

Summary of learning goals

- The resource introduces the key idea of multiplication as a Cartesian product, using the language of 'for each'.
- Students explore the total number of different robots that can be made using three heads, three bodies and three sets of legs. The students represent the different combinations for the robots as arrays.

Australian Curriculum: Mathematics (Year 2)

ACMNA031: Recognise and represent multiplication as repeated addition, groups and arrays.

Summary of lessons

Who is this sequence for?

- Students will need to have some familiarity with multiplication concepts and early strategies for solving multiplication problems. The task draws on students' knowledge that multiplication can be represented as equal groups, repeated addition and an array formation.
- Students will apply and build on this knowledge and create connections to multiplication as a Cartesian product.

Lesson 1: Robot, Go Fish

Students think about the number of robots it is possible to make with three heads, three bodies and three sets of legs. Students start by making one robot and checking whether anyone else has made the same robot as them. They then play a game that shows that many unique robots can be made.

Lesson 2: How Many Robots?

Students are asked to consider how many different robots can be made using three heads, three bodies and three sets of legs. Students sort and classify their robot cards according to their body parts. From here, an array structure is introduced. Students will see that for each head there are three bodies, and that for each body there are three sets of legs. This means that for each head, nine unique robots can be made.

Reflection on this sequence

Rationale

Teaching multiplication in the classroom often focuses on creating equal groups and then extending this idea to the use of arrays. The concept of *equal groups* is a very important aspect of multiplication, but there are other concepts that need to be explored. Multiplication comes in multiple forms and all should be explored to build fluency with the operation. One of these forms is that of *for each*; for example, for each cake there are three candles or for each egg use half a cup of flour. This resource explores the concept of *for each* through Cartesian product, although the Cartesian product is not specifically mentioned in the *Australian Curriculum: Mathematics*.

The Cartesian product of multiplication is the product of two or more sets. In this case, students are presented with three robot heads, three bodies and three types of legs. They are asked to work out how many combinations are possible using these different sets. Students will see that *for each* head there are three bodies, and that *for each* body there are three sets of legs. An array structure is used to help students work out the total in the collection.



reSolve mathematics is purposeful

- This sequence focuses on the substantial mathematical idea of multiplication as a Cartesian product and explores and builds the language of *for each*.



reSolve tasks are inclusive and challenging

- Access for students to this task is achieved through using what is familiar to introduce a new concept: the tasks in this sequence pull together the familiar representation of the array structure with the new concept of Cartesian product.
- Extending prompts are provided to allow students to look beyond the original problem to consider different numbers of heads, bodies and legs.



reSolve classrooms have a knowledge-building culture

- The concept of Cartesian product is introduced in the context of a game played in small groups. Collaborative play develops students' language and their understanding of concepts in a fun and encouraging environment. These groups then work together to find all possible robot combinations.

Robot, Go Fish

Y2

About this lesson

Students think about the number of robots it is possible to make with three heads, three bodies and three sets of legs. Students start by making one robot and checking whether anyone else has made the same robot as them. They then play a game that shows that many unique robots can be made.

Australian Curriculum: Mathematics (Year 2)

ACMNA031: Recognise and represent multiplication as repeated addition, groups and arrays.

Mathematical purpose

- To explore the key idea of multiplication as a Cartesian product, using the language of *for each*.

Learning intention

- Estimate how many different robots can be made using three heads, three bodies and three sets of legs.



Time

Two lessons of approximately 1 hour each.



Vocabulary

- for each



Resources

- reSolve PDF *1a Robot Parts Pictures* (two copies for whole class printed on **paper**)
- reSolve PDF *1b Robot Template* (two per student)
- Decks of Robot Playing Cards (see reSolve PDF *1c Robot Playing Cards*, one deck per four students, printed on **card**)

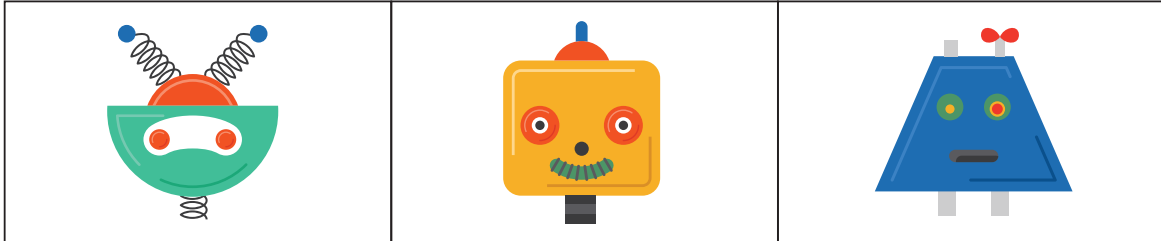
Create your own robot



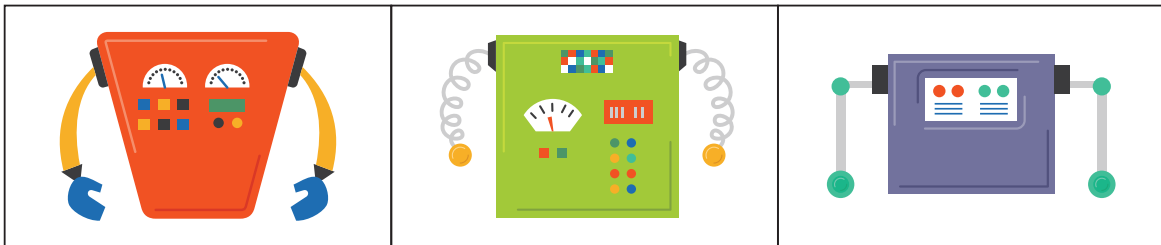
Resources: Provide students with a collection of robot parts on paper (reSolve PDF *1a Robot Parts Pictures*) and a copy of the robot template (reSolve PDF *1b Robot Template*).

Introduce the context of building robots.

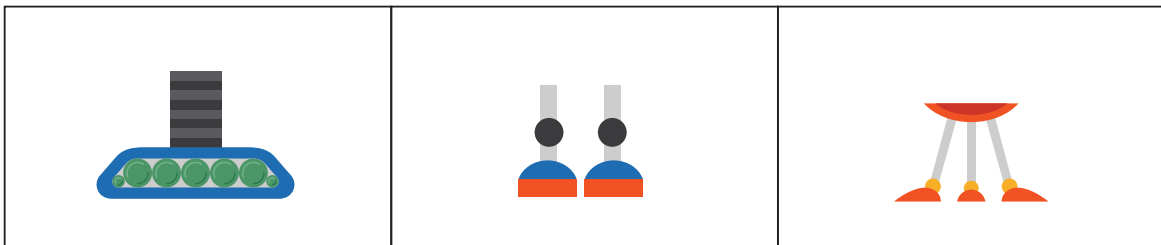
1. *A robot must have a head.* Show the students the different heads.



2. *A robot must have a body.* Show the students the different bodies.



2. *A robot must have legs.* Show the students the different sets of legs.



Have the students construct their own robot from the different body parts available and glue them onto the template.

Ask the students to see if they can find someone else with the same robot as them and, if so, to group themselves. Have a look at the different robots that have been made.

Ask students to estimate how many different robots they think it might be possible to make.

Reflection

Divide students into groups of about four.



Resources: Provide them with a deck of Robot Playing Cards (see reSolve PDF *1c Robot Playing Cards*).

Introduce the game Robot, Go Fish, which is a modified version of the popular children's card game Go Fish.

Game rules

1. The aim of the game is to make as many unique robots as possible.
2. Students are each dealt seven cards. The remaining cards are placed in a pile in the centre.
3. Students take turns to go first. On their turn, students may choose to put down a robot or 'fish' for a new card.
 - a. **Putting down a robot:** a robot must have a head, body and legs. To put down a robot, it must be different to any other robot already played.
 - b. **'Fish' for a card:** students name a person in the group and ask them if they have a body part card; for example, "Nadia, do you have a body/head/legs card?". If the player holds one of these cards, they must hand it over. If the player doesn't have the card, they say "Go fish" and the asking player must draw one card from the centre pile.
3. Every time a student plays a robot they must pick up three new cards. This ensures that the game continues until all the cards are used. The game ends when all the cards have been used and/or no more unique robots can be made.
4. The winner of the game is the person who made the most unique robots.

After the first game of Robot, Go Fish ask the students to use the robot part pictures and templates to make copies of each of the robots that they have made in the game. After the second game, ask them to make copies of any new robots that they have made.

Reflection

Ask students to consider: *Has your group made all the possible robots?* If students do discover a new robot, allow them to make it and add it to their collection.

Ask students to refine their estimate of how many robots they think can be made.

Keep the robot cards and pictures, as they will be used in the next lesson.

Where to next?

In Lesson 2: How Many Robots? students will use the robot templates that they have made to find all possible unique robots. This introduces the multiplicative concept of *for each*.

How Many Robots?

Y2

About this lesson

Students are asked to consider how many different robots can be made using three heads, three bodies and three sets of legs. Students sort and classify their robot cards according to their body parts. From here, an array structure is introduced. Students will see that for each head there are three bodies, and that for each body there are three sets of legs. This means that for each head, nine unique robots can be made.

Australian Curriculum: Mathematics (Year 1)

ACMNA031: Recognise and represent multiplication as repeated addition, groups and arrays.

Mathematical purpose

- Exploring the key idea of multiplication as a Cartesian product using the language of *for each*.
- Students represent the total number of different robots that can be made using three heads, three bodies and three sets of legs, using arrays.

Learning intention

- To find all the unique robots that can be made using three heads, three bodies and three sets of legs.



Time

Two lessons of approximately 1 hour each.



Vocabulary

- for each



Resources

- Leftover Robot Parts Pictures from Lesson 1: Robot, Go Fish (from reSolve PDF *1a Robot Parts Pictures*)
- Robot pictures glued to templates, as prepared in Lesson 1
- reSolve PDF *1b Robot Template* (two per student)

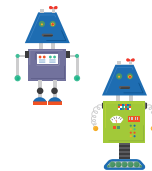
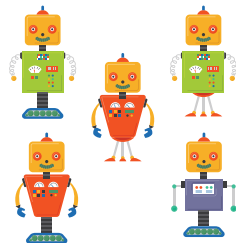
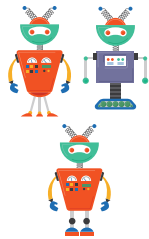
Sorting robots

To complete this task, students will continue working in their groups from Lesson 1.
Ask the students to sort their completed robots.

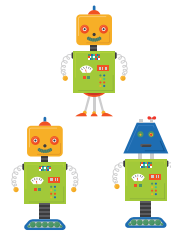
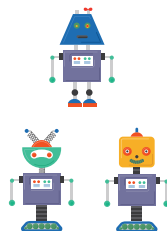
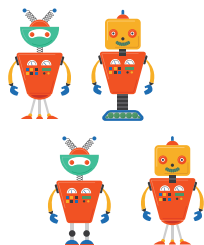


Possible student responses:

Students are likely to sort based on one of the specific body parts; for example:



We have sorted into groups of robots with the same head.

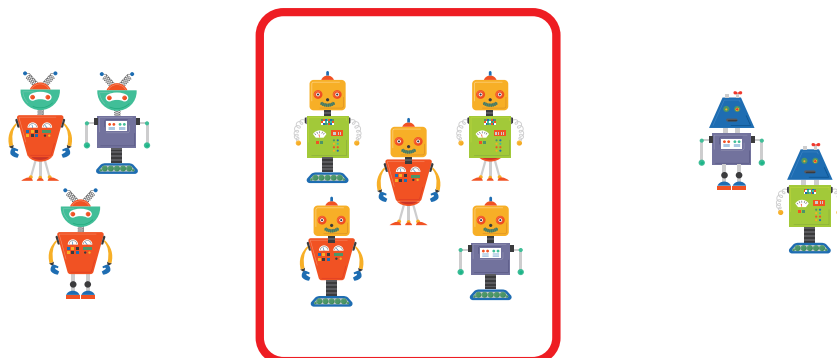


We have sorted into groups of robots with the same body.

Discuss the different ways that students have sorted their robots and observe that sorting by a body part (i.e. head, body or legs) creates three groups. Give students the opportunity to re-sort their robots by a selected body part, if needed.

Ask the students to pick one of the smaller groups created from sorting the robots according to a body part. It would be helpful for students to use the group for which they have the largest collection of robots. This chosen group will be looked at in more detail.

For example, the following robots are sorted based on their heads. Let's consider the middle group in more detail.



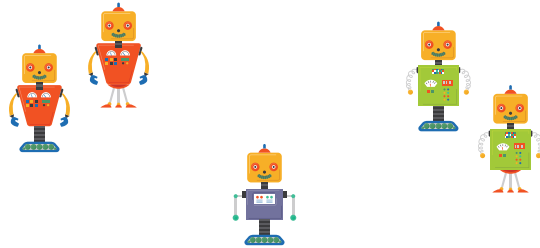
Focusing on one group makes the problem smaller. Explain that mathematicians will look at a smaller problem to help them solve a larger problem.

Pose the questions: *Do you have all the possible robots for the group that you have selected?
Which robots are missing?*

Allow students time to explore their smaller group of robots. Have them make any new robots that they identify as missing.

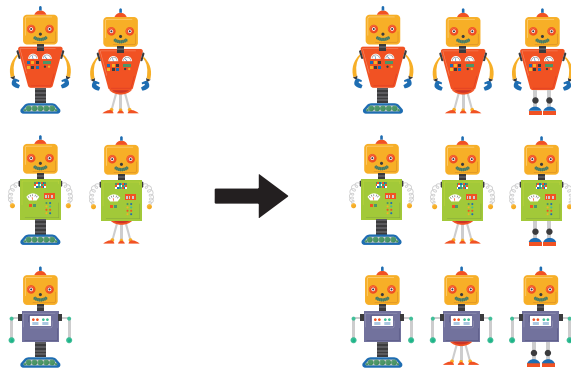
Ask the students: *How could you sort and organise the robots to show which robots are missing?*

For example, the robots were originally sorted according to their heads. The yellow heads were chosen as the group to consider in more detail. These robots have now been sorted by their body. This makes it clear that there are some robots missing:



To help work out which robots are missing, have the students arrange the robots in an array.

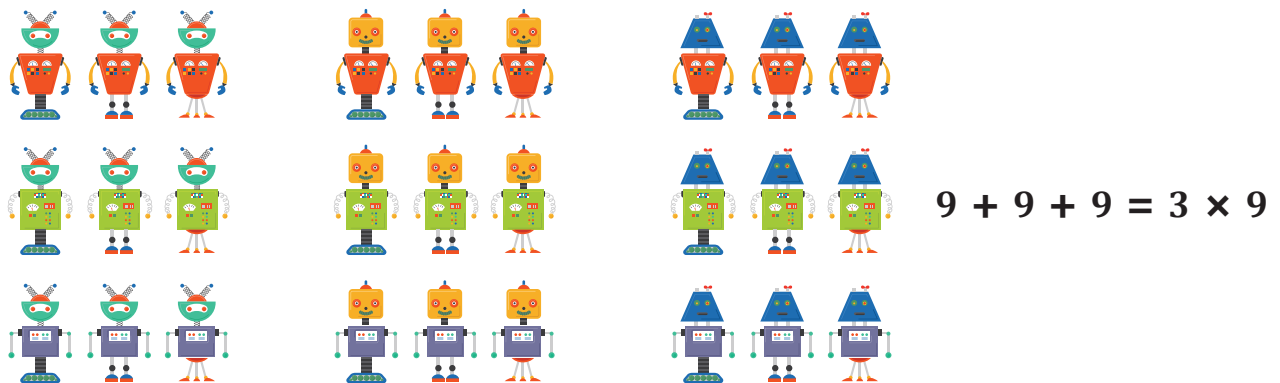
In the following example, the row shows the same body and the column shows the same legs.



This means that for each head, nine different robots can be made.

Pose the question: *If **nine** robots can be made for each head/body/legs, how many robots can be made in total?*

The total number of robots can be thought of as three groups of nine (as shown below).



Extending prompt:

- *How many different robots can be made with:*
 - ◊ three heads, three bodies and four sets of legs?
 - ◊ four heads, three bodies and three sets of legs?
- *Why are both answers the same?*

Reflection

The focus of this reflection is to use students' work to explore the *for each* idea in multiplication.

Select students to share work samples to the class. Look at different examples that show:

- For each head there are three bodies and three sets of legs.
- For each body there are three heads and three sets of legs.
- For each set of legs there are three bodies and three heads.

Ask the students to consider why the answer is the same regardless of the way they are ordered.

Consider what would happen if you built different robots using two heads, three bodies and two sets of legs? Students choose two heads and two sets of legs to use and see if they can represent the problem. Ask them to record how many different combinations can be made.