reSolve MATHS BY INQUIRY Tree Biomass

Y9

Summary of learning goals

- This resource addresses a practical environmental problem of the number of trees that need to be cut down to supply all the paper the school uses in one year. Students use their knowledge of similar triangles, Pythagoras' theorem and algebra to design and construct a Biltmore stick used to measure the diameter and height of a tree. They measure some trees, calculate their volume and use the density of the tree to find the dry mass of the tree. They estimate the amount of paper used in the school and, hence, estimate the number of trees that need to be cut down to supply the school's paper needs.

Australian Curriculum: Mathematics (Year 9)

ACMMG217: Calculate the surface area and volume of cylinders and solve related problems.

ACMMG221: Solve problems using ratio and scale factors in similar figures.

Summary of lessons

Who is this sequence for?

- This sequence follows the reSolve Year 8 resources Circumference and Area of a Circle. The sequence uses similar triangles and the formula for the volume of a cylinder. Significant algebraic skills are assumed when students are asked to work out the diameter markings on a Biltmore stick or they may use the d-tape from Lesson 3 of the Circumference sequence.

Lesson 1: A Biltmore Stick

Students make a Biltmore stick that measures both the diameter and height of a tree, using knowledge of similar triangles, Pythagoras' theorem and some challenging algebra.

Lesson 2: How Many Trees?

Students use a d-tape (see reSolve Year 8 Circumference) or a Biltmore stick to measure the diameter and height of a tree. They use known wood densities to calculate the biomass of a tree and, hence, estimate the number of trees needed to provide the paper that is used by their school in one year.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Australian Academy of Science

Reflection on this sequence

Rationale

Students use mathematics to investigate an environmental problem. In doing so, they combine their knowledge of several areas of mathematics, including similar triangles, volume of a cylinder, circle properties and algebra.

reSolve mathematics is purposeful

- This sequence draws on an important environmental context.
- Students use their mathematical knowledge to design and make an instrument that has traditionally been used to measure trees.

reSolve tasks are inclusive and challenging

- The task involves students collecting real data from the environment.
- All students are able to participate in a task that is meaningful and significant in their world.
- The derivation of the formula showing how to mark diameter on the Biltmore stick involves sophisticated algebraic and geometric reasoning.

reSolve classrooms have a knowledge-building culture

- The sequence builds on ideas from the reSolve Year 8 resources Circumference and Area of a Circle. Students develop a plan for working out how many trees need to be cut down to supply the school's paper and make sensible assumptions and approximations to model the real world.

LESSON 1

reSolve MATHS BY INQUIRY Tree Biomass

A Biltmore Stick

Y9

About this lesson

Students make a Biltmore stick that measures both the diameter and height of a tree, using knowledge of similar triangles, Pythagoras' theorem and some challenging algebra.

Australian Curriculum: Mathematics (Year 9)

ACMMG221: Solve problems using ratio and scale factors in similar figures.

Mathematical purpose

- Students use their knowledge of similar triangles, Pythagoras' theorem and algebraic expansions to determine where to put the graduations on a Biltmore stick that measures both the height and diameter of a tree. This enables them, in a subsequent lesson, to calculate tree volumes and mass and, hence, to investigate a significant environmental issue.

Learning intention

- To make a Biltmore stick, which is used by foresters, to measure the diameter and height of a tree.

Time

A lesson of approximately 1 hour.

Vocabulary

- Biltmore stick
- diameter at breast height (DBH)
- similar
- tangent

Resources

- reSolve PowerPoint *1a Biltmore Stick*
- reSolve Excel Spreadsheet *1b Biltmore Stick*
- 1 metre ruler without markings or covered to hide markings, or similar
- measuring tape or trundle wheel

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Making a Biltmore stick

Resources: Show slide 2 of reSolve PowerPoint *1a Biltmore Stick*.

Explain that the Biltmore stick was invented in the early 1900s to help manage a forestation project on the Biltmore Estate in North Carolina, USA. One side or edge of the stick measures a tree's diameter at breast height (DBH), and the other measures a tree's height or, sometimes, the number of logs of a given length that can be cut from the tree. In the days before chainsaws, the scales were sometimes inscribed on the handle of the forester's axe.

Measuring height

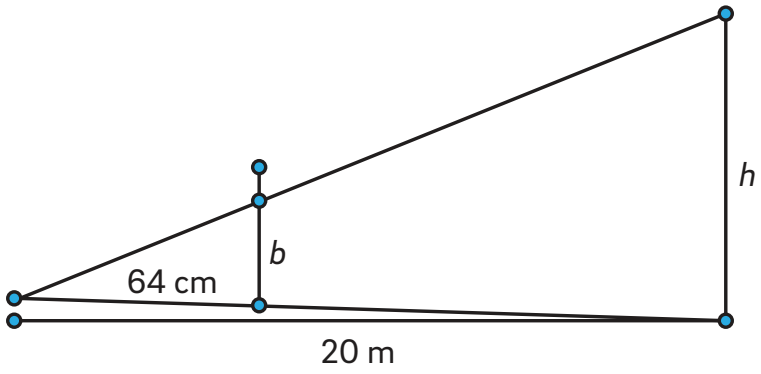
Show slide 3.

To measure a tree's height:

1. Stand at a predetermined distance from the tree. We will use 20 metres.
2. Hold the stick vertically at arm's length (typically 64 cm from the eye). Align the stick so that your eye, the bottom of the stick and the base of the tree form a straight line.
3. The desired height is read directly off the scale and the stick.

Note that foresters will often measure the tree using the 'stump height', or height of the stump if the tree were cut, as the base (rather than where the tree meets the ground) and the 'merchantable top' or uppermost usable part of the trunk as the position at which to measure the height. This ensures that the height measurement relates to the usable part of the tree.

The diagram shows the relationship between the Biltmore stick, the tree and the measurements.



Ask students: Why is $\frac{b}{0.64} \approx \frac{h}{20}$? What assumption are you making?

Give students a 1 metre ruler without markings or a ruler with the markings covered. Ask them to mark the stick so that the markings enable the user to directly read the height of the tree when the stick is held at arm's length 20 m from the tree.

Enabling prompts:

- What would be the value of b when $h = 10$ m? Where should the 10 m mark be on the Biltmore stick?

Teacher note:

- Using the formula above, $b \approx 0.032h$. So the 10 m mark should be at 32 cm on the Biltmore stick.

Measuring diameter

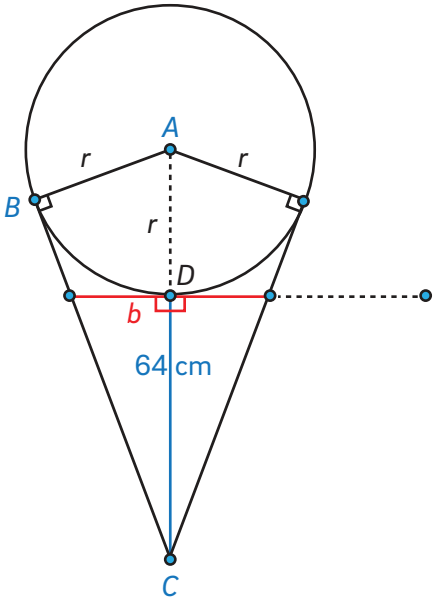
Teacher note:

- This part of the lesson involves use of Pythagoras' theorem, more complex calculations with similar triangles, knowledge that the tangent to a circle is perpendicular to the radius, and some challenging algebra. It is not essential for students to use a Biltmore stick in Lesson 2: How Many Trees.

Show slide 4 of reSolve PowerPoint *1a Biltmore Stick*.
To measure a tree's diameter, the Biltmore stick is held horizontally at arm's length (typically 64 cm from the eye) at chest height, against the trunk of the tree. The left end of the stick is held so that it aligns with the left edge of the trunk, and the measurement for the diameter of the tree is read directly from the scale on the stick at the point that aligns with the right edge of the trunk.

The diagram on slide 5 shows the relationship between the Biltmore stick, the tree and the measurements.

Ask students to choose values for the diameter of the tree and calculate the corresponding values of b in order to mark the stick. Note that several values need to be found, as the relationship is non-linear. Ask students to make tree diameter marks at the corresponding positions on their Biltmore stick.



Teacher notes:

- The calculation of b for a given diameter d (radius r) is shown below. It could be generalised for different distances from the eye.

  ◊ **Resources:** reSolve Excel Spreadsheet *1b Biltmore Stick* shows how a spreadsheet can be used to find the values of b when given d.

- Triangles ABC and EDC are similar since the tangent to a circle meets the radius at right angles. Hence, $\frac{AB}{BC} = \frac{ED}{DC}$.

- Hence: $\frac{r}{\sqrt{(r+64)^2 - r^2}} = \frac{b/2}{64}$

$$\frac{r}{\sqrt{64(2r+64)}} = \frac{b}{128}$$

$$b^2 = \frac{128^2 r^2}{64(2r+64)} = \frac{64^2(2r)^2}{64(2r+64)} = \frac{64d^2}{d+64}$$

$$b = \frac{8d}{\sqrt{d+64}}$$

- More generally, $b = \frac{d\sqrt{p}}{\sqrt{d+p}}$, where p is the perpendicular distance from the eye to the stick.

Enabling prompts:

- What would be the value of b when $d = 36$ cm? Where should the 36 cm mark be on the Biltmore stick?
- What would be the value of b when $d = 105$ cm? Where should the 105 cm mark be on the Biltmore stick?

Reflection

Students should check the accuracy of their Biltmore stick and measurement technique by measuring the height and diameter of known objects or ones that can be easily measured.

LESSON 2

re(Solve) MATHS BY INQUIRY Tree Biomass

How Many Trees?

Y9

About this lesson

Students use a d-tape (see reSolve Year 8 Circumference) or Biltmore stick to measure the diameter and height of a tree. They use known wood densities to calculate the biomass of a tree and, hence, estimate the number of trees needed to provide the paper that is used by their school in one year.

Australian Curriculum: Mathematics (Year 9)

ACMMG217: Calculate the surface area and volume of cylinders and solve related problems.

Mathematical purpose

- Students calculate the volume of a cylinder, use the volume and density to calculate mass, and make sensible estimations to investigate a significant environmental issue.

Learning intention

- To find out how many trees are needed to make the paper we use at school.

Time

One to two lessons of approximately 1 hour each.

Vocabulary

- Biltmore stick
- biomass
- density
- d-tape
- form factor
- tree basal area

Resources

- d-tape made in Lesson 3 of reSolve Year 8 Circumference or Biltmore stick made in Lesson 1: Tree Biomass.
- measuring tape or trundle wheel
- set of scales to measure up to 5 kg
- ream of A4 paper
- clinometer (optional)

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Measuring the volume of a tree

Measuring a tree

Choose a suitable location where students can measure the diameter and height of a number of trees. Ideally, the trees should have straight trunks so that it is easy to identify the usable portion of the tree.

Ask students to measure the diameter of their tree at breast height (DBH) using a d-tape (see reSolve Year 8 Circumference Lesson 3: Measuring Tree Trunks) or a Biltmore stick (see Lesson 1: Tree Biomass). Measuring the DBH gives a reasonable measure for the diameter of the tree above the stump.

Ask students to measure the height of the tree using a Biltmore stick. Alternatively, students could use the shadow of the tree and the shadow of a 1 metre ruler, and calculate the height using similar triangles, or use a clinometer and trigonometry.

Tree basal area

Tree basal area (*TBA*) is defined as the cross-sectional area at breast height (typically 1.3 m). It is used to calculate the area of ground covered by trees in a forest and, hence, as a measure of tree competition.

Ask students to use their diameter measurements to calculate the tree baxsal area for the trees they have measured.

Extending prompts:

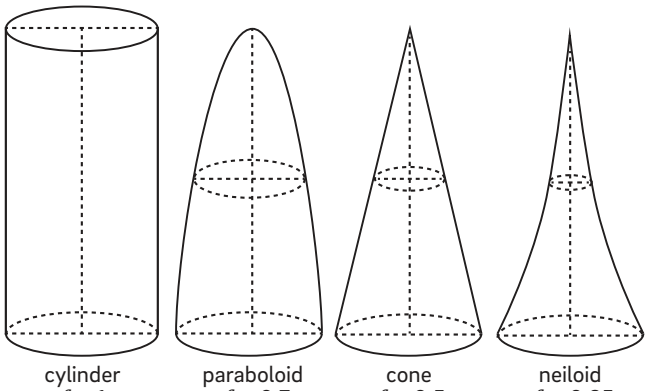
- In Australia, where measurements are in metric units, *TBA* can be calculated from the formula $TBA = 0.00007854 \times DBH^2$. This gives an area in metres when the diameter is in centimetres. Why does this formula work?
- Work out a similar formula for the USA, where *DBH* is measured in inches and *BA* is measured in square feet. The multiplication factor used is called the forester's constant.

Tree volumes

Ask students to calculate the volume (in m^3) of the usable portion of tree from their height measurements and their calculation of the tree's basal area. What assumption is being made in your calculation of volume?

Extending prompts:

- Foresters take into account the tapering of the trunk by describing the 'form factor', *f*, of a tree. The calculated volume for a cylinder is then multiplied by the form factor for the shape that most closely resembles the tree.



- Ask students to use the most appropriate form factor to obtain a better estimate for the volume of their trees.

Calculating the biomass of the tree

Ask students to find the density of the timber for the particular type of tree they measured or for one that is similar. The website http://www.fpc.wa.gov.au/about-us/publications/species gives the green density, the air-dry density and the basic density for a number of different Australian timbers.

- Green density is the density of the wood in the living tree.
- Air-dry density is the density when assuming 12% moisture content.
- Basic density is the density of oven-dried wood.

Have each student calculate the mass of the tree they have measured using the basic density, and use class measurements to find the mass of a 'typical tree' in the location chosen.

How many trees to supply the school's paper?

Ask students to estimate how many of the trees measured by students would need to be cut down to supply all the paper the school uses in one year.

T Teacher notes

- This is an example of a **Fermi problem**, named after Enrico Fermi (1901–1954), an Italian-American physicist.
- Fermi questions require students to make reasonable assumptions and estimates about the situation in order to come up with an approximate answer. The famous problem attributed to Fermi is: How many piano tuners are there in Chicago?

Some useful estimates for the number of trees needed to supply the school's paper might include:

- the number of sheets of paper handed to each student each day
- the number of students in the school
- the mass of one ream of paper (by measurement or by multiplying the gsm by the area of one ream of paper).

When presenting their solutions, students should be reminded of the need to explain and justify what they did to work out their estimate. Take advantage of opportunities to discuss students' different solution strategies and their assumptions.

Reflection

The number of trees needed to supply one school for a year might not seem that many. However, paper is used in every home, school and workplace in Australia. Students should discuss the environmental implications of their calculations.