re(Solve) MATHS BY INQUIRY Rule of Twelfths

Y10

Summary of learning goals

- Students learn that tidal patterns closely match a sine curve and use a well-known rule of thumb to approximate tidal heights. They develop a deeper understanding and appreciation of the properties of the sine curve and its real-life occurrences.

Australian Curriculum: Mathematics (Year 10A)

ACMMG274: Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

Summary of lessons

Who is this sequence for?

- Students should be familiar with the unit circle and should have examined how the vertical distance from the circle to the *x*-axis varies to plot a sine curve. They should understand that the sine of an angle can be evaluated for angles greater than 90°.

Lesson 1: Rule of Twelfths

Students learn about the rule of twelfths: a practical way to calculate tidal heights that is used by many sailors. Students apply the rule to graph tidal heights and observe that the graph resembles a sinusoidal curve. They extend their knowledge by comparing the results obtained from using the rule of twelfths with the actual values of the sine of various angles.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Australian Academy of Science

Reflection on this sequence

Rationale

The focus of this lesson is on modelling the real world with mathematics. A key element of Year 10 and 10A mathematics is developing and consolidating an understanding of functions as mathematical entities, and of how different types of functions model different real-world situations. Although transformations of the sine curve are not explicitly introduced until Year 11 mathematics, students' experiences with sine curves in the real world help set the scene for this further work. Appropriate levels of accuracy for real-life purposes are also examined.

reSolve mathematics is purposeful

- Through the use and analysis of a real-life context, students see mathematics as a way of modelling the real world, and use mathematical ideas to make decisions of practical importance.

reSolve tasks are inclusive and challenging

- Students work on a meaningful task that builds on existing knowledge of trigonometric functions, and builds deeper knowledge of the properties and shape of the sine curve.

reSolve classrooms have a knowledge-building culture

- Students use technologies to see how well the rule of twelfths approximates a sine curve and work together to develop better approximations.

LESSON 1

re(Solve) MATHS BY INQUIRY **Rule of Twelfths**

Rule of Twelfths

Y10

About this lesson

Students learn about the rule of twelfths: a practical way to calculate tidal heights that is used by many sailors. Students apply the rule to graph tidal heights and observe that the graph resembles a sinusoidal curve. They extend their knowledge by comparing the results obtained from using the rule of twelfths with the actual values of the sine of various angles.

Australian Curriculum: Mathematics (Year 10A)

ACMMG274: Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

Mathematical purpose

- Students learn that tidal patterns closely match a sine curve and use a well-known rule of thumb to approximate tidal heights. They develop a deeper understanding and appreciation of the properties of the sine curve and its real-life occurrences.

Learning intention

- To observe and approximate the sine function in a real-life context.

Time

Three lessons of approximately 1 hour each.

Resources

- tidal data, prepared as described in Teacher background information
- graphing software

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Teacher background information

This task explores the relationship between tidal heights and the sine curve. If students are interested in why changing tidal heights resemble a sine curve, you can direct them towards the website https://www.ausmarinescience.com/marine-science-basics/tides/, which discusses the reasons for tidal heights in more detail, or Activity 3: Lunar and solar tides.

This lesson plan focuses on tidal heights at Shale Island in Western Australia. Before running the lesson, visit https://tides.willyweather.com.au/wa/kimberley/shale-island.html and record all tide points for the current date, as shown below.

You will also need to make note of a tidal height that is within the bottom sixth of the daily range. In the example above, on January 5 this might be 2.5 m. If we were using January 9 instead, we might choose 3 m or 3.5 m. Note that larger tidal ranges are ideal.

The importance of tracking tidal heights

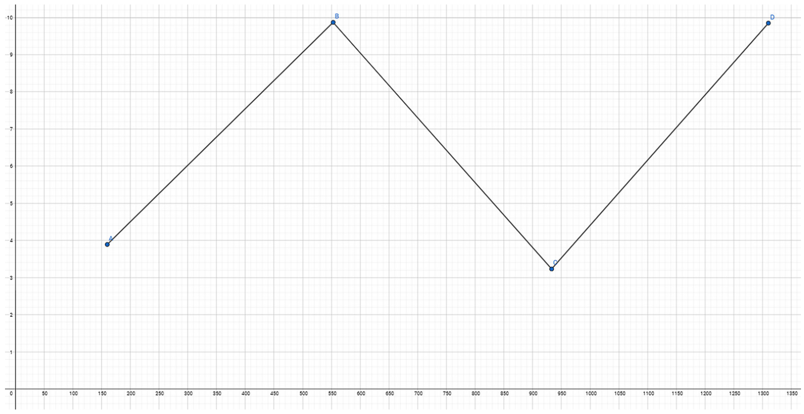
Show students this article ('Tide pulls the plug on yacht', *The West Australian*, Saturday 18 June 2016).

Explain that these sailors were not aware of the rate of change of tidal heights when they set out to sea on 2 June 2016. All they had were the following times of high tide and low tide at nearby Shale Island, an area known for dramatic tidal changes:

Thursday, 2 June 2016	2:40 am 3.89 m	9:13 am 9.87 m	3:33 pm 3.23 m	9:50 pm 9.85 m

Have students mark the times and heights of high tide and low tide on their graphs. The sailors might have simply connected the dots with straight lines (as shown below) to work out what the tide height would be at any given time. What might be wrong with this strategy?

According to the article, the yacht was stranded for nine hours 'to about nine o'clock at night'. What depth of water do you think they needed under the boat in order to sail safely?



Give students the more recent data collected, as prepared in the Teacher background information (TBI). Ask them to draw the graph that they think the novice sailors might have drawn using just the high and low tide marks. They might want to use 24-hour time or represent the time in minutes, along their x-axis.

Ask students: *Imagine I sail out to Shale Island in the morning to go fishing. It will take me 45 minutes to get home and I need at least [tidal height in bottom sixth of range, as prepared in TBI] metres of water below my boat to sail safely. At what time do I need to stop fishing and leave?*

Students should use their graphs to provide a range of similar answers. Emphasise the importance of leaving as late as possible to maximise fishing time.

Possible student response:

- It will take me 45 minutes to get home and I need at least 4 metres of water beneath my boat to sail safely. At what time do I need to leave?
  - ◊ According to the graph above, the tide will be at 4 metres at 2:50 pm, so I will need to leave at 2:05 pm.

Introduce students to WillyWeather or any other similarly interactive tidal graph resource. Do these graphs resemble the graphs shown above? How are they different? Students should observe that the graph resembles a sinusoidal curve. Trace along the curve using the moving bubble to find out the times when there will be sufficient depth of water. Make a note of the times that you find and compare it with your earlier answer.

- For example, on 2 June 2016, according to WillyWeather, the tide was at 4 metres at 2:03 pm. If I waited to leave at 2:05 pm, as originally planned, I would have been stranded!

Understanding the rule of twelfths

Out on the reefs, internet access is not always reliable. If we had no access to these websites or any graphing technology, how could we calculate the tidal height? The rule of twelfths was designed by sailors to give an accurate estimate of tidal height at any given time and requires minimal information. According to the rule of twelfths:

Hour 1	$\frac{1}{12}$
Hour 2	$\frac{2}{12}$
Hour 3	$\frac{3}{12}$
Hour 4	$\frac{3}{12}$
Hour 5	$\frac{2}{12}$
Hour 6	$\frac{1}{12}$

- In the first hour after low tide/high tide, the height increases/decreases by one-twelfth of its total range.
- In the second hour, the height increases/decreases by two-twelfths.
- And so on, following the table on the left.

To familiarise students with using the rule of twelfths, use your tidal data to ask questions about the height of the tide at certain times. For example:

Thursday, 2 June 2016	2:40 am 3.89 m	9:13 am 9.87 m	3:33 pm 3.23 m	9:50 pm 9.85 m

- What was the height at 4:40 am? At 6:40 am? At 4:40 pm?
- At approximately what times was the height 5 metres?

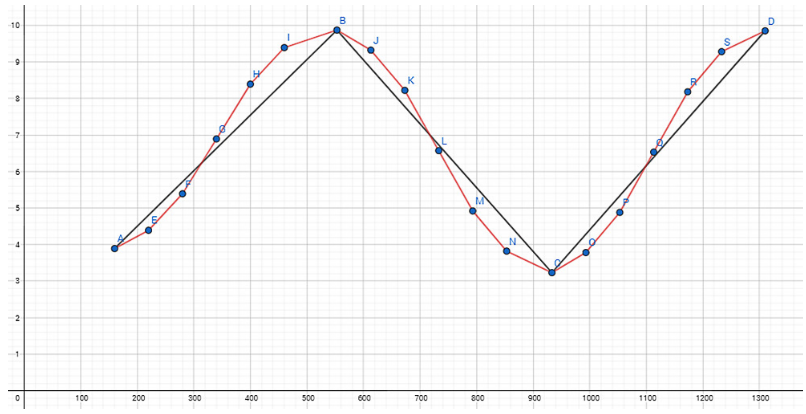
Graphing with the rule of twelfths

Ask students to trace over their novice graph, using the rule of twelfths to find the tidal height every hour and then connecting the points. Suggested discussion points (avoid giving these as explicit instructions, but allow the students time to consider the best way to graph on their own) are:

- *What calculations do we need to do before we can graph?*
  - ◊ Students will need to calculate the tidal range, work out the relevant fractions, and then add the value to low tide or subtract from high tide.
- *What is on the* y*-axis? The* x*-axis? What units are we using?*
  - ◊ The y-axis will chart height and the x-axis will chart time. Of the different units we can use to mark time, such as minutes, 24-hour time or hours, discuss which is best to use for this graph.

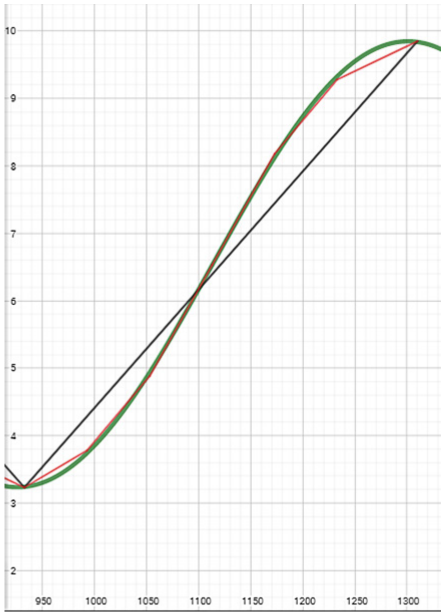
Ask: *According to your new graph, what time will you need to stop fishing? How does this compare to your earlier answer and to the answer you found online?*

Possible student response:



Teacher notes:

- GeoGebra has a 'FitSin' function that can generate a sine curve to fit a set of points.
- Using this function allows us to see how well a sine curve might match with the rule of twelfths. We are not attempting to model the tide variations with a sine curve; the GeoGebra function needs at least four points to fit a sine curve and if we used high and low tide times, the sine curve would not fit well due to day-to-day variations. The key finding here is that the sine curve models a 6-hour period quite well, which is enough for sailors to make decisions.
- The graph at right shows a sine curve (green) generated to fit the points found using the rule of twelfths over a 6-hour period (rule of twelfths plotted in red, corresponding sine curve in green).



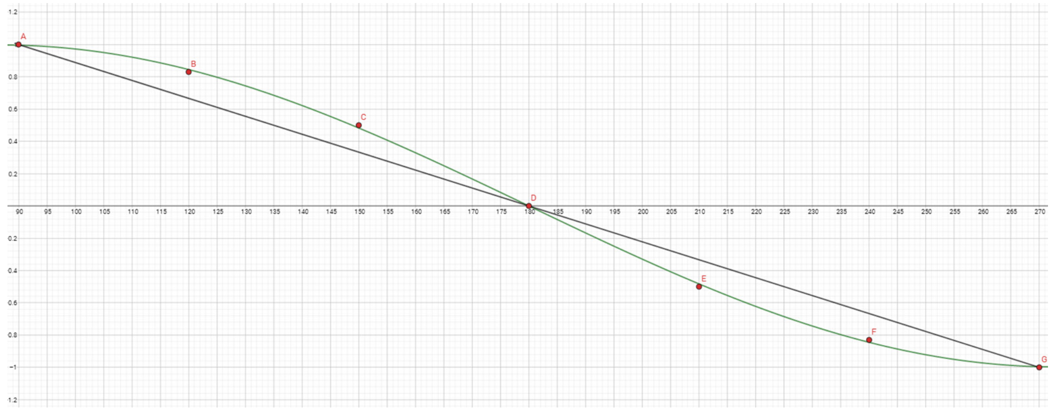
Comparing the rule of twelfths with the sine curve

Ask students to imagine that high tide has a height of 1 and occurs at a time of '90'. Low tide has a height of -1, which occurs at a time of '270'. Ask them to plot these and join the points with a straight line.

Now ask them to work out the 'hour intervals' by dividing the *x*-axis between 90 and 270 into six equal intervals, and to use the rule of twelfths to work out the 'height' at each point. Have students plot this.

Now ask them to look up sin (90°), sin (120°), sin (150°), etc. and compare these values with those obtained using the rule of twelfths. Which are the most accurate? Which are the least accurate?

Ask students to plot the sine curve and the values obtained by the rule of twelfths on their graph, and comment.



Further activities

Activity 1: Rule of tenths

Some seafarers swear by the rule of tenths instead of the rule of twelfths. The rule of tenths is shown in the following table.

Hour 1	$\frac{1}{10}$
Hour 2	$\frac{15}{100}$
Hour 3	$\frac{25}{100}$
Hour 4	$\frac{25}{100}$
Hour 5	$\frac{15}{100}$
Hour 6	$\frac{1}{10}$

Try tracing a tidal curve using the rule of tenths on top of the curve you drew using the rule of twelfths.

Are the two graphs significantly different? Which was more convenient to draw? Why? Can you think of a situation or conditions under which the other rule of thumb would be convenient?

Activity 2: Finding an even better rule of thumb

What if we wanted even greater accuracy, particularly near high and low tide times? Can we improve on the rule of twelfths or tenths?

One suggestion is to use a rule of fifteenths. What fractions would you use at each hourly (30°) interval? Is this better than the rule of twelfths or tenths?

Can we find a rule of thumb that will work well in half-hour (15°) intervals?

Are these rules likely to be of any practical use?

Activity 3: Lunar and solar tides

For students who are interested in the science behind tidal heights, SciencePrimer.com has a simulator here that shows the effect of the Moon and Sun on tidal patterns. The simulator shows why the pattern is not a perfect sine curve.