LUNCH LAP
Lesson 3: Shortest Distance

Australian Curriculum: Mathematics (Year 9)

ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles
ACMMG224: Apply trigonometry to solve right-angled triangle problems

Lesson abstract

Students observe that the shortest lunch lap forms a parallelogram, justify this using symmetry, and show that for any rectangle of given length and width the shortest path is always twice the length of the diagonal.

Mathematical purpose (for students)

To use findings from previous lessons to prove 400m is the shortest possible lunch lap.

Mathematical purpose (for teachers)

Students use symmetry to show how to construct the shortest path. There are opportunities to use dynamic geometry.

Suggested Presentation One lesson of approximately one hour each.

Lesson Materials

- reSolve Powerpoint *1a Lunch Lap*
- Tracing/light paper (optional)
- Access to dynamic geometry software for constructing digital models (optional—alternatively, student access to this GeoGebra app)

We value your feedback after these lessons via <link to be advised>

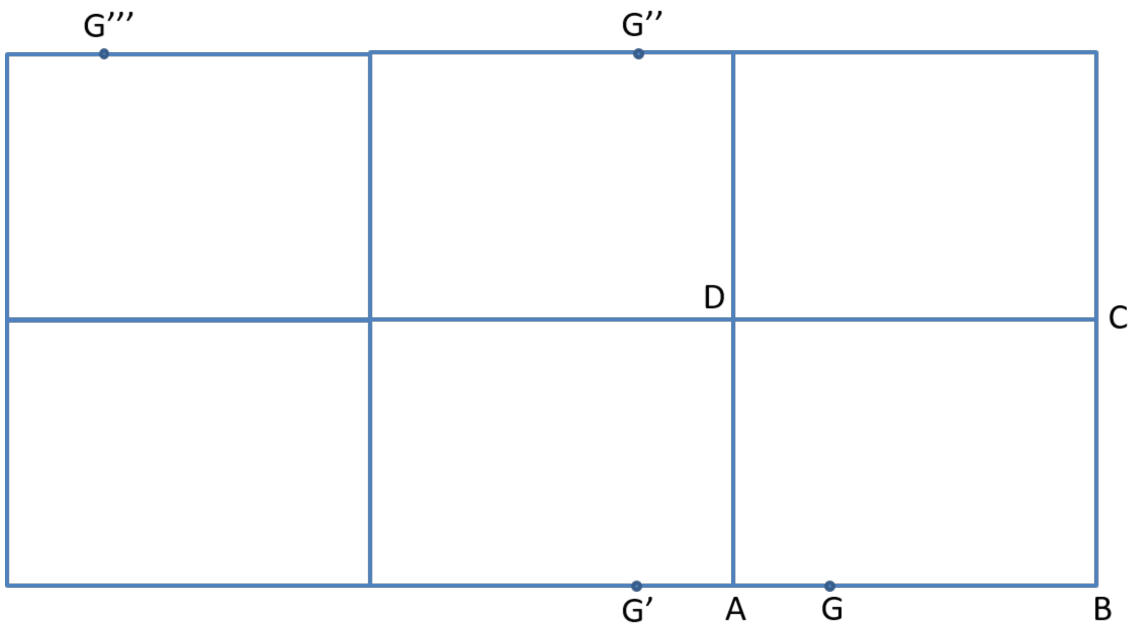
Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Reflections

Review findings from previous lesson. How can we use what we have learned about similar triangles and reflections to find the shortest possible lunch lap?

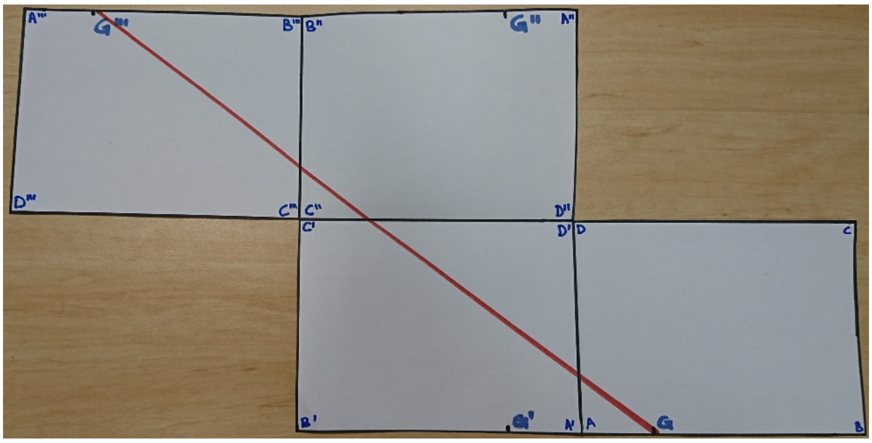
Ask students to use line reflection to show the shortest possible lunch lap. They can do this using paper and a ruler or dynamic geometry software as appropriate. You may use this diagram as a prompt (slide 7 of *1a Lunch Lap*):

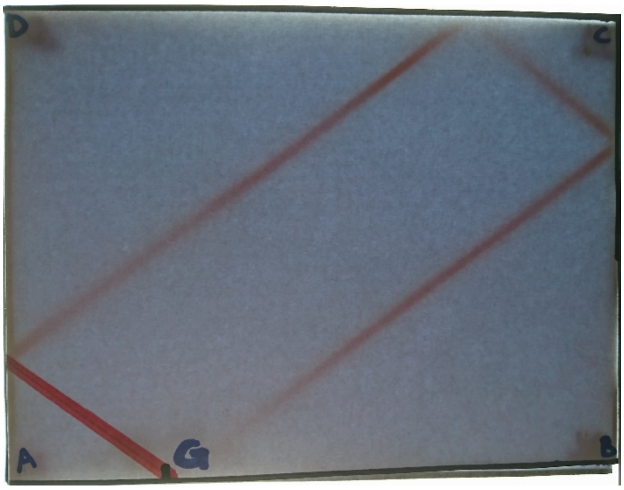


You can also prompt students to construct this diagram themselves using the following key points:

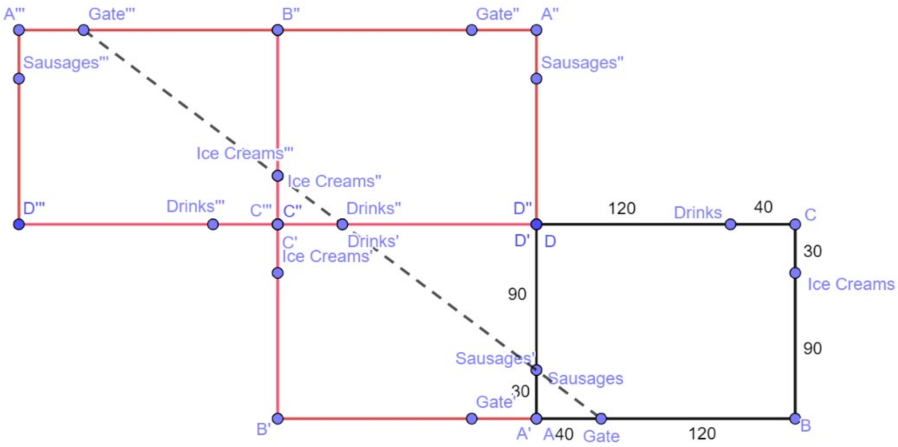
- *We need to draw a straight line that starts at G and ends at a reflection of G.*
- *On its way between G and its reflection, it must pass through each side of the rectangle (so that when reflected it will "bounce" off each wall), in order.*
- *Therefore the line constructed must start at G, pass through AD, pass through a reflection of DC, a reflection of BC, and then return to a reflection of G.*

If students construct this diagram on light paper (e.g. tracing) they can fold the piece of paper for each reflection, then hold it to the light to see the parallelogram constructed:

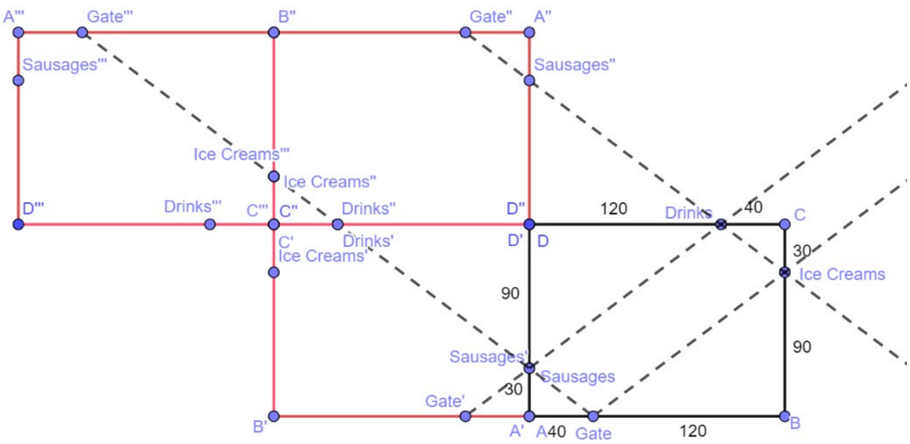




Alternatively, students can use geometry software to construct multiple reflections of their original lunch lap diagram. They can construct a straight line connecting G to its reflection, then move each cart so that its reflection intersects the straight line. This will show the positions of the carts required for a minimum length lap:



Alternatively, reflect line GG''' through each edge of the rectangle:



Discussion

- *Have we proven that the 400 m route we found earlier is the shortest possible lunch lap?*
- *Can you use this reflected diagram to show why the length of the lunch lap is twice the diagonal of the rectangle?*
- *How might we apply this strategy to other shapes e.g. a pentagon?*

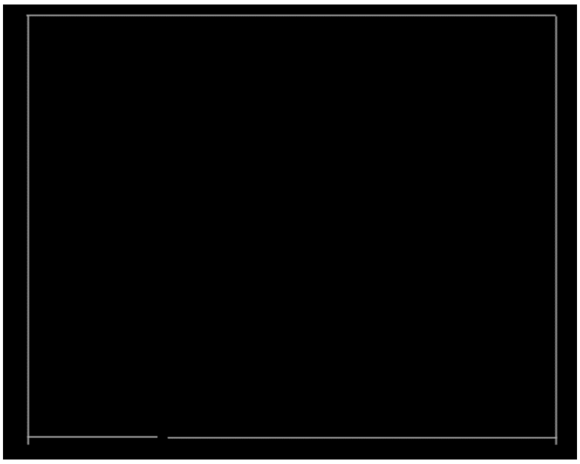
- *Using dynamic geometry software, it is possible to find multiple 400 m routes using different positions for carts. Are different positions for the same length path possible? Why/why not? Why might the software show multiple routes?*
    - There is only one route that will give exactly 400 m, as it is the only reflected straight line. Any other route that says it is 400 m is a rounding error.
- *What effect would moving the gate have? If we moved the gate to 20m away from A, what would change? Would the shortest possible route still be 400m?*

Reflection (get it!)

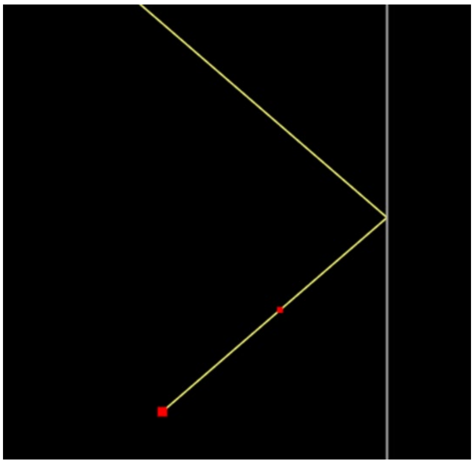
When a ray of light reflects of a surface, the angle it reflects off the surface is the same as the angle it hits the surface at (this is the **law of reflection**). Now doesn't that sound familiar?

Direct students to https://ricktu288.github.io/ray-optics/simulator/, a light simulator. Challenge them to reconstruct the Lunch Lap problem using the mirrors>segment and Ray tools. A rough reconstruction is fine.

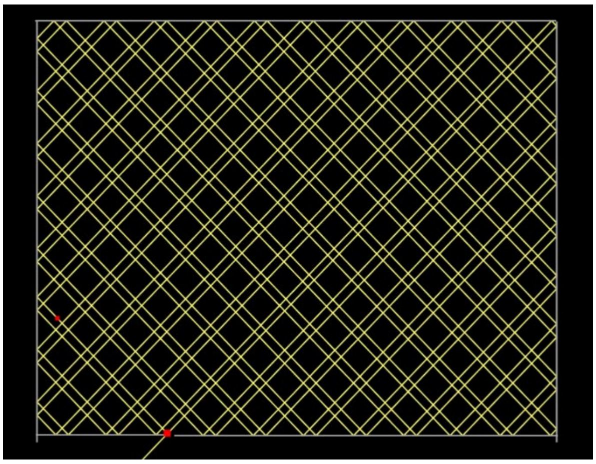
The grey lines are mirrors (note space in bottom mirror to represent gate).



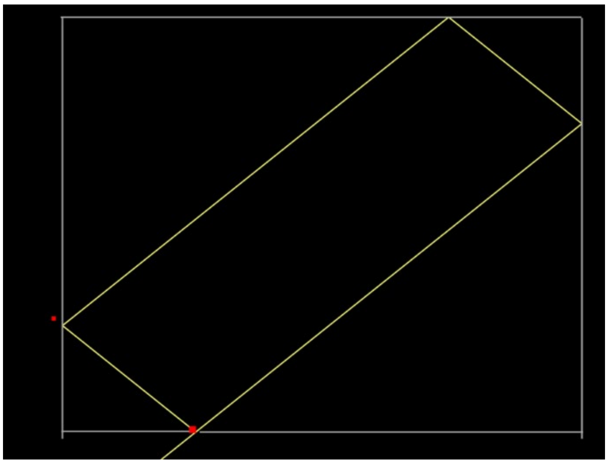
The large red square shows the origin of a beam of light and the red square at the left shows the angle of the beam of light (can be moved around)



Students will find that a beam of light, placed at the gate, will be reflected around the mirrors many times before it is reflected out of the gate.



There will be only one angle at which the beam will reflect off each wall once and then exit through the gate.



Compare this finding to the lunch lap result and discuss. You may want to investigate making other shapes using the light reflector (e.g. a pentagon), and compare to other shapes made in GeoGebra.