INTRODUCTION

re(Solve) MATHS BY INQUIRY

Polygon Pieces

Y8

Summary of learning goals

- Students explore different ways to cut convex polygons into two new polygons and investigate the different shapes that can be produced. They then use algebra to generalise their findings.

Australian Curriculum: Mathematics (Year 8)

ACMNA192: Simplify algebraic expressions involving the four operations.

ACMMG202: Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Summary of lessons

Who is this sequence for?

- Students with some knowledge of names of polygons. Students must be able to use algebra to make generalisations.

Lesson 1: Polygon Pieces

- Students use one straight cut to cut a convex polygon into two new polygons. They make generalisations about the total number of sides of the two new polygons, and about the number of different combinations of polygons that can be made.

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Reflection on this sequence

Rationale

This sequence focuses on developing mathematical reasoning by investigating the number of sides that result from cutting a convex polygon.

It links geometry, algebra and combinatorics through the generalisation and justification of a geometric result and by considering the number of different possible pairs of polygons that result from the cut.

reSolve mathematics is purposeful

- The sequence shows the interconnectedness of mathematical concepts, and illustrates how ideas from one strand of mathematics can be used to investigate and generalise results from another strand.

reSolve tasks are inclusive and challenging

- The initial task of cutting a convex polygon is low floor and high ceiling in that it commences with a simple problem but concludes by generalising to find the number of pairs of polygons resulting from different types of cuts.
- Students engage in sustained inquiry, guided by teacher questioning.

reSolve classrooms have a knowledge-building culture

- Students build success and understanding by examining an open question, 'What do you notice?', which focuses attention on salient and invariant features.
- The lessons are likely to challenge students' perceptions of what polygons look like.

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LESSON 1

re(Solve) MATHS BY INQUIRY

Polygon Pieces

Polygon Pieces

Y8

About this lesson

Students use one straight cut to cut a convex polygon into two new polygons. They make generalisations about the total number of sides of the two new polygons, and about the number of different combinations of polygons that can be made.

Australian Curriculum: Mathematics (Year 8)

ACMNA192: Simplify algebraic expressions involving the four operations.

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Mathematical purpose

- To explore the possible results that come from cutting convex polygons, and then to use algebra to generalise the findings.

Learning intention

- What new shapes can be made from cutting a convex polygon with one straight cut? What can we say about these shapes?

Time

A lesson of approximately 1 hour.

Resources

- reSolve PowerPoint *1a Polygon Pieces*

Vocabulary

- convex polygon
- dodecagon
- heptagon
- polygon

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Introduction and exploration

Resources: Show slide 2 of reSolve PowerPoint *1a Polygon Pieces*.

Pose the problem: *Use one straight cut to cut a convex polygon into two polygons. What do you notice?*

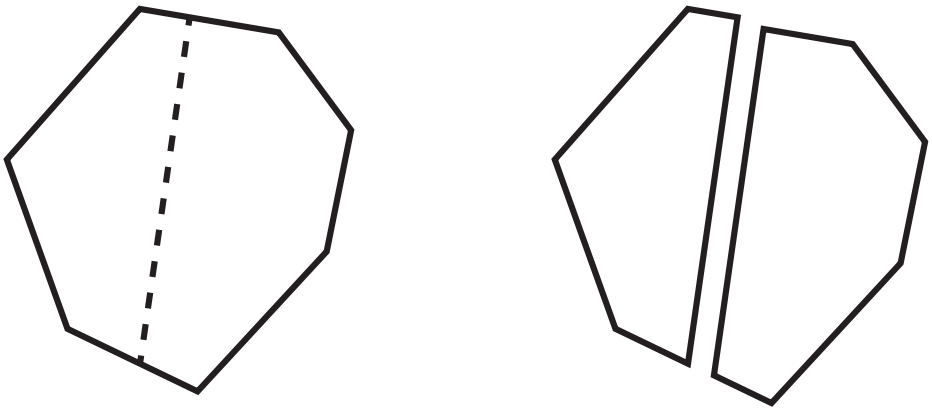
Students draw their own convex polygons and experiment with different ways they can be cut. Allow time to explore the problem and make some hypotheses/generalisations.

Teacher notes:

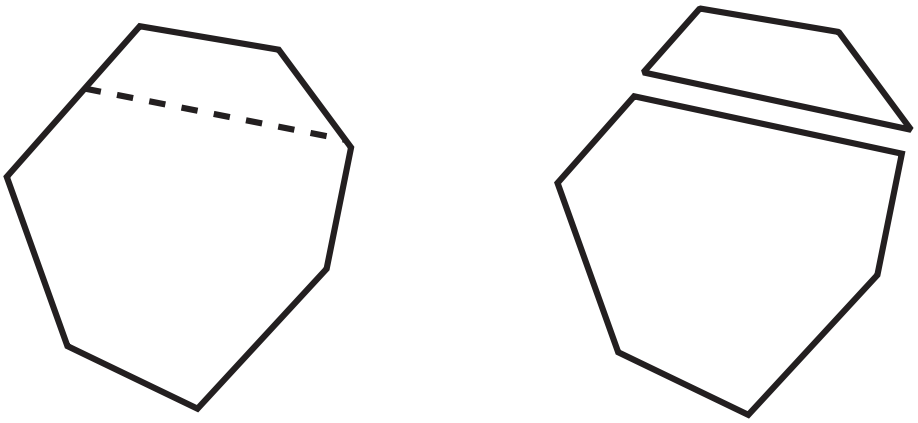
- The question 'What do you notice?' encourages students to focus their attention on salient features of the problem.
- Cutting particular polygons may result in new shapes with special features but, in the general case, the predictable result is the total number of sides that results from the different types of cuts.
- This resource is an extension of the reSolve Year 4 resource Trapezium Pieces.

Possible student responses for cutting a convex heptagon:

- This cut results in a pentagon and a hexagon. The total number of sides is 11.



- This cut results in a quadrilateral and a hexagon. The total number of sides is 10.



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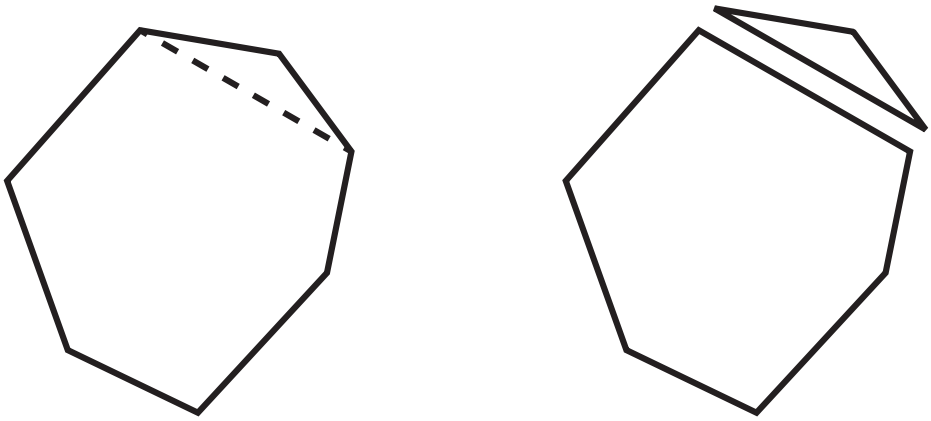
- This cut results in a triangle and a hexagon. The total number of sides is 9.



The following table shows all the possible pairs of polygons that can be made with one straight cut of a convex heptagon.

Polygon 1 (sides)	Polygon 2 (sides)	Total number of sides
triangle (3)	hexagon (6)	9
quadrilateral (4)	pentagon (5)	9
triangle (3)	heptagon (7)	10
quadrilateral (4)	hexagon (6)	10
pentagon (5)	pentagon (5)	10
triangle (3)	octagon (8)	11
quadrilateral (4)	heptagon (7)	11
pentagon (5)	hexagon (6)	11

Encourage students to experiment with other convex polygons and to make some generalisations regarding the total number of sides. For example, cutting a convex quadrilateral can result in:

- two quadrilaterals or a triangle and a pentagon (8 sides total)
- a triangle and a quadrilateral (7 sides total)
- or two triangles (6 sides total).

Extending, generalising and explaining

Ask students to express algebraically the number of sides that can be created when given a convex polygon with n sides.

Teacher note:

- The total number of sides increases by 2, 3 or 4 (except when cutting a triangle, for which an increase of 2 is not possible). Thus, the total number of sides in the new pair of polygons is $n + 2$, $n + 3$ or $n + 4$.

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If students have not observed how the position of the cut determines the total number of sides, prompt them:

- *Look carefully at two different cuts that produce different polygons but have the same total number of sides. What is the same?*
  - For example, a cut that produces a triangle and a hexagon, and a cut that produces a quadrilateral and a pentagon.
- *Look carefully at two cuts that produce a new pair of polygons, each with a different total number of sides. What is different?*

Students should make the generalisations:

- Cutting through two corners increases the total number of sides by 2.
- Cutting through a corner and a side increases the total number of sides by 3.
- Cutting through two sides increases the total number of sides by 4.

T Teacher notes:

- For an n-sided convex polygon:
  - Cutting through two corners creates two polygons with a total of $n + 2$ sides.
  - Cutting through a corner and a side creates two polygons with a total of $n + 3$ sides.
  - Cutting through two sides creates two polygons with a total of $n + 4$ sides.
- This can be explained as follows:
  - The cut itself produces two new sides, one in each polygon.
  - Cutting through a side divides an existing side into two, so the total number of sides increases by 1.
  - Cutting at a corner does not divide an existing side; therefore, the number of sides does not increase.

Finding all possibilities

For the convex heptagon, there are three ways to obtain a total of 11 sides, three ways to obtain a total of 10 sides, and two ways to obtain a total of nine sides.

Ask students to use their earlier findings to work out the number of different combinations of polygons resulting from one straight cut of a dodecagon (12-sided polygon), without drawing the shapes.

From the results above:

- Cutting through two corners gives a total of 14 sides.
- Cutting through a corner and a side gives a total of 15 sides.
- Cutting through two sides gives a total of 16 sides.

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Now we need to work out how many different pairs of polygons have a total of 14, 15 or 16 sides.

- If the total number of sides is 14, the possible numbers of sides in the pairs of polygons are: (3, 11), (4, 10), (5, 9), (6, 8) or (7, 7). Note that the smallest number of sides in a polygon is 3.
- If the total number of sides is 15, the possible numbers of sides in the pairs of polygons are: (3, 12), (4, 11), (5, 10), (6, 9) or (7, 8).
- If the total number of sides is 16, the possible numbers of sides in the pairs of polygons are: (3, 13), (4, 12), (5, 11), (6, 10), (7, 9) or (8, 8).

So, there are 16 possible pairs of polygons.

T Teacher notes:

- For a convex polygon with n sides, where $n > 3$, we need to find the number of partitions of $n + 2$, $n + 3$ and $n + 4$, excluding any partitions for which the smaller number is 0, 1 or 2.
  - For an even number of total sides, say $2m$, the partitions will be $(3, 2m - 3), (4, 2m - 4), \ldots, (m - 1, m + 1), (m, m)$.
    The total number of partitions is therefore $m - 2$.
  - For an odd number of total sides, say $2m + 1$, the partitions will be $(3, 2m - 2), (4, 2m - 3), \ldots, (m - 1, m + 2), (m, m + 1)$.
    The total number of partitions is therefore also $m - 2$.
  - For the dodecagon:
    - The number of partitions of 14 sides into polygons is $7 - 2 = 5$, as above.
    - The number of partitions of 15 sides into polygons is also $7 - 2 = 5$.
    - The number of partitions of 16 sides into polygons is $8 - 2 = 6$.

Reflection

Discuss with students how representing a result algebraically can succinctly and clearly express a mathematical generalisation.

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