re(Solve) MATHS BY INQUIRY

Cartoon Counting

Y4

Summary of learning goals

- Students will participate in an exploration of a different number base to build a deeper understanding of and appreciation for our base-10 number system.
- Students will explore grouping by 8s to create a place-value system based on 8 and investigate the ways in which base 8 can be represented on a number chart, highlighting patterns.
- They will explore the similarities and differences between the two number systems.

Australian Curriculum: Mathematics (Year 4)

ACMNA073: Apply place value to partition, and rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems.

Summary of lessons

Who is this sequence for?

- It is anticipated that students will have a developing understanding of the ways in which our number system is built around the number 10.
- Students should be able to unitise groups to make tens, hundreds, thousands and beyond. They should also be able to partition numbers into their place-value parts and recognise that the value of a digit is determined by its place in a number.

Lesson 1: Cartoon Counting

Students consider what counting would be like for cartoon characters, who generally have only eight fingers. Our number system uses 10 as its base, as we have 10 fingers. Students are asked to think about the ways in which numbers might be grouped using a base of 8 and so reinvent base-8 numbers.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

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Reflection on this sequence

Rationale

The purpose of this task is not primarily to learn a new base number system but, rather, to build a better understanding of our own base-10 number system. Working with a different base is an effective way to draw attention to the structure of our own number system. Students can become very familiar with the mechanics of working with numbers and place value without having a deep understanding of numbers. Working with a different base moves students out of their comfort zone as they need to restructure numbers, and helps them to see the power of 10 in our system.

reSolve mathematics is purposeful

- This task employs students' creativity and imagination to restructure our number system.
- Students use a different base to better understand the power of 10 in our number system.

reSolve tasks are inclusive and challenging

- Working with an unfamiliar base moves students from what is familiar to something very unfamiliar, creating a high level of challenge.
- Access is provided through enabling prompts that explicitly link the task back to base 10. This prompt still requires students to work on the same mathematical purpose as those working with base 8.

reSolve classrooms have a knowledge-building culture

- This task supports students to apply their existing knowledge, challenge their thinking and explore misconceptions in a collaborative environment.
- A 'gallery walk' invites groups to share their counting methods and answer questions from other class members.
- Students are invited to re-count their collection, adopting new learning received from others in the class.

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Cartoon Counting

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About this lesson

Students consider what counting would be like for cartoon characters, who generally have only eight fingers. Our number system uses 10 as its base, as we have 10 fingers. Students are asked to think about the ways in which numbers might be grouped using a base of 8 and so reinvent base-8 numbers.

Australian Curriculum: Mathematics (Year 4)

ACMNA073: Apply place value to partition, and rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems.

Mathematical purpose

- Students explore a different number base to build a deeper understanding of and appreciation for our base-10 number system.

Learning intention

- To investigate how cartoon characters would count a collection.

Time

A lesson of approximately 1 hour.

Resources

- 45–60 Unifix cubes per student (at least)

Vocabulary

- base

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Teacher background information

Understanding base-8 numbers

Our number system is based on 10. We have 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Using these 10 digits we can write all possible numbers. The value of a digit is determined by the place it holds in a number. This value increases by powers of 10 as the digit moves from right to left. This is demonstrated in the table below.

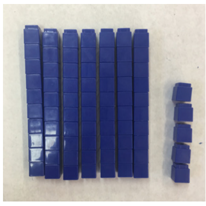
1000s			1s		
10 groups of 10,000 100,000s	**10 groups of 1000 10,000s**	**10 groups of 100 1000s**	**10 groups of 10 100s**	**Groups of 10 10s**	**Ones 1s**
10^5	10^4	10^3	10^2	10^1	10^0
100 000	10 000	1000	100	10	1

In a base-8 number system, there are only eight digits: 0, 1, 2, 3, 4, 5, 6 and 7. In base 8, the value of a digit increases by powers of 8 as it moves from right to left, as shown in the table below.

512s			1s		
8 groups of 4096 32,768s	**8 groups of 512 4096s**	**8 groups of 64 512s**	**8 groups of 8 64s**	**Groups of 8 8s**	**Ones 1s**
8^5	8^4	8^3	8^2	8^1	8^0
32 768	4096	512	64	8	1

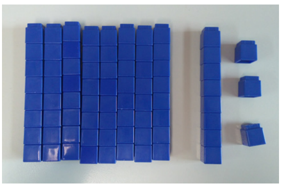
Let's explore 75, 36 and 106.

Base 10 | **Base 8**



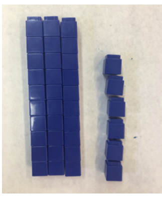
75 has 7 groups of 10 and 5 ones left over.

Hundreds 10^2	Tens 10^1	Ones 10^0
	7	5



75 has 1 group of 64, 1 group of 8 and 3 ones, and so it would be written as 113.

64s 8^2	8s 8^1	1s 8^0
1	1	3

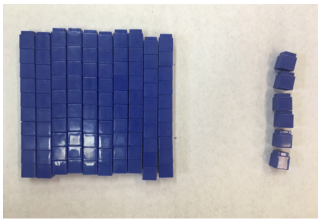


36 has 3 tens and 6 ones.

Hundreds 10^2	Tens 10^1	Ones 10^0
	3	6

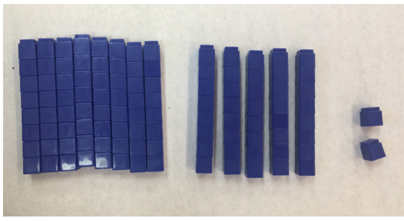
36 has 4 groups of 8 and 4 ones left over, so 36 is written as 44.

64s 8^2	8s 8^1	1s 8^0
	4	4



106 has 1 group of 100, zero tens and 6 ones.

Hundreds 10^2	Tens 10^1	Ones 10^0
1	0	6



106 has 1 group of 64, 5 groups of 8 and 2 ones, and is written as 152.

64s 8^2	8s 8^1	1s 8^0
1	5	2

Number systems and bases

Start the lesson by discussing bases for number systems. We use base 10 now, but this has not always been the case. For example, the Babylonians used base 60 for their number system and the Mayans used base 20. The ancient Romans didn't use a base at all. They had different symbols to indicate different numbers and didn't have a symbol for zero. As numbers got larger, the ancient Romans needed to invent new symbols.

Base 10 is not the only number system used in our lives. Computers use binary, which is base 2. They only use the digits 0 and 1. We also use base 60, base 7 and sometimes even base 12 in our everyday lives — can any students think of examples? (Teacher note: These are related to measurements of time.)

Pose the question: *Why do you think we use 10 as the base for our number system?*
Why don't we use another number?

- Hindu-Arabic numbers are base 10 because we have 10 fingers. This also links to the term 'digit': our fingers are referred to as digits, and digit is the term used for the individual numbers within a number; for example, 24 is made up of the digits 2 and 4.

There are 10 digits in our number system. Write the digits 0–9 on the board.

Nearly all cartoon characters have eight fingers because it is easier to draw four distinct fingers on each hand. Show the students a variety of images of cartoon characters, each clearly showing the four fingers on each hand.

Explain that in Cartoon Land only eight digits are used. Remove the 8 and 9 from the board, leaving 0–7.

Pose the challenge: *What would the number system in Cartoon Land look like?*
Can you create a number system that is based on 8?

Resources: Provide pairs of students with a bag of 45–60 Unifix cubes each.

Teacher notes:

- Ensure students do not receive more than 60 cubes. This keeps the count below 100 in base 8. Extra cubes will be added later.
- When speaking aloud numbers that are *not* in base 10, read out their digits rather than the number they resemble. For example, 10 in base 10 is written as 12 in base 8, and should be read aloud as 'one-two' or 'twelve base 8' rather than simply 'twelve'.

Counting in base 8

Ask students to count their collection of cubes using 8 as the base number, which means making groups of eight rather than 10 when counting.

Enabling prompts:

- Ask: *How many cubes are there? How could you keep track of your count?*
- Allow the students to arrange their cubes in groups of 10 and count them in base 10. Once they have completed the base-10 count and understand the groupings of 10, ask: *In what ways can we rearrange this collection to represent base 8?*

Prompt students to consider:

- When writing in base 10, after 9 (the last digit) the number is 10, which we write using the digits 1 and 0.
- When writing in base 8, after 7 (the last digit) we make 1 group of 8 but, rather than writing 8, we write 10 as in 1 group of 8 and no ones, reading it as 'one-zero'.

Counting in base 8 will take students a little while. They should be allowed time to experiment and make mistakes.

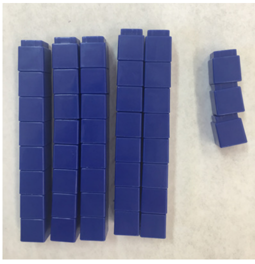
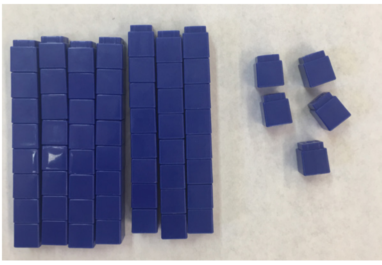
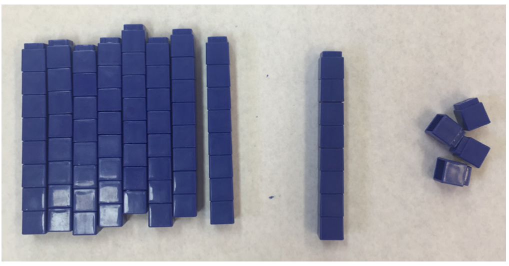
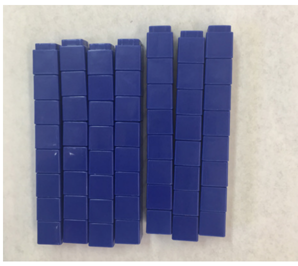
Partway through the lesson, have some students share their strategies. It is not necessary to share only their correct counting strategies. Consider which strategies are most helpful for calculating the total number of blocks using 'cartoon counting'.

Allow students to modify their count as needed.

Give the students additional blocks so that their count goes beyond 100 in base 8 (this means they will have more than 64 blocks). What happens when we get 8 groups of 8?

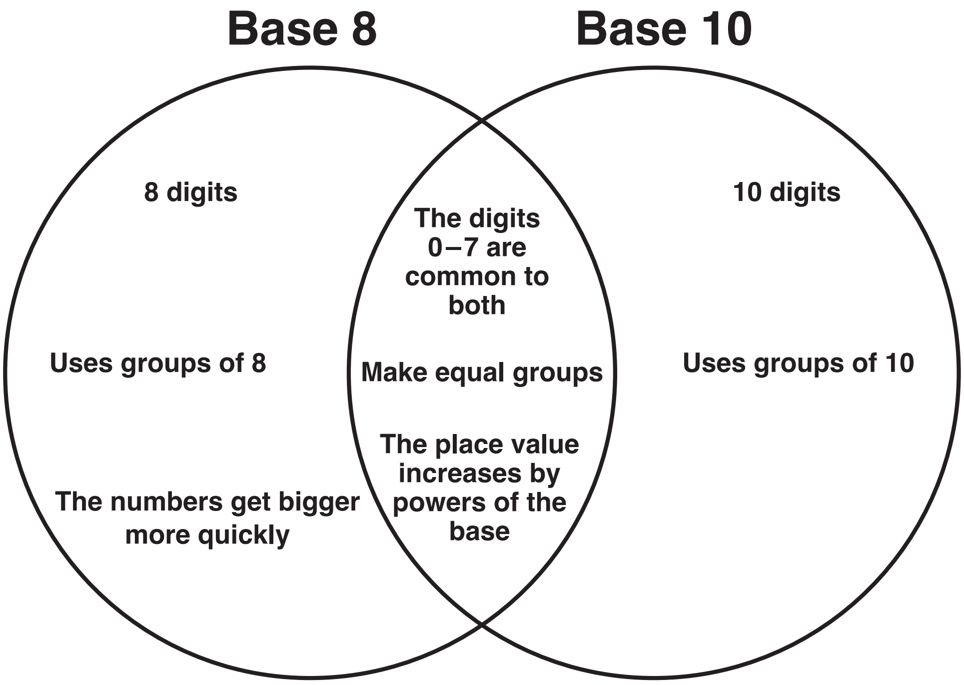
Reflection

Students share the ways in which they counted their blocks. Discuss the best way to record results. For example, using the following place-value chart.

	64s 8 groups of 8 8^2	8s Groups of 8 8^1	1s 0 groups of 8 8^0
		5	3
		7	5
	1	1	4
		7	0

Pose the question: *In what ways is this similar to and in what ways is this different from the way we normally count in base 10?*

The students' thinking can be recorded in a class Venn diagram.



Further activities

Activity 1

Using base 8, calculate the number of blocks in the whole class.

Activity 2

Show students a picture of the cartoon characters Minions, which have only three fingers on each hand.

Pose the questions:
- *In what ways would counting and their number chart be different from base 10 and base 8?*
- *In Minion counting, how many blocks do you have? How many in the class?*

Activity 3

In a base-2 number system, there are only two digits: 0 and 1. In base 2 the value of a digit increases by powers of 2 as it moves from right to left, as shown in the table below.

8s			1s		
2 groups of 16 32s	2 groups of 8 16s	2 groups of 4 8s	2 groups of 2 4s	Groups of 2 2s	Ones 1s
2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1

2 is represented as 10 in base 2, 4 is represented as 100, and 5 is represented as 101. Have students calculate their age in base 2.

Activity 4

Show the students a number chart with 0–9 on the top row and with numbers up to 119.

Ask: *What would a number chart for 'cartoon counting' look like?*

The chart should have eight columns. The highest two-digit number will be 77.