re(Solve) MATHS BY INQUIRY

Gradient and Tangent

Y9

Summary of learning goals

- Students make links between gradient and angle by applying their understanding of the tangent ratio to find the actual angles represented by a road grade sign or the angle of a street.

Australian Curriculum: Mathematics (Year 9)

ACMMG224: Apply trigonometry to solve right-angled triangle problems.

ACMNA294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.

Summary of lessons

Who is this sequence for?

Students who are familiar with:

- the use of the three basic trigonometric ratios to find unknown angles
- plotting points and lines on the Cartesian plane.

Lesson 1: How Steep is That?

Students are presented with examples of road grade signs, research what is a road grade and determine the actual angle of a road when given its grade. Students then construct their own road sign using the actual angle of elevation, and confirm their understanding by creating a sign for a ramp or slope around their school.

Lesson 2: The World's Steepest Street

Students use photos and YouTube videos of Baldwin Street, in New Zealand (the steepest street in the world at the time of publishing) to find the angle of elevation and tangent ratio of the street, then model the road system on a Cartesian plane. They analyse the elevation profile of the street and assess the accuracy of claims made about Baldwin Street.

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Reflection on this sequence

Rationale

Through this sequence students come to appreciate the importance of trigonometry in real-world applications. Their conceptions of angle and gradient are challenged when they realise that the actual angle represented on road signs is remarkably small.

reSolve mathematics is purposeful

- The sequence shows how mathematical ideas are represented and used in a real-world context.
- Mathematical ideas of gradient and tangent are connected.

reSolve tasks are inclusive and challenging

- The sequence activates students' knowledge of trigonometry through the investigation of a real-world context.
- Narrative concerning steepness of streets brings the investigation to life to motivate and engage students.

reSolve classrooms have a knowledge-building culture

- The sequence challenges existing conceptions as students realise that their initial estimates of angles represented by a road sign are grossly exaggerated.

LESSON 1

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Gradient and Tangent

How Steep is That?

Y9

About this lesson

Students are presented with examples of road grade signs, research what is a road grade and determine the actual angle of a road when given its grade. Students then construct their own road sign using the actual angle of elevation, and confirm their understanding by creating a sign for a ramp or slope around their school.

Australian Curriculum: Mathematics (Year 9)

ACMMG224: Apply trigonometry to solve right-angled triangle problems.

ACMNA294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.

Mathematical purpose

- To explore the relationship between road grades and angles of elevation.

Learning intention

- To investigate the meaning of road grades and explore the relationship between road grades and angles of elevation.

Time

A lesson of approximately 1 hour.

Resources

- reSolve PowerPoint *1a How Steep is That?*

Vocabulary

- angle of elevation
- gradient
- inclined plane
- road grade

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Interpreting road grade signs

Ask students if they have ever encountered a road grade sign. These are typically yellow caution signs with a diagram of an inclined plane and a percentage grade figure.

Resources: Display the photo below (included in reSolve PowerPoint *1a How Steep is That?*).

Teacher note:

- Sometimes these road signs use a ratio rather than a percentage (e.g. 1:3). This task will refer exclusively to percentages.

Ask students how they would interpret this sign. Discussion questions include:

- *What is the purpose of the road sign?*
- *What might this percentage mean?*
- *How would a driver or cyclist interpret this sign? Would the interpretation mean the same thing to a driver and a cyclist or even to a truck driver?*
- *Is the percentage grade the same as an angle in degrees? Is a 15% grade equivalent to 15°?*
- *Does the inclined plane depicted on the sign represent the grade given?*
- *What might the person who put the bike in the picture be trying to express?*

Investigation

Ask students to estimate the angle depicted by the slope on the sign. This should assist them in making an informed judgement about the meaning of 15% grade. Is grade the same as 'angle'?

Have students measure the actual angle of elevation of the inclined plane depicted on the sign in the photo.

Teacher notes:

- The method to measure the angle can be chosen to suit the class; for example:
  - ◊ Students can be given copies of the photograph to measure physically, using a protractor.
  - ◊ Students can use GeoGebra or its equivalent to measure the angle virtually.
    - ■ Using GeoGebra is an excellent opportunity to consolidate technology skills. A background image can be added to the Cartesian plane in GeoGebra and then an angle superimposed (shown at right).

Possible student responses:

- Students will observe that the angle is nearly 27°. So 15% is not 15°.
- Students might also note that every road gradient sign appears to show the same angle, but different gradient figures. So there seems to be little to no relationship between the picture, the road grade and the angle.

Pose the inquiry: *What would the road sign look like if it was a realistic representation of a 15% grade?*

Calculating the road angle

If necessary, explain that the road grade is actually a measure of the vertical rise per unit of horizontal distance. This can be represented as a ratio, a decimal or a percentage. Therefore, a 15% grade is the same as a gradient of 0.15 or 3:20.

Guide students to use the information in the road sign to create a right-angled triangle, using prompts:

- *What information can we use to make a triangle?*
- *What will making a triangle help us to do?*
- *What information do we need to calculate the angle of a slope?*

Have students use the right-angled triangle to calculate the actual angle corresponding to a grade of 15%.

Possible student responses:

- It is assumed that students are familiar with the three basic trigonometric ratios, and that they are able to use their scientific calculator to find the size of an angle for a given ratio. Although the $\tan^{-1}$ notation is not preferred, it is included here for procedural accuracy with the scientific calculator.
- The **gradient** is defined as the rise of a slope divided by the run.

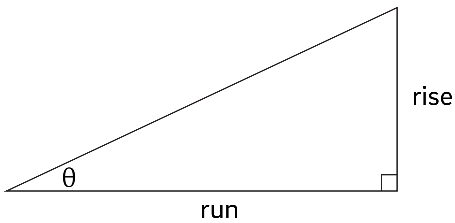
  $$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

  $$\tan\theta = \frac{\text{rise}}{\text{run}}$$

  $$\theta = \tan^{-1}\left(\frac{\text{rise}}{\text{run}}\right)$$

  Therefore:

  $$\theta = \tan^{-1}(\text{gradient})$$



- For example, when given a 15% road grade:

  $$\text{Gradient} = 0.15$$

  $$\theta = \tan^{-1}(0.15)$$

  $$\theta = 8.53° \text{ (to 2 decimal places)}$$

Students can now draw an accurate representation of a 15% grade. However, they will note that it appears very unimpressive.

Reflection

Find places around the school that use a slope to enable accessibility (e.g. a ramp for wheelchair access). Calculate the gradient of the ramp/slope and create an appropriate sign for this. Students might find it useful to research the Australian Standards for wheelchair access.

- Note that the Australian Standards give a ratio for slopes rather than a percentage grade. Why might this be more useful than a percentage for constructions such as ramps?
- How does the gradient of the ramp compare to the gradient of any steps or stairways in the school?
- Compare the length of the ramp to the depth (i.e. horizontal length) of the stairs. How much more distance is required to cover the different gradients?

If students are familiar with graphs of straight lines on the Cartesian plane, make the link between the gradient of the line and the angle the line makes with the x-axis.

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The World's Steepest Street

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About this lesson

Students use photos and YouTube videos of Baldwin Street, in New Zealand (the steepest street in the world at the time of publishing) to find the angle of elevation and tangent ratio of the street, then model the road system on a Cartesian plane. They analyse the elevation profile of the street and assess the accuracy of claims made about Baldwin Street.

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ACMMG224: Apply trigonometry to solve right-angled triangle problems.

ACMNA294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.

Mathematical purpose

- Students continue to explore the relationship between road grade and angle of elevation from Lesson 1, and investigate different representations of road slope grades.

Learning intention

- To find the road grade and angle of elevation of the world's steepest street.

Time

A lesson of approximately 1 hour.

Resources

- reSolve PowerPoint *2a The World's Steepest Street*

Vocabulary

- angle of elevation
- grade

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Teacher background information

Baldwin Street in Dunedin, New Zealand holds the Guinness World Record for the world's steepest residential street.

From 2001 to 2017, Dunedin hosted the running of the balls competition.

> *Thousands of Jaffas are released from the top of Baldwin Street in Dunedin for the annual Jaffa Race. 25,000 of the traditional red Jaffas, then 25,000 yellow and finally 25,000 of the pink sweets are rolled down Baldwin St, leaving behind a multi-coloured smear on the street – and a winning Jaffa in each category. Arguably the best position to watch the race is at the bottom, where hundreds of children and a surprising number of adults wait to collect the sticky treats.*
>
> *The race has raised more than $600,000 for charities such as Make-A-Wish, Surf Life Saving New Zealand and the Dunedin Parents Centre. Jaffas have been made in Dunedin since 1931, and New Zealanders consume an average of 66,000 of Jaffas a day.*
>
> https://www.stuff.co.nz/life-style/food-wine/food-news/70313844/75000-jaffas-hurled-from-worlds-steepest-street (N.B. Jaffas® – red, round orange-flavoured sweets with chocolate centre.)

Unfortunately, Cadbury, which used to make the Jaffa in New Zealand, closed their factory in Dunedin in March 2018 and there hasn't been an annual Jaffa Race since 2017.

Some data for Baldwin Street

- Road length is approximately 350 m.
- Change in elevation from start of road to cul-de-sac (top end of road) is approximately 70 m (from 30 m above sea level to 100 m above sea level).
- Maximum street grade is about 35%.
- Average street grade is around 19% but also reported as more than 1:5.

Introduction

Ask students where the steepest street in the world might be. Do they think they could cycle or run up it? What are some of the steepest tracks they've been up?

Have students estimate the angle of inclination of the world's steepest street and record their estimates for later reference.

Provide some background on the Jaffa Race in Dunedin, which used to occur annually up until 2017. Use a YouTube clip to introduce the race context; for example:

- https://www.youtube.com/watch?v=gxporA_0rYM (the 2015 Jaffa Race)
- https://www.youtube.com/watch?v=a9uO3KCJImA (a cyclist rides up Baldwin St)

Exploration

Resources: See slides 3 and 4 of reSolve PowerPoint *2a The World's Steepest Street*.

Students analyse one (or both) of these streetscape photos of Baldwin Street to estimate the angle of the street (the angle of elevation). They draw on principles from Lesson 1: How Steep is That? to estimate the percentage road grade.

Teacher note:

- The method to do so should be chosen to suit the class; the angle can be measured physically using a protractor on the photograph or it can be measured virtually using GeoGebra or its equivalent.

Students use the data to create and present a road grade sign for the street, as in Lesson 1.

Graphing a slope

Ask students to find the angle of elevation by graphing the slope:

1. Using Google Maps (or similar), find the length of Baldwin Street, and estimate how much higher the top of the road is than the bottom.
2. By drawing an appropriate diagram, calculate the *horizontal distance* and the *gradient* of the road.
3. Using all this information, construct an **appropriately labelled** linear graph (on a Cartesian plane) to model the road.
4. By analysing the graph, students can determine the value of the gradient, as a decimal fraction, and find the angle of elevation of the street (by measurement and/or calculation). Have students compare these results to their original estimate of the road grade.

One website reported on the 2015 Jaffa Race as follows:

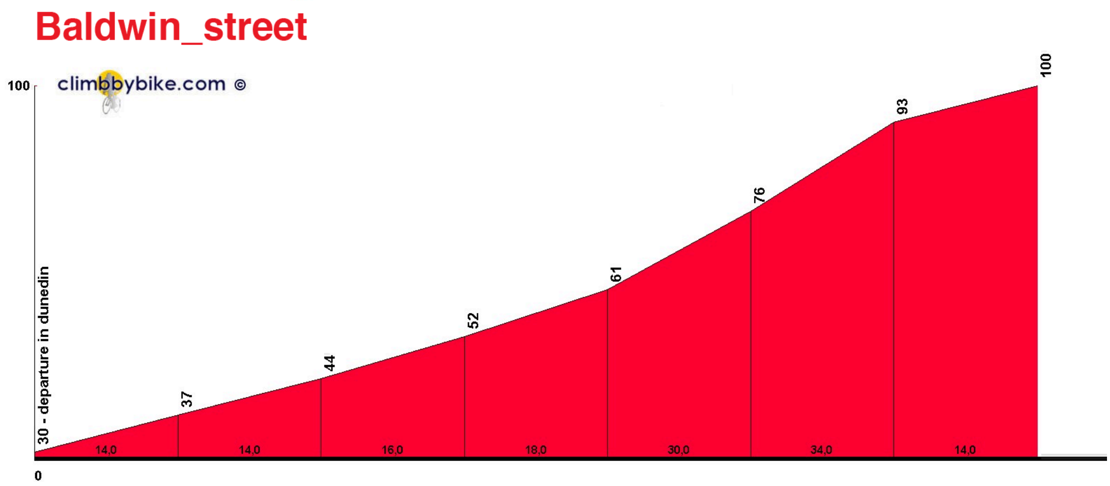
> *Each giant Jaffa weights 2.5 g, the distance travelled is 350 m, the velocity is 100 km/h (28 m/s), time is 25 seconds. And for those of you who are interested, Usain Bolt gets to a maximum speed of less than half that of a Jaffa rolling down Baldwin Street; about 12.27 m/s.*
> (https://www.instagram.com/p/5OGDwkpT0A/)

Are these claims plausible? Why or why not?

Reflection

Cycling website Climbbybike.com provides the following cycling profile for Baldwin Street. The information on the unlabelled vertical axis is clear but the information on the horizontal axis is less so. Discuss what information this graph is providing.

Ask: *How useful is the graph? What might be problematic about this graph?*



Teacher note:

- The horizontal axis provides percentage gradients for the different sections of the street. Note the European decimal format.

Further activities

Activity 1

Investigate the road profiles of the stages in the Tour de France, the Tour Down Under, the Giro d'Italia or other similar cycling races.

Activity 2

Investigate other steep streets around the world that might challenge Baldwin Street's status as the World's Steepest Street. Examine the Wikipedia article for Baldwin Street here. Explain how the controversy described came about. How might the maximum gradient be determined? Why might other streets also make the claim of being the steepest street?