Bunches of Balloons

Lesson 1: Discover Phase

Australian Curriculum: Mathematics (Year 2)

ACMNA032: Recognise and represent division as grouping into equal sets and solve simple problems using these representations.

- Dividing a collection of objects into equal-sized groups.
- Identifying the difference between dividing a set of objects into three equal groups and the same set of objects into groups of three.

Lesson abstract

Students are presented with the real-life context of decorating the classroom with 29 balloons. Students use counters to represent the balloons, as they collaboratively work to divide the balloons into equal groups. They record each attempt (draw or photograph) as evidence of their findings and clearly label their representations. They discover that 29 cannot be divided into equal-sized groups.

Mathematical purpose (for students)

Numbers can usually be sorted into equal groups.

Mathematical purpose (for teachers)

Students practise dividing a set of counters into equal groups, strengthening the ability to view a group as a unit which is important for later work on multiplication and division. Concrete materials help visualise the division of objects into equal groups, with some objects left over. Sharing a collection into equal sized groups in more than one way provides opportunities to highlight the difference between dividing into equal groups and groups of (3 equal groups or groups of 3). The inquiry skill of making well-labelled diagrams to record evidence is emphasised.

Lesson Length 60 - 90 minutes. May be split into 2 lessons.

Vocabulary Encountered

- groups of
- even (meaning 'uniform')
- equal
- divide
- left over
- evidence

Lesson Materials

- Picture of different sized bunches of balloons (optional)
- Packet of exactly 29 balloons (or other prime number), plus one extra balloon out of the packet for demonstration
- Blocks or counters (29 per group)
- Student workbooks
- Camera, or other technology for recording such as *Book Creator* (optional)

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Australian Academy of Science

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Introduce the Inquiry Question

Inquiry Question: What is the best way to evenly decorate the room?

Set the context

1. **Introduce the context and purpose** to promote enthusiasm and engagement: we are decorating the classroom for Grandparents' day (or another upcoming event).
2. **Display the packet of 29 balloons** to the class and suggest they use these to decorate the room. Blow up the extra balloon and hold it against something in the room (If you want to re-use the balloon, do not tie it off at this stage).
3. Draw attention to the fact that one balloon, on its own, isn't very appealing. They look better in bunches. Perhaps show a picture of bunches of balloons. For example: *We want the room to look balanced. The balloons need to be split into bunches, but I'm not sure which sized bunches will look the best. I also want to use all the balloons in the packet. We only have one packet of balloons so how will we decide on the best way to arrange them in bunches to decorate the room?*
4. **Introduce Inquiry question** and write it on the board so students can refer to it: *What is the best way to evenly decorate the room?*
5. **Raise the need to clarify** 'evenly decorate' and 'best'. '*Best*' is a relative term and initially subject to personal preference. Don't worry if responses are not mathematical at this stage. This term will be revisited and what is 'best' will depend on how well the student justifies their response later. Evenly in this sense means uniform – perhaps you need to distinguish from 'even and odd numbers'.
6. **Create a word wall** which will be added to during the inquiry. Encourage students to use this vocabulary.
   - '*Evenly decorate*': the balloons need to be in equal bunches (the same number in each bunch) in order to make the room look balanced. The bunches also need to be spread around the room.
   - '*Best*': we need to arrange the bunches around the room so that it looks nice.
7. **Revisit the inquiry question** on the board. Rephrase the task: *What do you think would look best? Where would you put the bunches of balloons so the whole room is decorated? How many balloons will be in each bunch? How many bunches will you need?*

Generating initial ideas

8. **Count the balloons** together with the class. How could we decorate the room with these 29 balloons? (Note: A prime number 29 has been selected so that the balloons cannot form equally sized bunches)
9. **Have students discuss and then draw the room, including a balloon arrangement,** with a partner. As you circulate, observe how students are approaching the task. If they are noticing that there is a problem sharing 29 equally with a selected bunch size, encourage them to try another bunch size or to write what the problem is. After about 5-10 minutes of work, move on to the Checkpoint.

Checkpoint

10. **Pause and ask the class to share initial ideas**: *Who has an idea for how we can group the balloons?* Students' ideas should be revoiced by the teacher as they share, informally substituting the words "divided" or "groups" as relevant. Ensure students respond using a full description of their groupings. *I made 7 groups of 4 but there was 1 balloon left over*. As mathematical vocabulary is introduced, add to the 'word wall' and promote its use in all discussions.
11. **Discuss common issues** (see examples below). A key point here is to encourage students to express obstacles openly and offer suggestions to others. This discussion creates a need to further investigate whether 29 balloons can be equally grouped. For example, students might discuss:
    - **Equal bunches/groups**. *Are all the groups equal? Does anyone have a drawing that shows equal groups?* Flag this as an issue that needs to be resolved.
    - **Leftovers**. *What can we do about the leftovers? Is there another way to group the balloons so there are no leftovers? Can you show me?*

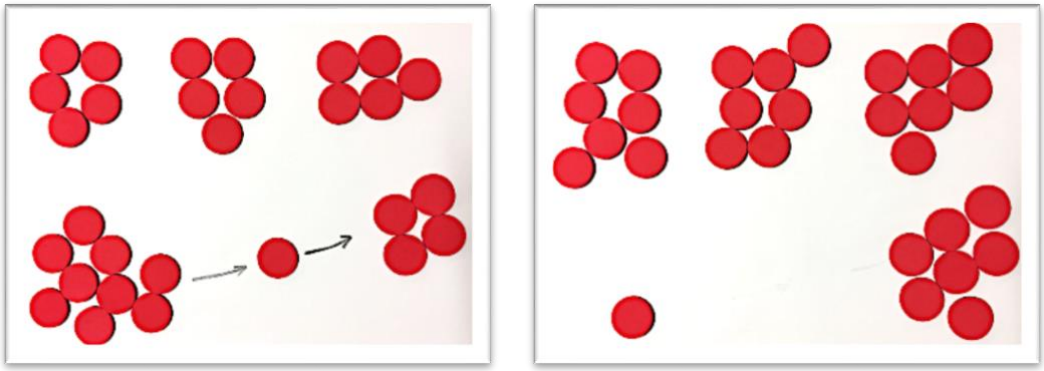
- **Not enough**. *What can we do if there are not enough to make a whole bunch? Is there another way to group the balloons so that every group has enough? Can you show me?*
- **Doesn't look good**. *What do you not like about it? Do you see another group's arrangement that you like better?*
- **Hard to draw because if one arrangement doesn't work it needs to be erased**. *Why? Is there another way we can show our ideas?* [e.g., use counters]
- Watch out for students who have dropped counters, so no longer have 29. Warn students to be careful!

Gather evidence

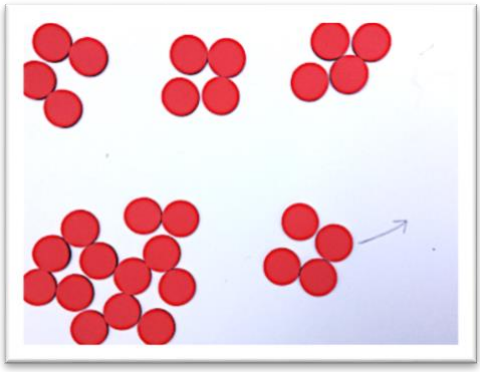
12. Begin a discussion about assembling evidence. An unlabelled representation cannot effectively communicate what has been discovered and it will be difficult for the final audience to interpret. Evidence is a way to show others what has been discovered. For example, you could link it to how police collect evidence to solve crimes. Encourage students to draw or otherwise record each arrangement, and clearly label their representations e.g. *7 groups of 4 balloons with 1 leftover*. If representations do not have clear labelling (e.g. of group size), re-counting is frequently required.

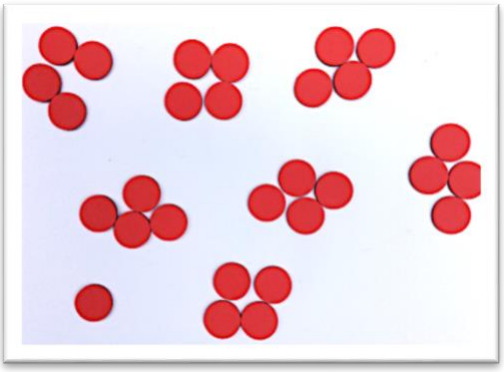
Investigating 29 Counters

13. **Organise students** to work with 1 or 2 partners and provide them with 29 counters. Explain that counters can represent the balloons, and students will work together to see if they can make equal groups from the 29 counters. Inform students that they are expected to gather evidence by recording each arrangement of balloons in their workbook (or by storing a photograph on a device) and labelling it.
14. **Observe the variety of strategies students use** to group their 29 counters equally. For example, they may:
    - Distribute counters one by one into a fixed number of groups (e.g. 5) until there are none left to share.
    - Decide on a group size and continue to make groups of that size until the counters are all used. (Repeatedly subtracting 4, with a remainder.
    - More advanced students may use trial multiplication. For example, if you have 4 in each group and there are 7 groups, we get 28. But there would be one balloon left over.



Number of groups fixed. Distributing one by one to create 4 groups of 7 with 1 left over ("remainder", if you choose to introduce this language).





Number in groups fixed. Create groups of a fixed size (here 4) until all counters are used.

15. **Prompt students to persevere through difficulties** as they try to divide the balloons into equal groups. Ask prompting questions based on their findings. For example:
    - Why can't you make equal groups of 2 from the 29 balloons? Are you sure it won't work? Have you tested it? Do you have the evidence to show me?
    - Maybe a different group size might work? What else could you try?
    - How many different ways are there to group the balloons?
    - I don't think anyone has tried groups of 7 yet, I wonder if that would work. Could you try groups of 7?

16. Check with the class to see that every possible group size (2, 3, 4, 5, 6, 7, 8,…, 20,..) has been investigated by someone.

Reflection

17. **Share ideas.** As many students come to the realisation that they cannot make equal groups of 29 balloons, if everyone is having the same problem. As students share, find opportunities for class or individual skip counting. e.g.: *Can you make groups of 3? Let's see: "3, 6, 9, 12, …..24, 27, 30!")*
18. **Revisit the need for clear evidence and labelling.** *Who has used labels on their diagram to show everyone how many in each group?*
19. **Discuss common issues as a class.** Ask prompting questions to seek further information. Prompting questions could include:
    - **Tried a fixed number of groups**

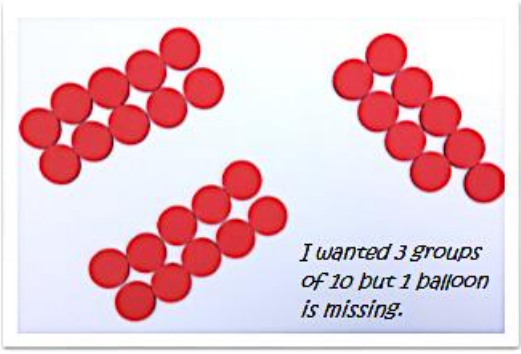
    Student: *I tried to make four groups, one for each corner of the room, but it didn't work.*
    Teacher: *Why didn't it work? Can you show me your evidence? Did anyone else try to make four groups? Did you have any left over?*

    - **Tried a fixed number in each group, but some were left over**

    *Student: **I tried to make groups of three but there were some left over**.*
    Teacher: Did anyone else try to make groups of 3? Did you have any left over? Show me your evidence. Is there anyone who made groups of 3 with none left over? Let's compare the representations of groups of 3. Why didn't it work?

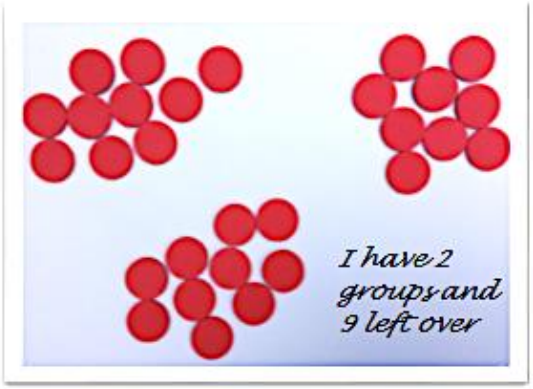
    - **Tried a fixed number in each group, with not enough for the last group**

    Student: I don't have enough balloons to make groups of 10.
    Teacher: *Did anyone else run out of balloons when they were trying to make equal groups? How many more would you need to make groups of…?*



Documenting evidence

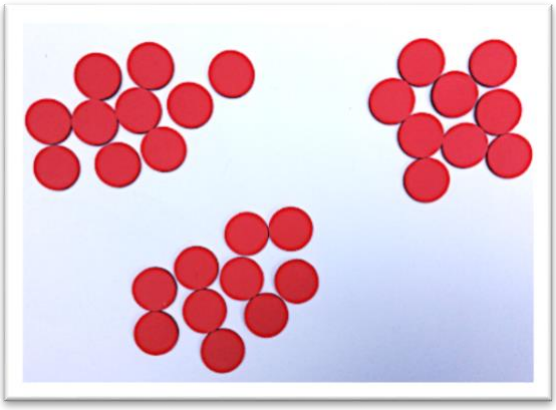
20. **Select one or two examples** of clearly labelled and unlabelled representations. Use the examples to highlight the need for clearly labelled evidence. Model how a poorly labelled representation could be made clearer. Further examples of student work are displayed at the end of this lesson.



Labels may be incomplete

'I have 2 groups and 9 left over'

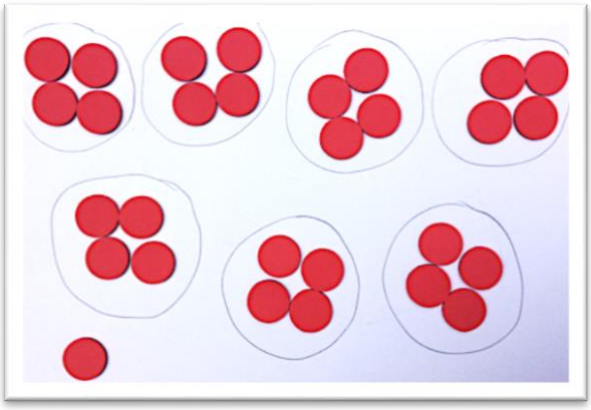
Teacher: 2 groups <u>of 10</u> and 9 left over



Labels may be missing completely:

Teacher: I can't see that they are groups of 10 without having to count the balloons. The groups all look the same size. Are there 3 groups of 10? Labelling your groups would make them easier to compare.

(Model how to label groups.)



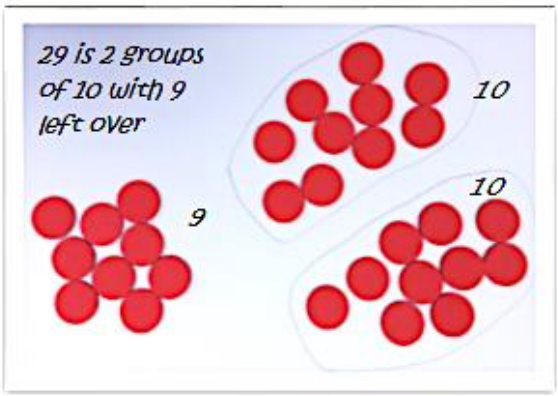
These labels show confusion between number of groups and number in a group.

'I have 4 groups of 7 and 1 left over'

Teacher: you have 7 groups <u>of 4</u> and 1 left over.

Teacher: Is 7 groups the same as groups of 7? Would the total number of balloons change if we made 4 groups of 7 or 7 groups of 4?

Show how the arrangement of balloons is different, but the total number is the same.



'29 is 2 groups of 10 with 9 left over' is the desired response as we are moving towards a multiplication focus.

Circling the complete groups improves the representation.

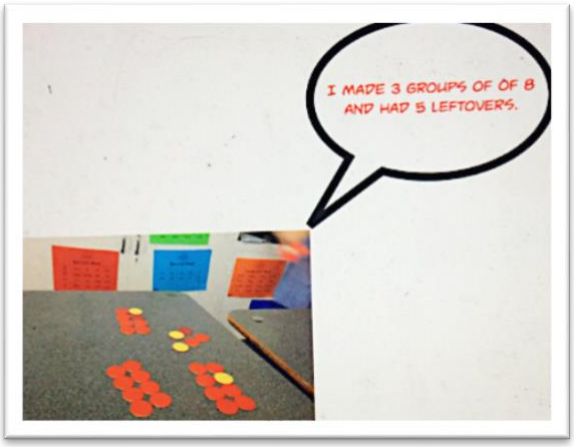
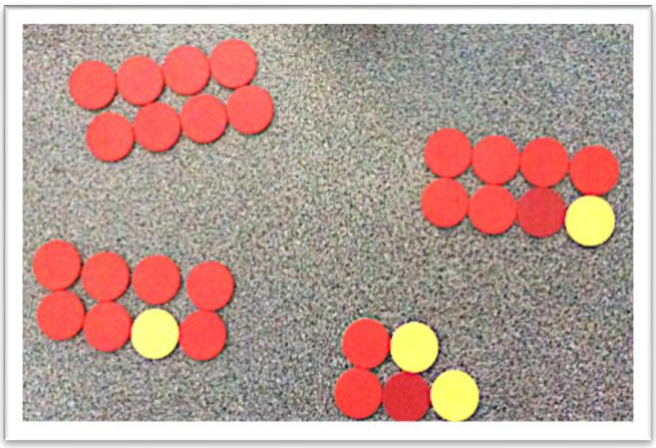
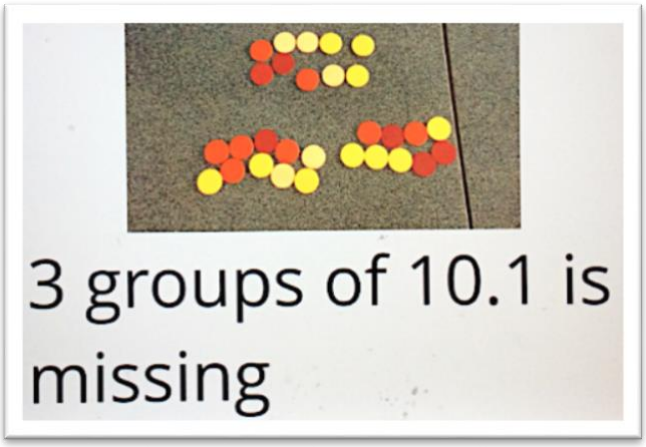
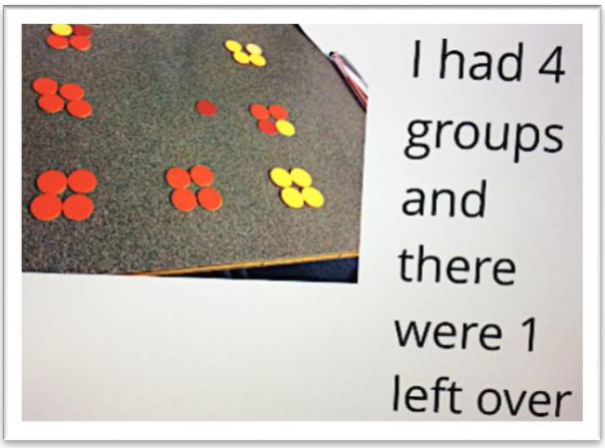
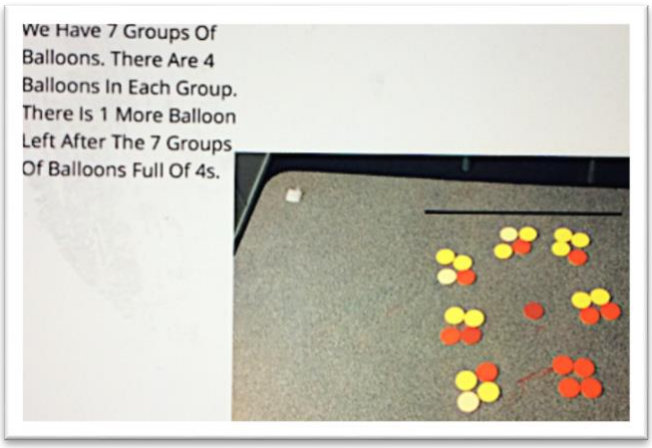
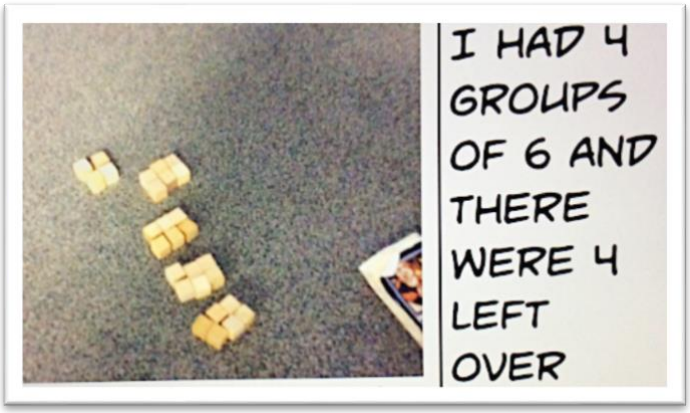
Other good alternatives are:

'3 groups of 10 with one missing'

'10 + 10 + 9 = 29'.

Conclusion

21. **Send students back to see if all of their representations are properly labelled.** Ask them to then check their revised representations with a partner.
22. **Recap** with students what they have done so far and foreground the next lesson where they will try and address the problems they've come across with finding groupings for 29 balloons.

Further examples of student responses

These were made on tablets using the app *Book creator*.
Note that the student making the last image had probably dropped one MAB block.