FIBONACCI MILES

Australian Curriculum: Mathematics (Year 8)

ACMNA188: Solve a range of problems involving rates and ratios, with and without digital technologies.

Lesson abstract

Students are introduced to a trick for quick conversion between miles and kilometres using the Fibonacci sequence and are challenged to explain why the trick works. They investigate using their knowledge of ratio and discover that the miles/kilometres conversion rate is close to the golden ratio.

Mathematical purpose (for students)

Why does this approximation for converting between miles and kilometres work?

Mathematical purpose (for teachers)

Students deepen their understanding of ratios as the multiplicative comparison of two numbers. Students apply this understanding to explore relationships between neighbouring numbers in the Fibonacci sequence. They generate the golden ratio.

Suggested presentation: One lesson of one hour

Vocabulary encountered

- conversion
- Fibonacci numbers
- rate
- ratio

Lesson materials

- *1a Fibonacci Miles* PowerPoint (for display)

We value your feedback after this lesson via our website.

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Introduction

Introduce the context of miles and kilometres and converting between the two measurements (Slide 1). If students are unfamiliar with miles you can spend some time estimating known distances in miles to establish familiarity.

Show students Slide 2 and explore the claim that you can convert between miles and kilometres by using consecutive Fibonacci numbers—for example, three miles is approximately five kilometres.

Pose the challenge: *What is happening? Why does this occur?*

Exploration

Allow students time to explore the problem.

Useful Prompts

- *How does each number in the Fibonacci sequence compare to the previous number?*
- *Are the numbers increasing linearly?*
- *Experiment with a few pairs of consecutive Fibonacci numbers. How can we describe the* ***rate*** *at which the numbers increase? What is the ratio of each number to the next?*
- *How much is one mile in kilometres?*

Teacher Notes

- Students will need to explore relationships in the Fibonacci sequence as part of this problem. They may do this using a spreadsheet or by hand. They will only need to work with the first fifteen or so numbers in the Fibonacci sequence to identify the pattern.

Come together to discuss findings.

Teacher Notes

- As the Fibonacci sequence continues, the ratio between consecutive numbers approaches 1.618 (also known as the "golden ratio").
- The ratio between miles and kilometres is 1.609.
- Because these ratios are so close, we can use numbers in the Fibonacci sequence to approximate the miles/conversion rate.

Application

Pose the challenge (Slide 4): *How could you use Fibonacci numbers to convert:*

- *25 miles to the nearest kilometre?*
- *58 miles to the nearest kilometre?*
- *100 miles to the nearest kilometre?*
- *Any number of miles to the nearest kilometre?*

Conclusion

Discuss as a class: *is this a convenient way of converting between miles and kilometres? Are there any other convenient ways to convert? Are there any other situations where the Fibonacci sequence could be used to convert values?* (for example, currency exchange rates).