussion, discussing the two temperature scales with reference to the weather forecast (for example) or a thermometer with both scales visible.

Enabling Prompts

- If the temperature is 20°C, what is it in °F?
- What is the freezing point of water in °C? What is this in °F?
- What is the boiling point of water in °C? What is this in °F?
- Make a table of values with °C in the first column and °F in the second.

Extending Prompts

- Represent the relationship graphically.
- What temperature is the same number, regardless of whether it is written in degrees Celsius or Fahrenheit?
- What are degrees K (Kelvin) ? Write a relationship between °F and °K.

Summarising

- Encourage students to share different strategies and solutions in groups and with the class.

Solutions

- There are an infinite number of solutions. Some are shown in Teacher Sheet 3 – Temperature Conversion.

re(Solve) MATHS BY INQUIRY — Expressing Relationships

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What might be the values of *a* and *b*?
(Give a range of possible answers)

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Teacher Sheet 1 - Expressing Relationships

Setting each term equal…

$$\frac{2a}{3} + \frac{3b}{4} = 120 \qquad 60 + 60 = 120 \qquad \frac{2a}{3} = 60 \quad a = 90 \qquad \frac{3b}{4} = 60 \quad b = 80$$

Setting a value for *a*:

$$\frac{2a}{3} + \frac{3b}{4} = 120 \qquad \text{Let } a = 45 \longrightarrow 30 + \frac{3b}{4} = 120 \qquad 3b = 360 \qquad \therefore b = 120$$

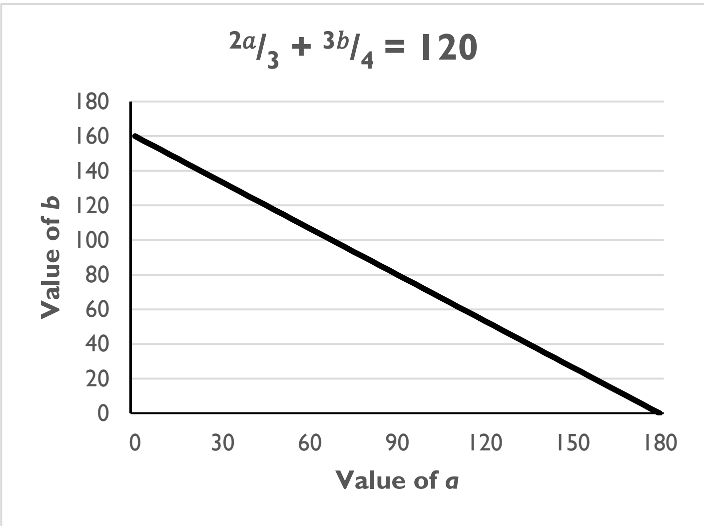
Setting a value for *b*:

$$\frac{2a}{3} + \frac{3b}{4} = 120 \qquad \text{Let } b = 40 \longrightarrow \frac{2a}{3} + 30 = 120 \qquad 2a = 270 \qquad \therefore a = 135$$

Common denominator and table of values or graph:

$$\begin{aligned} \frac{2a}{3} + \frac{3b}{4} &= 120 \\ 8a + 9b &= 1440 \\ 9b &= 1440 - 8a \\ b &= 160 - \frac{8}{9}a \end{aligned}$$

a	0	9	18	27	…	180
b	160	152	144	136	…	0



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What Can It Be?

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Teacher Sheet 2 - What Can It Be?

Solution 1

Use 3 as the coefficient of *a* on the RHS…

$$3(a + \_\_) - \_\_ = \underline{3}a + \_\_$$

We now have a range of possible sets of values that could complete the equation, such as

$$3(a + \underline{1}) - \underline{3} = \underline{3}a + \underline{0}$$

$$3(a + \underline{2}) - \underline{3} = \underline{3}a + \underline{3}$$

Solution 2

Use 2 as the coefficient of *a* on the RHS…

$$3(a + \_\_) - \_\_ = \underline{2}a + \_\_$$

To get 3*a* on the RHS we could use *a* in the last box.

$$3(a + \_\_) - \_\_ = \underline{2}a + \underline{a}$$

This now means that the LHS must be 3*a* giving a range of possible solutions all of the form:

$$3(a + \underline{1}) - \underline{3} = \underline{2}a + \underline{a}$$

$$3(a + \underline{2}) - \underline{6} = \underline{2}a + \underline{a}$$

where the second box must be 3 times the first box.

Solution 3

Use *a* as the missing value in the first box.

$$3(a + \underline{a}) - \_\_ = \underline{3}a + \_\_$$

We now have 6*a* on the LHS, which we could balance by adding 3*a* on the RHS. This gives

$$3(a + \underline{a}) - \underline{0} = \underline{3}a + \underline{3a}$$

Alternatively we could use 2*a* on the LHS and *a* on the RHS:

$$3(a + \underline{a}) - \underline{2a} = \underline{3}a + \underline{a}$$

There is a range of other possible solutions.

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Temperature Conversion

There is a rule for converting Fahrenheit temperature (used in the US) to Celsius temperature (used almost everywhere else):

$$\mathbf{F = \frac{9}{5}C + 32.}$$

What might be the values for F and for C?

(Give a range of possible answers)

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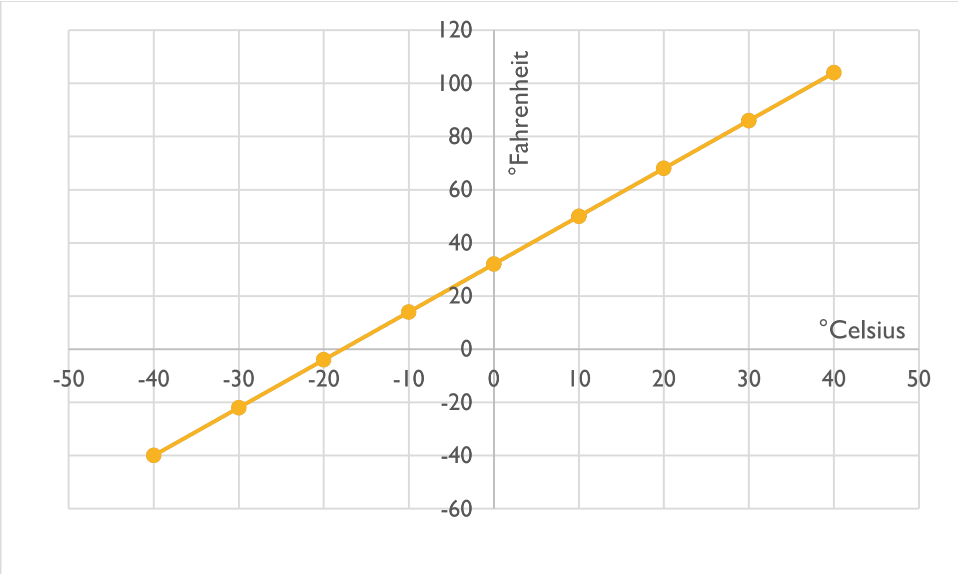
Teacher Sheet 3 - Temperature Conversion

Choose values of C that are multiples of 10:

C	⁻40	⁻30	⁻20	⁻10	0	10	20	30	40
F	⁻40	⁻22	⁻4	14	32	50	68	86	104

A graphical representation:



Let the temperature with the same number of degrees in Fahrenheit and Celcius be F degrees in Fahrenheit.

Then $F = \frac{9}{5}F + 32$

$-\frac{4}{5}F = 32$

$\therefore F = -40$

K stands for degrees Kelvin, and is the temperature measured using the same units as Celsius, above absolute zero, the temperature at which all thermal motion ceases. 0°K is -273.15°C.

So C = K – 273.15, hence $F = \frac{9}{5}K - 459.67$

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