re(Solve) MATHS BY INQUIRY

Area and Perimeter

Y5

Summary of learning goals

- Students consolidate their understanding of and skills in calculating the area and the perimeter of rectangles. They observe patterns, reason mathematically and make generalisations.

Australian Curriculum: Mathematics (Year 5)

ACMMG109: Calculate perimeter and area of rectangles using familiar metric units.

ACMNA098: Identify and describe factors and multiples of whole numbers and use them to solve problems.

ACMNA291: Use efficient mental and written strategies and apply appropriate digital technologies to solve problems.

Summary of lessons

Who is this sequence for?

- This sequence assumes that students know how to calculate the perimeter and area of rectangles.
- Students should be familiar with the metric units that are used to measure perimeter and area.

Lesson 1: What is the Area?

Students explore the relationship between area and perimeter using the context of bumper cars at an amusement park. They design a rectangular floor plan with the largest possible area when given a perimeter of 50 m, and observe that the area of a rectangle increases as the sides get closer in length. Students find that the rectangle with the greatest area for a set perimeter is a square.

Lesson 2: What is the Perimeter?

Students explore the possible perimeter of a bumper car ride that has a floor area of 48 m^2. They recognise that factors of the area form the dimensions of the possible rectangles. Students graph the results and form the generalisation that the smaller the difference in side lengths, the smaller the perimeter.

Lesson 3: Designing Bumper Car Rides

Students design three different floor plans for bumper car rides: (i) where the area is numerically greater than its perimeter; (ii) with a perimeter numerically greater than its area; and (iii) with a perimeter numerically equal to its area.

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Reflection on this sequence

Rationale

Students often confuse perimeter and area. It is important that they know the differences between the two. Perimeter is a linear measurement, measuring the boundary of a figure. Area is a two-dimensional measurement, measuring the space inside the boundary of a figure. A common misconception about perimeter and area is that there is a fixed relationship between the two. This sequence challenges this misconception, showing students that rectangles with a fixed perimeter do not have the same area and vice versa. Students learn how to maximise area when given a fixed perimeter and how to minimise perimeter with a fixed area.

reSolve mathematics is purposeful

- These lessons help students build a mathematically sound definition for perimeter and area. They develop skills in calculating the perimeter and area of rectangles.
- The sequence addresses the misconception that there is a fixed relationship between perimeter and area.

reSolve tasks are inclusive and challenging

- Nearly all students will be familiar with bumper car rides.
- Each lesson asks students to find different rectangles that meet certain specifications. While some students might find all the possible rectangles, other students make a valuable contribution to the investigation if they find only one or two rectangles.

reSolve classrooms have a knowledge-building culture

- Students' data from across the class is used to see regularities and form generalisations.
- Students' current conceptions of area, perimeter and rectangles are challenged.
- The class collaboratively forms mathematically sound definitions.

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LESSON 1

re(Solve) MATHS BY INQUIRY

Area and Perimeter

What is the Area?

Y5

About this lesson

Students explore the relationship between area and perimeter using the context of bumper cars at an amusement park. They design a rectangular floor plan with the largest possible area when given a perimeter of 50 m, and observe that the area of a rectangle increases as the sides get closer in length. Students find that the rectangle with the greatest area for a set perimeter is a square.

Australian Curriculum: Mathematics (Year 5)

ACMMG109: Calculate perimeter and area of rectangles using familiar metric units.

Mathematical purpose

- Students consolidate their understanding of and skills in calculating the area and perimeter of rectangles. They observe patterns and make generalisations.

Learning intention

- To make generalisations about the area of rectangles that have a set perimeter.

Time

A lesson of approximately 1 hour.

Vocabulary

- area
- perimeter
- rectangle
- square

Resources

- reSolve PowerPoint *1a Bumper Cars* (for display)
- reSolve Excel Spreadsheet *1b Area Graph*

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Teacher background information

The following definitions can be used in the lesson:

- **Perimeter:** The distance around the outside of a shape.
- **Area:** The amount of space inside a shape.
- **Rectangle:** A quadrilateral with four right angles. This means that the opposite sides will be parallel.
- **Square:** A square is a special type of rectangle — it is a *regular* rectangle.

In this lesson, students explore the different areas of rectangles with a perimeter of 50 m. Students see that the area of a rectangle is maximised when the rectangle is a square. This challenges the misconception that a square is not a rectangle. A square is a rectangle; it is a regular rectangle.

This concept can be taken further to explore other regular shapes (as suggested in the Further activities section). When given a set perimeter, the greater the number of sides of a regular shape, the greater the area will be. As the number of sides increase, the shape gets closer and closer to becoming a circle. The circle is the shape with the maximum area with a given perimeter.

The lesson uses Excel to tabulate and graph the results. The Excel file has two sheets. The first sheet, 'Bumper car 50 m perimeter', has a table and graph showing the area of rectangles with a perimeter of 50 m.

The second sheet, 'Any perimeter', allows for any set perimeter to be used. It illustrates that no matter what the perimeter, the shape of the graph does not change. To use the 'Any perimeter' sheet, type a chosen perimeter into the orange cell, then highlight the values in the 'Side 2' and 'Area' columns, go to Insert → Recommended Charts and choose the line graph chart.

A perimeter of 50 m

Resources: Show students slides 2–4 of reSolve PowerPoint *1a Bumper Cars*.

Most often, the bumper car track at an amusement park is rectangular in shape. Around the outside of the floor is a rail. Explain to students that the floor space is the **area** and the rail around the outside is the **perimeter**.

Pose the question: *The rail around the outside of a rectangular bumper car ride is 50 m. The length of each side is a whole number. What is the area of the floor?*

Allow the students time to explore the question.

Enabling prompt:

- Provide students with:
  - 1 cm grid paper and explain that 1 cm = 1 m
  - square tiles and explain that each tile represents 1 m^2.

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Noticing students' working

Calculating perimeter and area

- It is possible that there will be confusion around some of the terms in the question, such as perimeter and area. Allow the students the opportunity to recognise their confusion and need for clarification.
  - ◊ **Questioning:** *What is the difference between perimeter and area? Give a definition for the terms.*

Trial and error

- Students guess different measurements with little evidence of number sense.
  - ◊ **Questioning:** *How could you refine your previous working to help find rectangles with a perimeter of 50 m?*
- Do the students refine the side lengths based on previous calculations?
  - ◊ **Questioning:** *Can you see a pattern to help you find all the possible rectangles that have a perimeter of 50 m?*

Recognising patterns

- Students recognise that possible rectangles have adjacent sides that add to 25. This is because the perimeter $(P) = 2(a + b)$. In this case $P = 50$, so $50 = 2(a + b)$, therefore $25 = a + b$.
  Students can systematically find all combinations of whole numbers that add to 25.
  - ◊ **Questioning:** *Can you organise your data in a way that demonstrates you have found all the possible rectangles? What patterns can you see?*

Allow students time to find multiple rectangles. Ask them to sort the data/rectangles to help work out those rectangles that they are still to find or to show that they have found them all. Have students create a poster or display of possible floor plans.

Extending prompt:

- *What if the side lengths are not whole numbers? What might the rectangle look like?*

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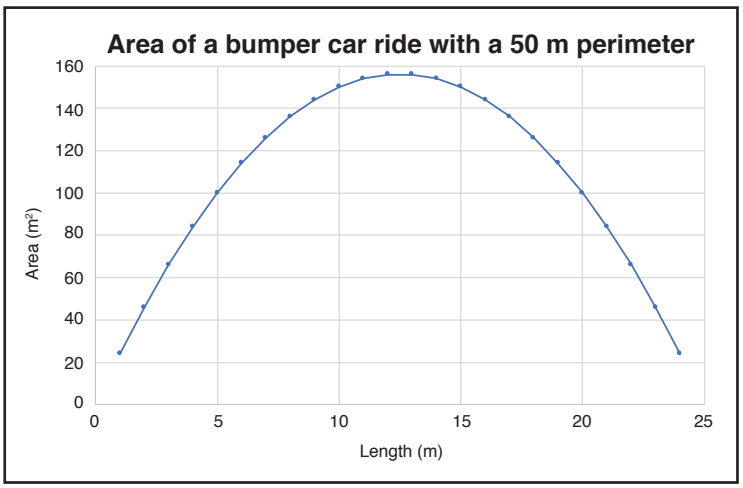
Graphing results

Pose the question: *What is the largest area possible while maintaining the rectangular shape?*

The students will need to work out a way to organise and order their data. Mathematicians organise the data they collect so they can look for patterns and make generalisations. Tables and graphs are useful tools to do this.

Resources: reSolve Excel Spreadsheet *1b Area Graph* tabulates and graph the results, as shown below (also included as slides 5–9 of reSolve PowerPoint *1a Bumper Cars*).

Side 1 (m)	Side 2 (m)	Area (m^2)
1	24	24
2	23	46
3	22	66
4	21	84
5	20	100
6	19	114
7	18	126
8	17	136
9	16	144
10	15	150
11	14	154
12	13	156
13	12	156
14	11	154
15	10	150
16	9	144
17	8	136
18	7	126
19	6	114
20	5	100
21	4	84
22	3	66
23	2	46
24	1	24



Discuss: *Describe the shape of the graph. What does this tell you?*

- The highest point of the graph is in the middle, which means that when the sides are closest in length, the area is maximised.

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Reflection: Making generalisations

Look at the table and graph. Discuss some of the patterns that can be seen, such as:

- The two side lengths add to 25 (as previously noted).
- The area increases by 22, 20, 18, etc. This pattern then reverses.
- With whole number sides, the minimum area is 24 m^2 and the maximum is 156 m^2.

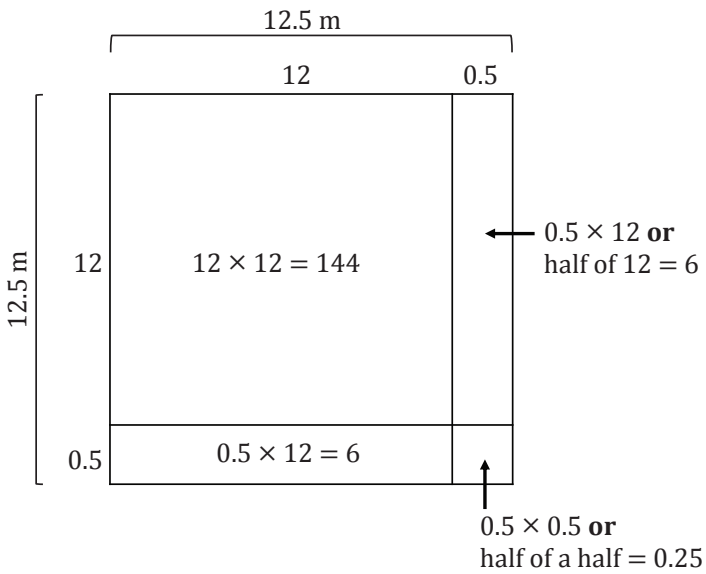
Pose the question: *What would be the highest point on the graph? In other words, for which rectangle will the area be the greatest?*

The rectangle with the greatest area will have side lengths of 12.5 cm × 12.5 cm. This is a square.

Pose the question: *Is a square a rectangle?*

Allow them the opportunity to discuss. Look at the fact that a rectangle is a quadrilateral with four right angles and that, therefore, a square is a regular rectangle.

To calculate the area of a rectangle that is 12.5 cm × 12.5 cm, an open array (or area model) can be used.



$$12.5 \times 12.5$$
$$= (12 \times 12) + (12 \times 0.5) + (12 \times 0.5) + (0.5 \times 0.5)$$
$$= 144 + 6 + 6 + 0.25$$
$$= 150.25 \text{ m}^2$$

Pose the question: *Do you think the same shaped graph will be formed for any set perimeter?*

Open sheet 2 'Any perimeter' in reSolve Excel Spreadsheet *1b Area Graph*. See Teacher background information for directions. Enter some different set perimeters (e.g. 20, 65, 150) into cell C3 (highlighted orange) and notice that the curve does not change.

Pose the question: *What generalisations can be made about the area of rectangles with a set perimeter?*

- As the side lengths get closer in length, the area increases. The rectangle with the greatest area is a square.

Further activity

Look at other regular shapes with a perimeter of 50 m. Use a grid to calculate the area of each shape. *Which shape has the greatest area with a set perimeter of 30 cm?*

Watch the TED-Ed Animation 'Why do honeybees love hexagons?'

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LESSON 2

re(Solve) MATHS BY INQUIRY

Area and Perimeter

What is the Perimeter?

Y5

About this lesson

Students explore the possible perimeter of a bumper car ride that has a floor area of 48 m^2. They recognise that factors of the area form the dimensions of the possible rectangles. Students graph the results and form the generalisation that the smaller the difference in side lengths, the smaller the perimeter.

Australian Curriculum: Mathematics (Year 5)

ACMMG109: Calculate perimeter and area of rectangles using familiar metric units.

ACMNA098: Identify and describe factors and multiples of whole numbers and use them to solve problems.

Mathematical purpose

- Students use factors to determine the possible dimensions of rectangles that have a set area.
- Students will apply their knowledge of and skills in calculating area and perimeter. They reason mathematically and form generalisations.

Learning intention

- To explore the perimeter of rectangles that have a set area and make generalisations.

Time

A lesson of approximately 1 hour.

Resources

- reSolve PowerPoint *2a Perimeter* (for display)
- reSolve Excel Spreadsheet *2b Perimeter Graph*

Vocabulary

- constant
- factor
- perimeter

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Perimeter

Resources: Show students slides 2 and 3 of the reSolve PowerPoint *2a Perimeter.*

Pose the question: *The floor area of a rectangular bumper car ride is 48 m².*
What might the perimeter be if the side lengths are all whole numbers?

Before the students explore, have them predict what they think they will observe based on the previous lesson. Have students justify their predictions. Some possible questions for the students to consider include:

- *Do you think the perimeter will remain constant? If not, when do you think the perimeter will be smallest/largest?*
- *If we graphed one side length against the perimeter (similar to the graph of side length and area in the previous lesson), what do you think the shape of the graph might look like?*

Allow the students time to solve the problem.

Noticing students' working

Finding factors

- Students are now looking for factor pairs of 48 rather than numbers that add to 48.
  - ◊ *What is different between the way you calculate area and the way you calculate perimeter? How will this affect the way you determine the dimensions of the possible rectangles?*
  - ◊ *How will you know that you have found all possible factor pairs for 48?*

Enabling prompt:

- *What arrays can be made with a total of 48 squares?* Provide the students with:
  - ◊ 1 cm grid paper and explain that 1 cm = 1 m
  - ◊ square tiles and explain that each tile represents 1 m².

Graphing the results

Ask the students to consider if they think a graph of the perimeter would look similar or different to the area graph from the previous lesson. Have the students justify their answers.

Possible student responses:

- A curve is expected as the perimeter values symmetrically decrease then increase.
- The curve will be upside down compared to the previous lesson because the smallest values are in the centre of the data.
- The curve might look different because factors are used rather than consecutive numbers.

Resources: reSolve PowerPoint *2a Perimeter* (slides 7–9) and reSolve Excel Spreadsheet *2b Perimeter Graph* present graphs of the data.

Alternatively, create a graph as a class. Discuss the similarities and differences between this graph and the area graph from Lesson 1. Discuss the shape of the perimeter graph.

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Teacher notes:

- The graph plots the length of one side against the perimeter. The first four factors are consecutive and then the difference between the numbers steadily increases. This causes the non-symmetrical shape of the curve.

Reflection: Making generalisations

Pose the question: *If the set area is 48 m^2, when is the perimeter at its greatest and when is it at its smallest?*

- The perimeter is greatest when the difference in side lengths is greatest.
- The smaller the difference in side lengths, the smaller the perimeter. The smallest perimeter will be a square.

To see if this is true for any rectangle, ask students to explore another set area of their choosing. Show students the tables created by Kevin and Nisha on slide 10 of reSolve PowerPoint *2a Perimeter.* Ask students to choose one set of data and create a graph. The point of this exercise is not to find factors but to look at the shape of the curve.

Teacher notes:

- Numbers with many factors have been chosen. All rectangles will form the same curve but numbers with only a small number of factors, including primes, would need to include decimals to form the same curve.
- **Pose the question:** *What generalisations can be made about the perimeter of rectangles that have a set area?*
  - ◊ The smaller the difference in side lengths, the smaller the perimeter.

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LESSON 3

re(Solve) MATHS BY INQUIRY

Area and Perimeter

Designing Bumper Car Rides

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About this lesson

Students design three different floor plans for bumper car rides: (i) where the area is numerically greater than its perimeter; (ii) with a perimeter numerically greater than its area; and (iii) with a perimeter numerically equal to its area.

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ACMNA291: Use efficient mental and written strategies and apply appropriate digital technologies to solve problems.

Mathematical purpose

- Students apply their understanding of perimeter and area to design rectangles that meet particular specifications.

Learning intention

- To design rectangles that meet particular area and perimeter specifications.

Time

A lesson of approximately 1 hour.

Resources

- reSolve PowerPoint *3a Designing Rides* (for display)

Vocabulary

- area
- equable shapes
- is greater than
- is less than
- perimeter

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Teacher background information

This lesson asks students to find rectangles with areas numerically equal to, less than, and greater than their perimeters. It is important to note that area and perimeter are measured using different units and so it is not the measurements that are equal to, less than or greater than — it is only the numerical value that can be compared.

There are only two rectangles with integer side lengths that have areas numerically equal to their perimeter: 4×4 and 3×6. These are called **equable shapes**. There are other rectangles whose side lengths are not integers; for example: 2.5×10, 2.25×18, 2.125×34, etc.

This can be calculated algebraically:

Area $= xy$

Perimeter $= 2x + 2y$

$$xy = 2x + 2y$$

$$xy - 2y = 2x$$

$$y(x - 2) = 2x$$

$$y = \frac{2x}{x - 2}$$

Values can be substituted for x. For example, when $x = 10$:

$$y = \frac{2 \times 10}{10 - 2}$$

$$y = \frac{20}{8}$$

$$y = 2.5$$

Designing rides

Resources: Show reSolve PowerPoint *3a Designing Rides*.

Present the challenge to the students: *Design some different rectangular bumper car rides where:*

1. *The numerical value of the ride's perimeter is greater than its area.*
2. *The numerical value of the ride's area is greater than its perimeter.*
3. *The numerical value of the ride's perimeter is equal to its area.*

Allow students time to create designs.

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Noticing students' working

Notice whether students draw on the generalisations formed from the previous two lessons to help trial and modify their designs.

- **As the side lengths get closer in length, the area increases. The rectangle with the greatest area is a square.**
  - ◊ A rectangle with an area numerically greater than its perimeter will have sides that are equal to or close to equal to in length.
- **The smaller the difference in side lengths, the smaller the perimeter.**
  - ◊ A rectangle with a perimeter numerically greater than its area will have sides that are significantly different in length.

Pose the question: *Look at the dimensions of the rectangles that you have found. At what point does the numerical value of the area become larger than the perimeter?*

Reflection

Have students share their solutions.

Discuss: *Which design would make the best bumper car ride? Why?*

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