re(Solve) MATHS BY INQUIRY Circumference

Y8

Summary of learning goals

- Students develop a strong intuitive sense that the ratio of circumference to diameter is the same for all circles and use a variety of approaches to find a more accurate value for the ratio. They then apply their knowledge by making a diameter measuring tape, from which they can read off the diameter of a cylindrical object.

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference (and area).

Summary of lessons

Who is this sequence for?

- These lessons assume only elementary knowledge of circles—that all the points on the circumference are the same distance from the centre—and an intuitive understanding of ratio.

Lesson 1: Spheres in a Cylinder

Students compare the height of a cylinder containing three tennis balls to its circumference. They ask what would happen if the balls were smaller or larger and conclude that it is always the case that the circumference is a little more than three times the diameter. They apply this to estimations relating to real-world circular objects.

Lesson 2: A Better Value for π

Students are reminded of the conclusion from the previous lesson: the ratio of circumference to diameter is '3 and a bit'. Students use a variety of contexts to find the ratio of circumference to diameter and, hence, find a better value for π. They discuss how the accuracy of the value they find could be improved.

Lesson 3: Measuring Tree Trunks

Students apply their knowledge of the ratio of circumference to diameter to make a d-tape, which can be wrapped around a cylindrical object, such as a tree trunk, to instantly measure its diameter.

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Reflection on this sequence

Rationale

This sequence introduces properties of circles, focusing on the relationship between the circumference and the diameter (and radius) rather than on calculation. An initial value of 'a bit more than 3' is refined through measurements in a range of contexts to introduce the concept of π as the ratio of circumference to diameter. Students apply their knowledge to the practical problem of measuring the diameter of a tree.

reSolve mathematics is purposeful

- The sequence emphasises real-world applications of mathematics, encouraging students to estimate and consider issues of accuracy in measurement.

reSolve tasks are challenging yet accessible

- The initial activity provides a common experience that allows all students to access subsequent tasks.
- Physical materials and measurements in a range of contexts provide all students with the opportunity to refine and consolidate their understanding

reSolve classrooms have a knowledge-building culture

- The emphasis on sense-making rather than on measurement or calculation lays the foundation for a deep understanding of the relationship between the circumference and diameter of a circle.
- The use of a range of contexts develops understanding of the circle as a geometric shape, as a limit of polygons, and as a locus of points equidistant from the centre.

LESSON 1

re(Solve) MATHS BY INQUIRY

Circumference

Spheres in a Cylinder

Y8

About this lesson

Students compare the height of a cylinder containing three tennis balls to its circumference. They ask what would happen if the balls were smaller or larger and conclude that it is always the case that the circumference is a little more than three times the diameter. They apply this to estimations relating to real-world circular objects.

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles, such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Mathematical purpose

- Students determine that the ratio of circumference to diameter of a circle is a little more than 3. They learn to approximate the **circumference** of a circle when given its **radius** or **diameter**, and to approximate the diameter or radius when given the circumference.

Learning intention

- To estimate the circumference of a circle when you know its diameter, and vice versa.

Time

A lesson of approximately 45 minutes.

Vocabulary

- circle
- circumference
- cylinder
- diameter
- radius

Resources

- reSolve PowerPoint *1a Spheres in a Cylinder*
- a cylinder containing three tennis balls
- a roll of masking tape (optional)
- string
- ruler

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The inquiry

Resources: Show students slide 2 of reSolve PowerPoint *1a Spheres in a Cylinder*.

This shows a cylinder containing three tennis balls.

Ask: *Is the height of the cylinder greater than or less than its circumference? Are you sure?*

Encourage students to commit to a yes or no answer, perhaps by asking those who think the height is greater than the circumference to move to one side of the classroom and those who think it is less to move to the other.

Ask students to check their answer using an actual cylinder of tennis balls. Encourage students to think of a way to check the answer using direct comparison, rather than measurement. The focus should not be on numbers and calculation. One way might be to wrap string around the cylinder and compare the length of string to the height of the cylinder. Another is to roll the cylinder exactly once and compare this to the height of another cylinder.

Follow-up question (slide 3): *Some tennis balls are sold in packs of four. Will the height of a four-tennis ball cylinder be greater than or less than its circumference?*

The follow-up

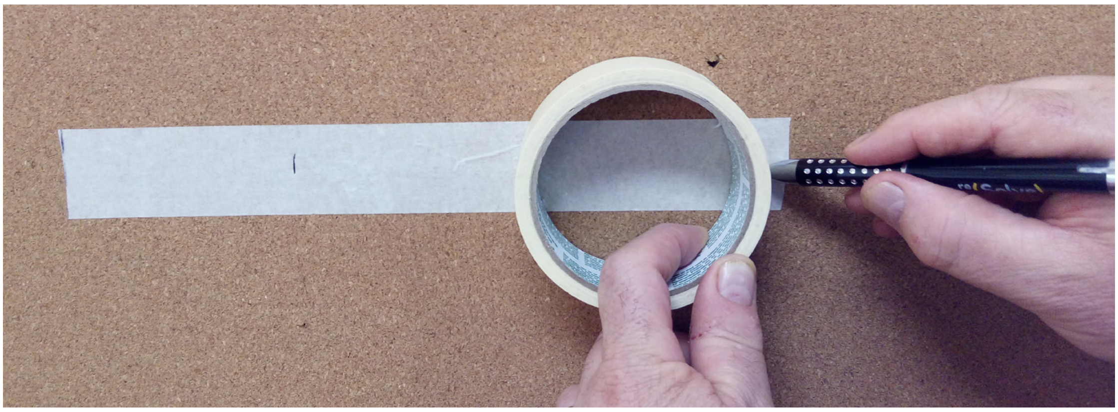
Ask students: *How much bigger or smaller would we have to make the balls so that the height of a cylinder containing three balls is equal to its circumference?*

Teacher note:

- This question is intended to develop a strong intuitive sense that all spheres are similar and all circles are similar; that is, the size of the sphere or circle has no effect on the relationship between circumference and diameter, which can be expressed as the circumference is always a little more than three times the diameter.

Another demonstration

A nice additional demonstration of the relationship between the circumference and diameter of a circle is to unroll exactly one layer of masking tape and stick it to the whiteboard. The roll can then be placed along this piece of tape and the diameter marked as it is moved along. Students will see that the diameter fits into the circumference a little more than three times.



Although many students will already know the formula $C = \pi d$, this lesson places the emphasis on developing a sense of what this means, rather than focusing on the mechanics of calculation. Lesson 2 develops a more accurate value for π.

Further activity

Go through slides 4–15 of reSolve PowerPoint *1a Spheres in a Cylinder* as a class. These slides show a series of photos of real-world circular objects along with a question related to the diameter or circumference of the object. Each object has three slides with progressively more information on each slide:

1. The first slide describes a scenario.
2. The second slide provides limited visual information.
3. The third slide provides a frame of reference.

Encourage brief sharing of ideas and give students the opportunity to adjust their answer after each slide, in response to the new information.

Teacher notes:

- Students could prepare a sheet of paper with 'is greater than' and the $>$ symbol written on one side and 'is less than' and the $<$ symbol written on the other side.
- Students respond to the first slide of each set by holding up the symbol that reflects their estimate.
- When new information becomes available on subsequent slides, students can make a better estimate and, if desired, can change their symbol.

Some reasonable estimates:

- If the man next to the wheel of the penny farthing bicycle is 1.8 m tall, then the diameter of the wheel is around 1.6 m. So the circumference would be greater than 3×1.6 m; that is, more than 4 m.
- The circumference of the wheel of a mine dump truck is about the height of two people or around 3.5 m. So the circumference is greater than 3×3.5 m; that is, more than 10 m.
- Since the circumference of the MILO® tin is a bit more than three times the diameter, the diameter is a bit less than one-third of the circumference. So if the circumference is around 60 cm, then the diameter will be less than 20 cm.
- The length of a pen is around 15 cm, which is about the length of the minute hand. So the diameter of the circle traced by the minute hand will be a bit more than 3×30 cm; that is, very close to 1 m.

LESSON 2

re(Solve) MATHS BY INQUIRY Circumference

A Better Value for π

Y8

About this lesson

Students are reminded of the conclusion from the previous lesson: the ratio of circumference to diameter is '3 and a bit'. Students use a variety of contexts to find the ratio of circumference to diameter and, hence, find a better value for π. They discuss how the accuracy of the value they find could be improved.

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles, such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Mathematical purpose

- Students learn that the ratio of circumference to diameter of a circle is the same for all circles and is called π. They understand that π is an irrational number and can be approximated as 3.14 but cannot be written as an exact fraction or as a terminating or repeating decimal.

Learning intention

- The ratio of circumference to diameter of a circle is π.

Time

One to two lessons of approximately 1 hour each.

Vocabulary

- π (pi)
- circumference
- circumscribed
- diameter
- inscribed
- irrational
- polygon
- radius
- ratio

Resources

- *Circle Runner* GeoGebra animation: https://www.geogebra.org/classic/EDZrFfZ9
- Activity 1:
  - ◊ objects with a circular cross-section: some large, some small (e.g. plates, drink bottles)
  - ◊ tape measure, or string and ruler
  - ◊ scientific calculator
- Activity 2:
  - ◊ pair of compasses
  - ◊ ruler
  - ◊ protractor
  - ◊ scientific calculator
- Activity 3:
  - ◊ rope (approx. 5 m)
  - ◊ large open space, preferably with a pole (e.g. netball court)
  - ◊ scientific calculator

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Teacher background information

This lesson suggests three activities to improve the '3 and a bit' estimate for the ratio of circumference to diameter. Activity 1 builds on students' familiarity with circles as geometric shapes through measuring the circumference and diameter of several circular objects. Activity 2 uses inscribed polygons, following a method of Archimedes which emphasises that a circle is the limiting case of a regular polygon with a very large number of sides. Activity 3 involves pacing the circumference of a circle using a rope as the radius. It emphasises that a circle is the locus of all points equidistant from the centre.

It is important that students are exposed to all three activities, either through doing all three themselves or doing a single activity themselves and listening to the explanations of the other activities. In each case, encourage students to record results systematically, both individually and as a class. Discuss the importance of finding the average when there are many measurements, and how the accuracy can be improved using larger objects or polygons with more sides.

A brief history of π

The Babylonians (1900–1680 BCE), the ancient Egyptians (c. 1650 BCE), the ancient Greeks (300–200 BCE) and the early Chinese (400–500 CE) had all established reasonably accurate approximations for the ratio of circumference to diameter. Mathematicians began using the Greek letter π in the 1700s. The symbol π, the Greek letter 'p', is used to represent this ratio because the ratio concerns the perimeter (circumference) of a circle. The Greek word for perimeter is περίμετρος (perimetros).

π is an irrational number, meaning that it cannot be expressed exactly as a fraction. Its decimal representation does not end and has no obvious pattern in the digits. Other irrational numbers that students may have encountered include all of the square roots and cube roots that cannot be evaluated exactly; for example, $\sqrt{2}$, $\sqrt{3}$ and $\sqrt[3]{5}$ are irrational numbers.

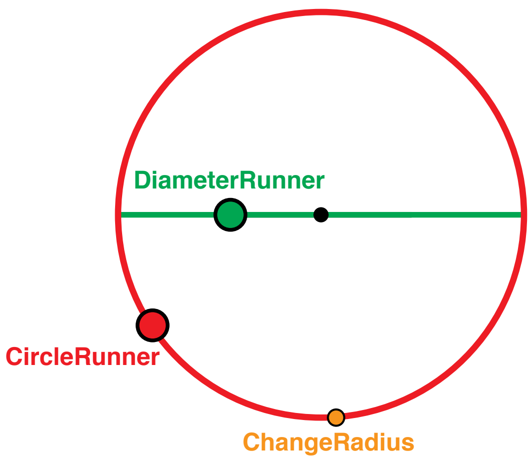
Today π is known to more than one trillion digits. Scientific calculators store a value of π correct to 10 decimal places, which saves us from having to memorise the value. However, students should remember that its value is about 3.14. Here are a couple of references to brief histories of pi:

- www.exploratorium.edu/pi/history_of_pi/
- http://www.pcworld.com/article/191389/a-brief-history-of-pi.html

Getting started

Resources: Use the GeoGebra Circle Runner animation at https://www.geogebra.org/classic/EDZrFfZ9 to remind students that the circumference of a circle is a bit more than three times the length of the diameter.

Note that the animation shows the correct ratio only because the speeds of both runners are the same.



LESSON 2

Finding a better value for π

Activity 1: Plates, balls and other round things

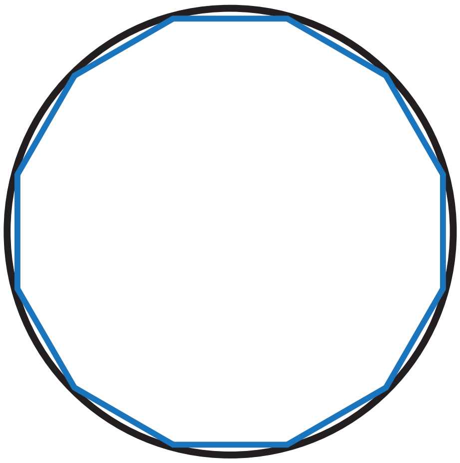
Measure a variety of circular objects, such as drink bottles and paper plates, to find the ratio of circumference to diameter.

Discuss:

- *How do we know where to measure the diameter?*
- *How should we organise our data?*
- *How do we deal with a variety of different ratios?*
- *How can we improve the accuracy of our calculated ratio?*

Activity 2: Polygons in the circle

Draw a circle and construct a regular polygon inside the circle so that the vertices are on the circumference of the circle. In the diagram below, the inscribed polygon has 12 sides.



Measure the perimeter of the polygon to find an approximation for the circumference of the circle and, hence, calculate the ratio of circumference to diameter. Discuss:

- *How do we deal with different ratios found by different students?*
- *How can we improve the accuracy of our calculated ratio?*

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Teacher note:

- This is an adaptation of the method of Archimedes (c. 250 BCE). Archimedes used inscribed and circumscribed polygons to find a lower and upper bound for circumference.

Activity 3: Walk around the circumference

Using a rope as the radius of a circle, students count the number of shoe lengths as they walk around the circumference. They find the ratio of the number of shoe lengths of the circumference to the number of shoe lengths along the radius.

Discuss with students:

- *Does it matter whether we use shoe lengths or more formal units of measurement?*
- *How do we deal with a variety of different ratios?*
- *How can we improve the accuracy of our calculated ratio?*

LESSON 3

re(Solve) MATHS BY INQUIRY Circumference

Measuring Tree Trunks

Y8

About this lesson

Students apply their knowledge of the ratio of circumference to diameter to make a d-tape, which can be wrapped around a cylindrical object, such as a tree trunk, to instantly measure its diameter.

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles, such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Mathematical purpose

- This lesson reinforces the proportionality of circumference to diameter by asking students to design and make a d-tape, from which they can instantly read the diameter of a cylindrical object. Students use their knowledge of the ratio of circumference to diameter of a circle to make a practical measuring instrument.
- Note that the reSolve Year 9 resource Tree Biomass makes use of a d-tape to find the volume and biomass of trees. If students are familiar with similar triangles, the Year 9 lesson could be used immediately following this lesson.

Learning intention

- To use your knowledge of circumference and diameter to make a tape that measures the diameter of a tree trunk.

Time

One to two lessons of approximately 1 hour each.

Vocabulary

- d-tape
- diameter at breast height (DBH)

Resources

- reSolve PowerPoint *3a Measuring a Tree Trunk*
- rolls of paper, such as those used in EFTPOS machines (available at stationery stores) or similar
- rulers or measuring tape
- scientific calculators

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Designing, making and using a d-tape

Resources: Show students slide 2 of reSolve PowerPoint *3a Measuring a Tree Trunk*.

This shows a photograph of a d-tape being used to measure the diameter of a river red gum's trunk.

Explain to students that even though the tape is wrapped around the trunk, the measurement shown on the tape is the diameter of the trunk, not the circumference. The tape is called a d-tape, and it is used to measure the diameter of the tree's trunk at 'chest height', which is 130 cm by convention. This measurement is called the DBH or diameter at breast height, and is used to calculate tree volume and mass.

The DBH of the river red gum's trunk is 120 cm.

Ask students: *How would you design a tape that instantly shows you the diameter of a tree trunk?*

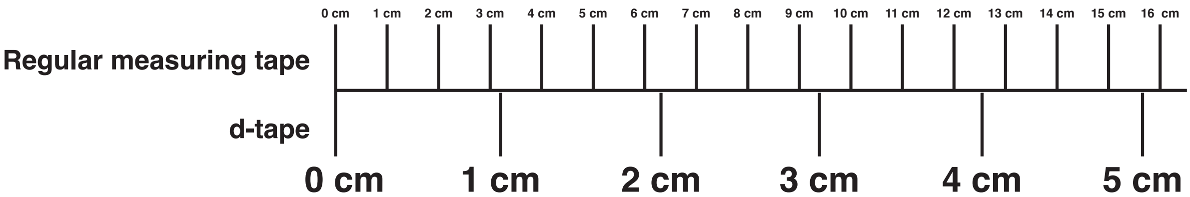
Enabling prompt:

- *If a tree trunk has a diameter of 1 m, what is its circumference?*
  How far from the start of the tape should you position the 1 m mark?

Resources: Give each group of students around 5 m of blank tape, such as that used in EFTPOS machines. Ask them to make their own d-tape. A more permanent d-tape could be made from ribbon.

Teacher note:

- d-tape is marked in intervals of π units rather than single units, as shown below.



- When wrapped around a cylinder with a circumference of 3.14 cm, the d-tape will show 1 cm (i.e. the diameter).

Ask students to measure the DBH of some tree trunks in their neighbourhood and compare answers. They could also use their d-tape to measure objects of known diameter, such as pipes, in order to check the accuracy.

Reflection

Ask students:

- *What property of circles did you use in making your d-tape?*
- *Why might foresters use a d-tape rather than a standard measuring tape?*
- *Why is it important for foresters to know the diameter of a tree trunk?*