LUNCH LAP
Lesson 2: One Corner

Australian Curriculum: Mathematics (Year 9)

ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations
ACMNA294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles
ACMMG224: Apply trigonometry to solve right-angled triangle problems

Lesson abstract

In the previous lesson students formed the hypothesis that the shortest lunch lap possible is 400 metres. In this lesson they investigate how it might be possible to prove that a distance is the shortest possible, using a simplified problem.

Mathematical purpose (for students)

To use our mathematics toolset to find the shortest distance between two points.

Mathematical purpose (for teachers)

The resource extends the Pythagoras-based exploration in the first lesson to give a geometric solution. Students see that the exploration involves too many variables, and hence that Pythagoras' Theorem is of limited value in finding the locations of the carts or justifying that the path is the shortest. They learn some alternative geometric strategies to use instead.

Suggested Presentation — One lesson of approximately one hour.

Lesson Materials

- reSolve PowerPoint *1a Lunch Lap*
- Graphing software

We value your feedback after these lessons via our website.

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Australian Academy of Science

Exploring the one corner problem

Observe that positioning the carts along the sides of the rectangle involves three different variables: the position of each cart. An important mathematical problem-solving technique is to reduce the number of variables to shed light on a possible solution.

Use slide 3 of *1a Lunch Lap* to pose the challenge:

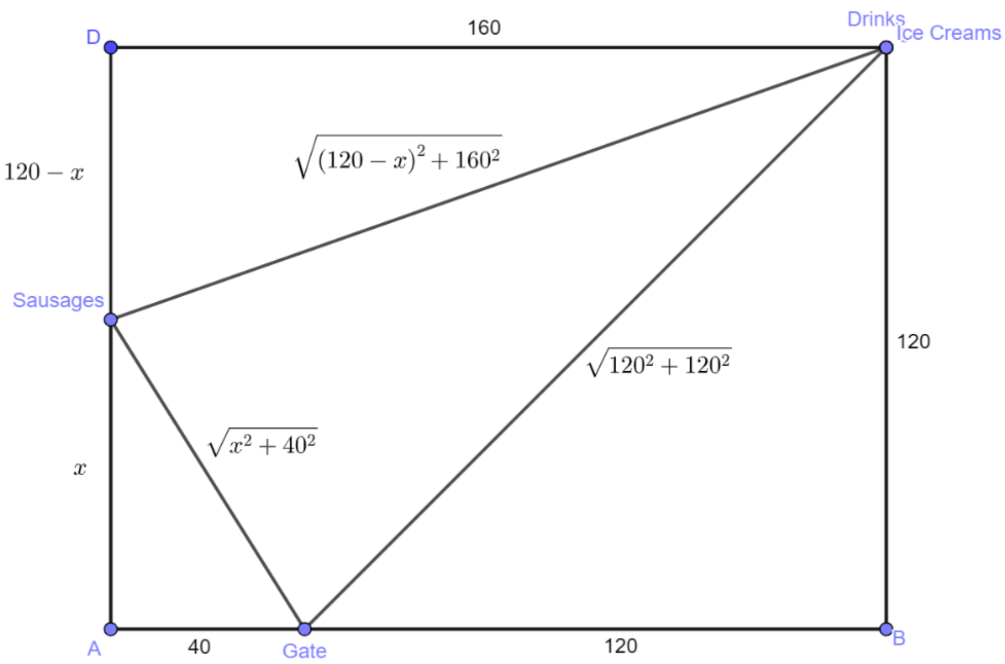
- *Could I put the drink cart and ice cream cart at C, and the sausage cart somewhere on AD? Where should the sausage cart be placed for the shortest possible lunch lap?*

Discuss briefly some strategies for finding the shortest possible lunch lap, e.g. giving each student a different location for the ice cream cart, having each student calculate the lap for their location, and then comparing to find the shortest lap. Observe that even this strategy will miss a lot of possible locations.

Ask students: *how many variables are there in this question?* Guide students to see that there is only one variable (the distance from A to the sausage cart), and explain that reducing the problem to a single variable means we can now use Pythagoras' theorem to create an algebraic formula for the length of the path. Challenge students to construct a formula for the distance of a lunch lap given any position of the sausage cart.

Teacher notes

- Let x represent the distance from A to the sausage cart. We can now use Pythagoras' theorem to create formulas for each stretch of the lap, as shown below.



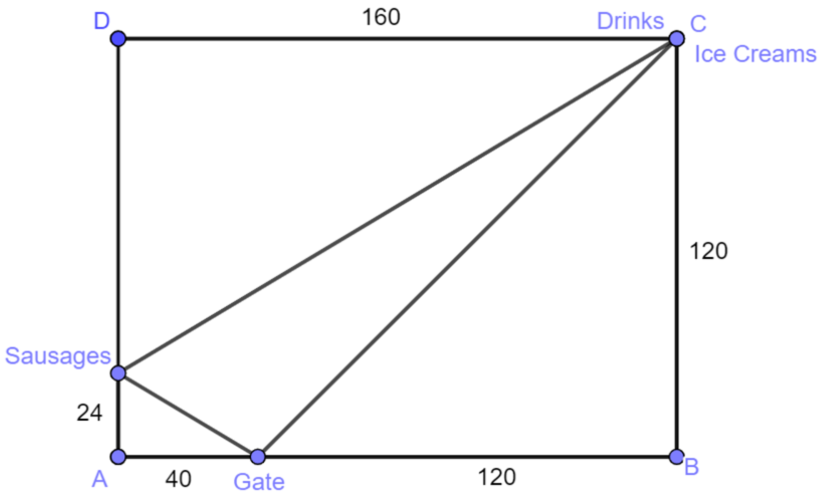
- Adding these three square roots fives us the total distance of the lap.

Once they have developed a formula for the length of the lap, check understanding by asking: *what are the possible values for* x? Students use graphing software (GeoGebra, Desmos or similar) to graph the formula and discuss their findings. Some questions to prompt reasoning:

- Where should the cart be placed for the minimum possible lap distance?
- Where could the cart be placed for the maximum possible lap distance (discuss what "maximum" means in this context)

Students will see that the parabola is at its minimum at x = 24, y = 402.944.

Ask students to construct a new diagram showing the lap created by this solution (either by hand or using digital geometry software):



Challenge students in groups to make as many observations about this solution as possible. Encourage students to apply trigonometry and Pythagoras' theorem to calculate all of the lengths and angles involved, and to experiment with manipulating the diagram in many different ways.

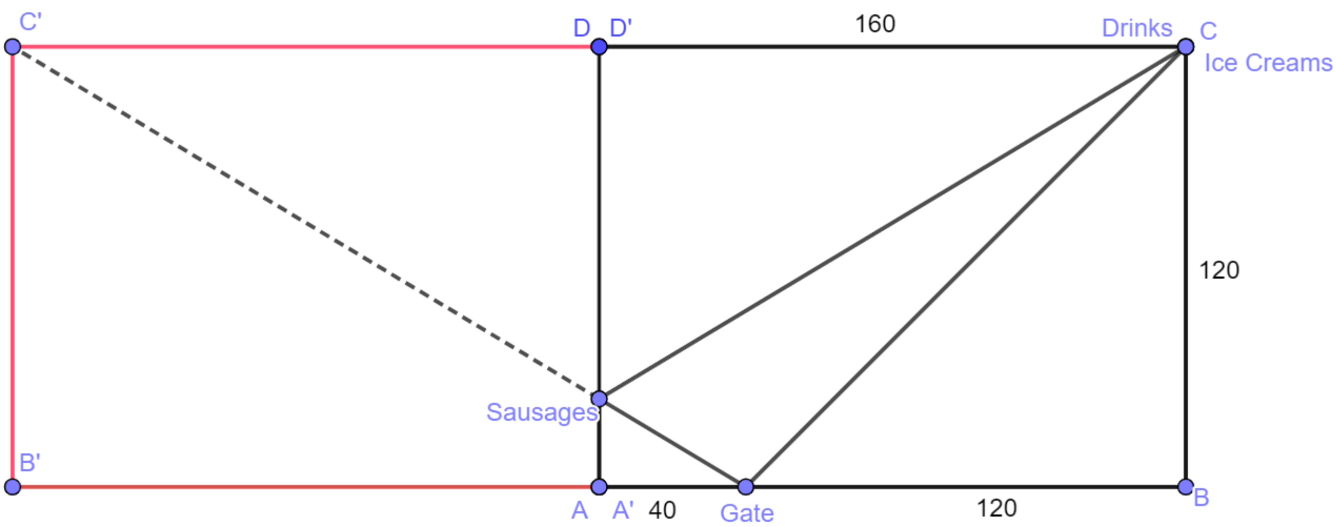
Some possible observations:

- *The distance from the sausage cart* (*S*) *to D is four times the distance from S to A*
- *The angle formed by DSC is the same as the angle formed by ASG*
- *The distance from S to C is four times the distance from S to G*
- *The distance CD is four times the distance AG*
- *The gradient of the lines SG and SC are exactly the same*
- (combining all of the above observations:) *The triangle SCD and the triangle SGA are **similar**.*

Prompt students:

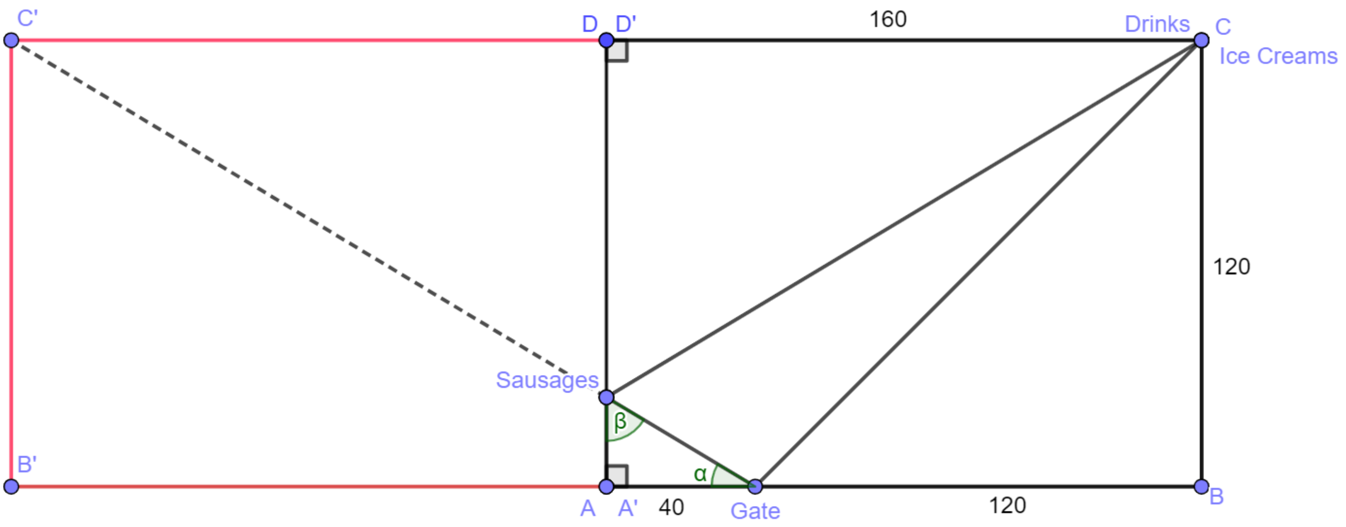
- *What is the shortest route between any two points?* (a straight line)
- *How do we define a straight line?* (180 degrees, gradient is constant)
- *Is there any way we could draw a straight line that connects G and C and also touches line AD?*

Use slide 4 of *1a Lunch Lap* to guide students to reflect the entire diagram through the line *AD*, and then connect *G* with the reflection of *C*:

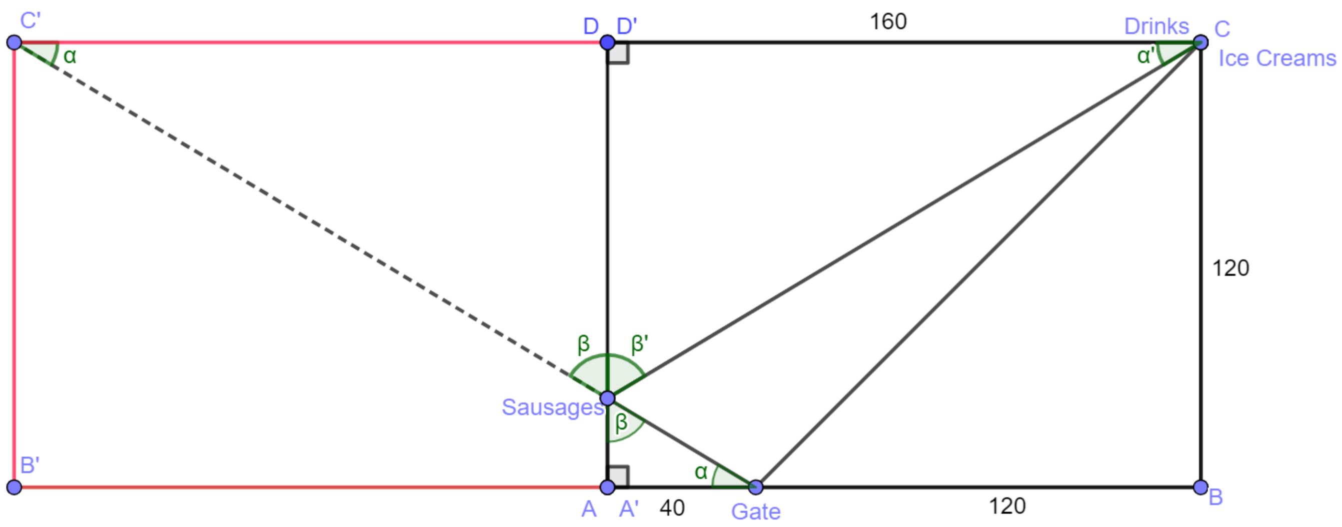


Look at slide 5 and discuss: why is CNG shorter than CMG? Conclude that the shortest distance between points *G* and *C* is a straight line that has been reflected

Challenge students: *can you use your understanding of geometry to show why reflecting the line C'G to create the line SC creates two similar triangles?* You can use this diagram (slide 6 of the PowerPoint) as a stimulus:



Solution, found by identifying alternate angles and vertically opposite angles:



Reflect: *in the next lesson we will apply this reflection strategy to the lunch lap problem. How do you think we could do this?*