LUNCH LAP

Lesson 1: How Far?

Australian Curriculum: Mathematics (Year 9)

ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles

Lesson abstract

Students participate in an investigation to find the length of a path that touches three sides of a rectangle, starting and finishing at the same point on the fourth side. They model the problem and gather data on possible solutions.

Mathematical purpose (for students)

To investigate multiple routes between four points on a rectangle.

Mathematical purpose (for teachers)

Students reason about a mathematical problem, construct diagrams, and experiment with different models to form an understanding of the problem.

Suggested Presentation Two lessons of approximately one hour each.

Lesson Materials

- reSolve PowerPoint *1a Lunch Lap*
- Supplies for constructing models (see TBI for options)
- Student Sheet 1 – Lunch Lap (one per student)
- Access to dynamic geometry software for constructing digital models (or alternatively, student access to this GeoGebra app)

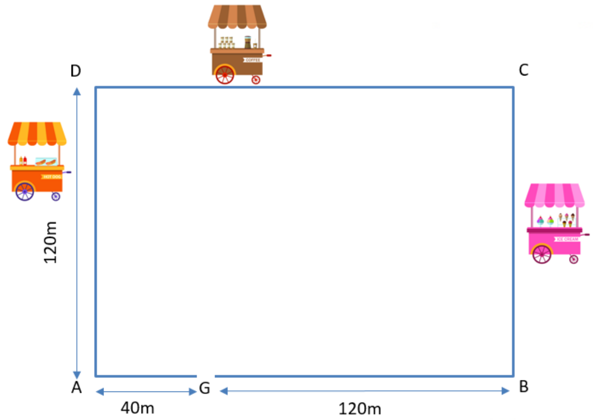
We value your feedback after these lessons via our website.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Teacher Background Information

The *Lunch Lap* sequence is constructed through multiple iterations of a single problem: in the diagram below, one cart is positioned on each side of a rectangle. We must construct a path that starts at G and visits each cart in order before returning to G. Where should the carts be placed to minimise the length of this path?



In this first lesson students get experience working with a hands-on model of the problem. Below are some suggestions for how to construct this model, however you and your students are encouraged to be creative with your models. We encourage having students construct their own model of the problem. The only requirement is that the model is hands on and not built using digital technology – we will move to that later on!

Some suggestions:

- Large scale: Mark out a 16 x 12 m rectangle in the schoolyard. Point G, the gate, is fixed and marked. Students will work in groups of 4-5 to place cones along the sides of the rectangle (representing carts), measure the location of the cones and measure the distance of the lunch lap using trundle wheels.
- Small scale: Mark out a 1.6 x 1.2 m rectangle for each group and repeat as above.
- Miniaturised: Mark out a 16 x 12 cm rectangle on a cork board or other surface. Use drawing pins, push pins or split pins to mark the location of the carts, and wrap string around the pins to measure a lap.

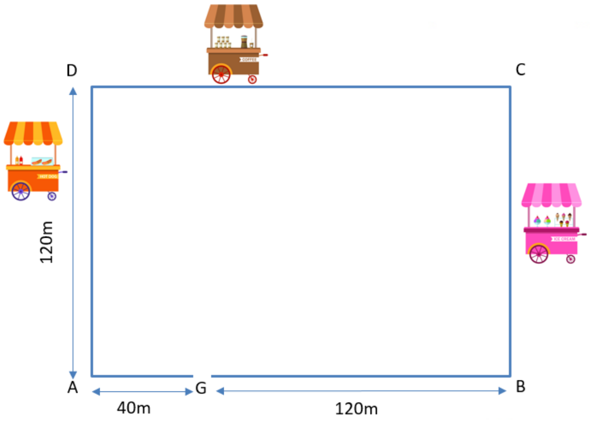
Students learning from home could create a model by marking a 1.6 x 1.2 cm rectangle with string and then using chair legs, jars or cups to represent each cart.

However models are constructed, students must ensure they are **to scale** with the original 160 x 120 m field. The values 160 and 120 have been chosen as they can be scaled down to many different dimensions e.g. 40 x 30 cm, 80 x 60 cm, 8 x 6 cm.

Introduction

Introduce the Lunch Lap context to the students using PowerPoint *1a Lunch Lap*.

It's Saturday, just after morning sport, and we all want our sausage, soft drink and ice cream for lunch. Fortunately, the reSolve food and drink carts are here to help!



As you can see in the diagram there is a rectangular field, measuring 120m by 160m. Forty metres along side AB is a gate G.

Sami the sausage vendor has a sausage cart somewhere on side AD. Deni the drink vendor has a drink cart somewhere on side DC. Ira the ice cream vendor has an ice cream cart somewhere on side CB.

To get my sausage, soft drink and ice cream, I need to enter through the gate, run to the sausage cart, run to the drink cart, run to the ice cream cart, and then run back out of the gate. This is my ***lunch lap****.*

How far might I have to run?

Teacher note

- At this stage students are investigating only *possible* lunch lap distances. The challenge of *what is the shortest possible lunch lap* will be introduced further into the investigation.

Ask students to make some quick predictions from looking at the diagram. Emphasise that each cart can be placed anywhere along its side.

Some prompts:

- Where do you want to try placing the carts? How might that change the path?
- What is a reasonable estimate for the length of the lunch lap?

Modelling

As outlined in the TBI, either present students with a model of the lunch lap and explain how they should use it to calculate their lunch laps, or challenge students to construct their own models.

If asking students to construct their own model, make sure to establish the following key points:

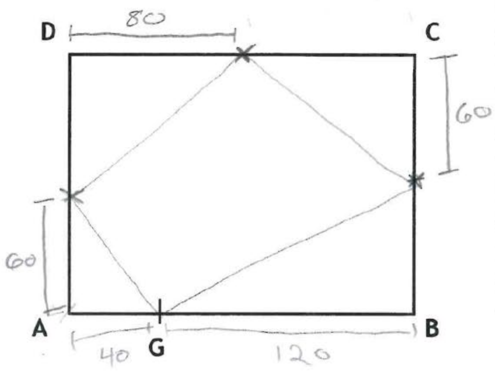
- *The model must be kept to scale. Why is this? What might a scaled down model look like?*
- *What parts of the model need to be moved around? Which parts don't need to move?*
    - The gate and the field do not need to move.
- *Which parts of the model will we need to measure in order to solve the problem?*
    - The length of the lap and the locations of the cart need to be measured.
- *How will you measure the distance of your lap?*

Once the form of the model has been determined:

1. Students, in groups, position the carts in different locations on each side of the rectangle.
2. Students record the positions of each cart on Student Sheet 1, marking the locations of the carts on the diagram and measuring their distance from each corner.
3. Students measure the length of their lunch lap. They record the length of each stretch of the lap and the total lap distance on their sheet under "Lunch Lap distances".

Allow students some time to measure at least 3 lunch laps in their groups. Emphasise the importance of precisely measuring the locations of the carts, so that they can confirm their answers later.

An example of a completed student sheet section:



Cart positions		Lunch Lap distances	
Distance from A to Sausage Cart	60	Distance from G to Sausage Cart	70
Distance from D to Drinks Cart	80	Distance from Sausage Cart to Drinks Cart	98
Distance from C to Ice Cream Cart	60	Distance from Drinks Cart to Ice Cream Cart	102
		Distance from Ice Cream Cart to G	138
		Total Lunch Lap distance	408m

Teacher notes

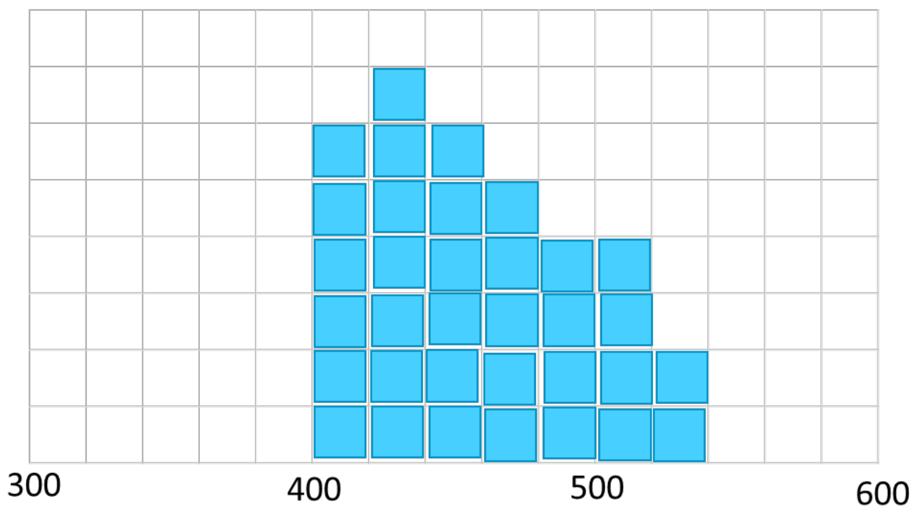
- If you are delivering this task over multiple lessons, this is a natural place to end the first lesson.

Confirming

Set aside the models and ask students to use the diagrams and tables on their worksheets to check the accuracy of their lap distance measurements. Do not direct students on how to do this, rather allow them to investigate themselves.

Students will need to use the recorded cart positions to calculate the distance travelled using Pythagoras' theorem. Update Student Sheet 1 as necessary. As they do this, encourage them to consider what shapes are formed by the lap. What sorts of shapes are made by long laps? By short laps?

Ask students in their groups to record the distance of each of their laps on a sticky note. Create a dot plot using these sticky notes on the whiteboard, to see how lunch lap distances vary. An example of what this might look like is shown below.



Identify the longest and shortest laps found in the classroom and have the groups that found these laps draw a diagram for the class showing where they located their carts. Discuss: what cart positions seem to result in short laps? What cart positions seem to result in long laps?

Ask: What do you think the **shortest possible distance** might be, and what **shape** might it take?

Modelling with technology

At this stage students are ready to move to the next level of abstraction: dynamic geometry software. If students are familiar with working with GeoGebra or similar (e.g. Geometer's Sketchpad, Desmos) direct them to construct their own model for a lunch lap, keeping in mind the key points for model construction outlined previously. If time is not available for this, a prepared model is available on GeoGebra.

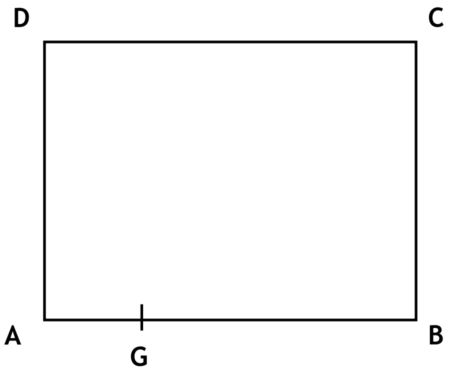
Challenge students to use their model to find the shortest possible lunch lap, and allow them time to experiment with their model until they agree that 400 m is the shortest possible lap. They will observe that this lap takes the shape of a parallelogram. Discuss: *are you surprised the shortest route forms a parallelogram? Would you expect the shortest lap to be a triangle or some other shape?*

Reflect: *can we **prove** that 400 m is the shortest possible lunch lap? Is it possible that there might be a 399.99 metre lap?* Inform students that you will be exploring geometric proofs in the next lesson.

Lunch Lap

Name: ____________

Scale:



Cart positions		Lunch Lap distances	
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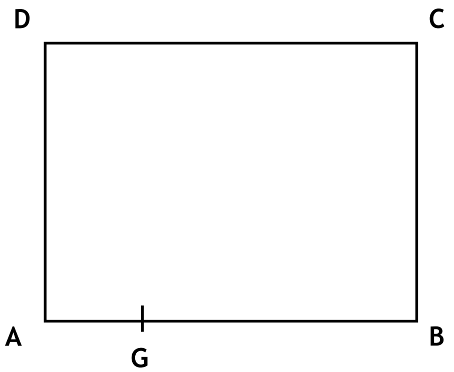
Distance from D to Drinks Cart		Distance from Sausage Cart to Drinks Cart	
Distance from C to Ice Cream Cart		Distance from Drinks Cart to Ice Cream Cart	
		Distance from Ice Cream Cart to G	
		Total Lunch Lap distance	



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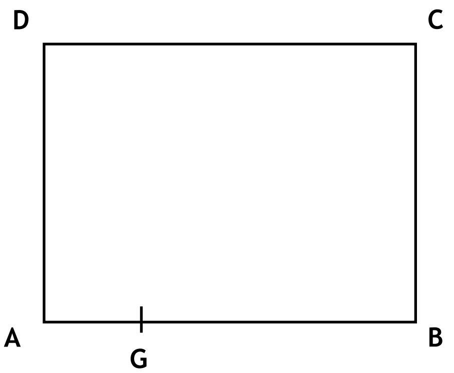

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