re(Solve) MATHS BY INQUIRY My Place in Space

Y6

Summary of learning goals

- Students learn to plot points on the Cartesian plane and find the coordinates of given points. They measure distance between points.

Australian Curriculum: Mathematics (Year 6)

ACMMG143: Introduce the Cartesian coordinate system using all four quadrants.

Summary of lessons

Who is this sequence for?

- This sequence extends the reSolve sequence Directed Number through plotting points in the four quadrants of the Cartesian plane and finding horizontal and vertical distances between points.
  It is assumed that students are familiar with basic concepts relating to negative numbers, and that they can plot points on the Cartesian plane when given the coordinates or find the coordinates of a given point.

Lesson 1: Hidden Squares

Students locate points on a Cartesian plane and describe their location using ordered pairs of coordinates. They search for squares whose vertices are points on a Cartesian plane and find the side lengths of the squares. They draw squares or rectangles when given a combination of the coordinates of one or more vertices and the side length(s).

Lesson 2: The Best Spot for the Depot

Students locate points on a Cartesian plane and describe their location using ordered pairs of coordinates. They find the vertical and horizontal distances between points on a Cartesian plane and use this to solve a problem of finding the best location for a depot.

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Reflection on this sequence

Rationale

The resource provides two interesting problems that develop fluency in locating points in the four quadrants of the Cartesian plane. It builds on the reSolve Year 6 Directed Number sequence, in which students learn about notation for positive and negative numbers, and distances between numbers on a number line.

reSolve mathematics is purposeful

- This sequence provides a context in which the Cartesian plane makes sense as a way of describing locations in 2-dimensional space.

reSolve tasks are inclusive and challenging

- The context of locating a depot for ice-cream deliveries is obviously contrived but is nevertheless an engaging problem. Students use different strategies to solve the problems in the two lessons, from counting spaces between two points, to visualising the points and finding distances without counting squares.

reSolve classrooms have a knowledge-building culture

- Students build on prior knowledge about the difference between a pair of numbers, which might be positive or negative, by considering horizontal and vertical translations in two dimensions. They share results as a class and generalise to build rules relating to the best location for a depot.

Acknowledgements

Lesson 2 is an adaptation of the chapter 'Locating a Depot' from *Mathematics at Work: Modelling Your World* by Ian Lowe, originally published in 1988 by the Australian Academy of Science.

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LESSON 1

re(Solve) MATHS BY INQUIRY

My Place in Space

Hidden Squares

Y6

About this lesson

Students locate points on a Cartesian plane and describe their location using ordered pairs of coordinates. They search for squares whose vertices are points on a Cartesian plane and find the side lengths of the squares. They draw squares or rectangles when given a combination of the coordinates of one or more vertices and the side length(s).

Australian Curriculum: Mathematics (Year 6)

ACMMG143: Introduce the Cartesian coordinate system using all four quadrants.

Mathematical purpose

- Students learn to plot points on the Cartesian plane and find the coordinates of given points. They use their knowledge of coordinates to identify squares with a horizontal and vertical orientation.

Learning intention

- To locate points on a Cartesian plane.

Time

A lesson of approximately 1 hour.

Resources

- Student Sheet 1 – Hidden Squares
- Student Sheet 2 – Squares Without Drawing

Vocabulary

- coordinates
- orientation

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Teacher background information

In the first part of the lesson all the squares have a horizontal or vertical orientation. Students should be encouraged to find side lengths, first by counting squares, but should then move onto visualising the horizontal or vertical distance between two points having the same *y*- and *x*-ordinates, respectively. In this way, the lesson builds on the reSolve Year 6 Directed Number sequence.

Finding squares

Resources: Give students Student Sheet 1 – Hidden Squares.

Pose the challenge: *Some squares are hidden in the Cartesian plane. Each square has sides that are parallel to the x-axis and y-axis (i.e. horizontal and vertical).*

There are 47 points marked in the plane. Three of these cannot be a vertex of a square. What are the coordinates of these three points? Why can't they be a vertex of a square?

The eight points shown in red indicate vertices that are used by two different squares.

Pose the questions: *How many squares are there?*

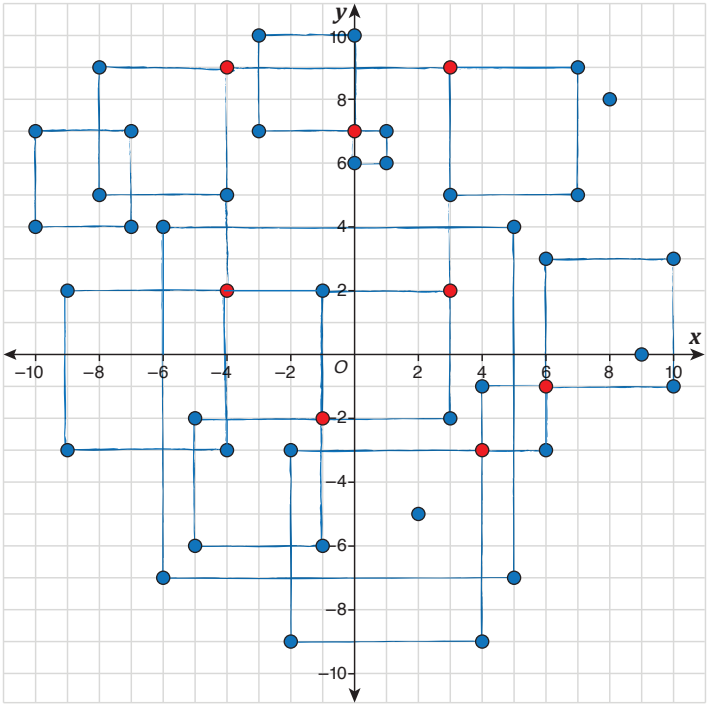
Draw in all of the squares.

What is the side length of the smallest square? What are the coordinates of its four vertices?

What is the side length of the largest square? What are the coordinates of its four vertices?

How many squares of side length 4 are there?

Possible student response:



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Enabling prompt:

- *One of the squares has a side length of 1 unit. Can you find it?*
  - ◊ *This square shares a corner with another square. Can you find it?*
  - ◊ *How can you use this thinking to help you find other squares?*
- *All the squares have side lengths that are parallel to the* x*-axis and* y*-axis. Look at the outer parts of each quadrant. Can you find any points that must be the corners of a square? Use this to help you find other squares.*
  - ◊ For example: points (–2, –9) and (4, –9).

Squares (and rectangles) without drawing

Resources: Give students Student Sheet 2 – Squares Without Drawing.

Note that all the squares and rectangles are positioned with side lengths that are parallel to the *x*-axis and *y*-axis, as in the previous activity. Encourage students to complete the sheet without drawing the squares.

Enabling prompt:

- Allow students to draw.

Further activities

Extending to squares that are not horizontal or vertical

Complete the nRich activity Ten Hidden Squares at https://nrich.maths.org/2654.

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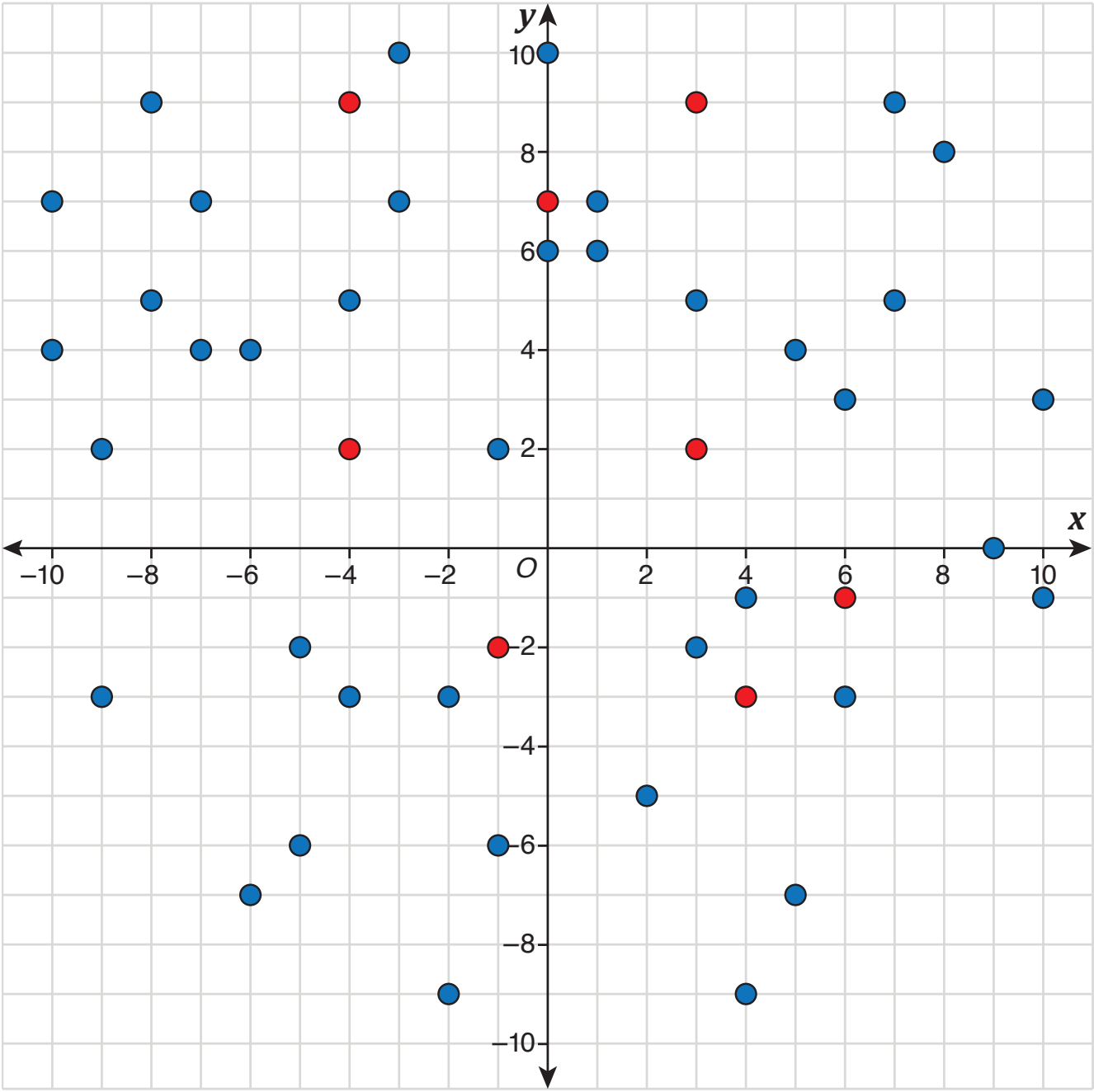
Hidden Squares

Name: _____



1. How many squares are there?

2. a. What is the side length of the smallest square?
   b. What are the coordinates of its four vertices?

3. a. What is the side length of the largest square?
   b. What are the coordinates of its four vertices?

4. How many squares of side length 4 are there?

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Squares Without Drawing

Name: _____

1. A square has three of its vertices at (–1, –3), (–1, 2) and (2, 3).
   a. What are the coordinates of the other vertex?
   b. How long is the side of the square?
2. A square has two of its vertices at (–3, 2) and (–3, –5).
   a. How long is the side of the square?
   b. What are the coordinates of the other vertices? Is this the only solution?
3. A square has one vertex at (–1, 3) and side length 4.
   a. How many different squares can you draw?
   b. What are the coordinates of the vertices of these squares?
4. A rectangle has three of its vertices at (–5, –3), (–5, –1) and (1, –1).
   a. What are the coordinates of the other vertex?
   b. How long are the sides of the rectangle?
5. A rectangle has two of its vertices at (–7, 1) and (–7, –5). One of the sides has length 2.
   a. How long is the other side?
   b. What are the coordinates of the other vertices? Is this the only solution?
6. A rectangle has one vertex at (–1, 3) and side lengths of 5 and 2.
   a. How many different rectangles can you draw?
   b. What are the coordinates of the vertices of these rectangles?

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LESSON 2

re(Solve) MATHS BY INQUIRY My Place in Space

The Best Spot for the Depot

Y6

About this lesson

Students locate points on a Cartesian plane and describe their location using ordered pairs of coordinates. They find the vertical and horizontal distances between points on a Cartesian plane and use this to solve a problem of finding the best location for a depot.

Australian Curriculum: Mathematics (Year 6)

ACMMG143: Introduce the Cartesian coordinate system using all four quadrants.

Mathematical purpose

- Students learn to plot points in the four quadrants of a Cartesian plane and find the coordinates of given points. They find horizontal and vertical distances between the points.

Learning intention

- To locate points and calculate distance on a Cartesian plane.

Time

A lesson of approximately 1 hour.

Vocabulary

- coordinates

Resources

- reSolve PowerPoint *2a The Best Spot for the Depot*
- Student Sheet 3 – Locating the Depot

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Teacher background information

In this lesson all movement is restricted to a horizontal or vertical direction. Students should be encouraged to find distances by comparing the *x*- and *y*-ordinates of pairs of points separately. In this way the lesson builds on the reSolve Year 6 Directed Number sequence.

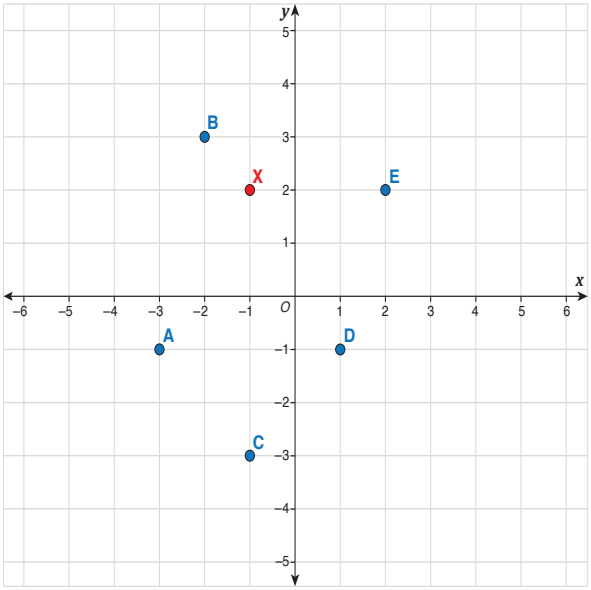
What is the shortest route?

Resources: Read through the story of Iris and Cian, shown on slides 2 to 9 of reSolve PowerPoint *2a The Best Spot for the Depot*.

On slide 8, students are asked to calculate the total delivery distance.

- The delivery distance from **X** to **A** is 10 units all together.
- To deliver from **X** to **B**, the driver must travel 1 unit west and 1 unit north, then return. The delivery distance from **X** to **B** is 4 units.
- The delivery distances from **X** to **C**, **D** and **E** are 10, 10 and 6 units, respectively.
- The total delivery distance is $10 + 4 + 10 + 10 + 6 = 40$ units.

Show slide 9.



Pose the problem: *Iris and Cian have employed you as a consultant to find the best location for the depot. Can you find a total delivery distance that is shorter than 40 units?*

Resources: Give students Student Sheet 3 – Locating the Depot.

Allow them to experiment with delivery routes. Encourage students to share their results, recording them so that they can compare them with those of other students. They should conclude that the shortest delivery distance is 26 units, when the depot is located at (–1, –1).

Iris and Cian open a new shop at location (3, –4). Ask students to think about whether Iris and Cian should move their depot.

Explain: *As a depot location consultant, you must be able to quickly and easily make recommendations about the best location for the depot for any number or locations of the shops. To do this, mathematicians often solve a simpler problem.*

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Solving a one-dimensional problem

One way to make the problem simpler is to consider only two shops on the same north–south or east–west road.

Show slide 10.

Pose the question: *In the diagram, shops* **A** *and* **B** *are located at (–3, –1) and (1, –1). Where should Iris and Cian locate the depot?*

Ask students to mark locations using the second diagram on their sheet.

Possible student response:

- The depot can be located anywhere between and including at **A** and **B**; that is, at (–3, –1), (–2, –1), (–1, –1), (0, –1) or (1, –1). Each of these locations has a delivery distance of 8 units.

Show slide 11.

Pose the question: *Now consider adding a third shop,* **C**, *at (4, –1) along the same east–west road. Where should the depot be located now?*

Possible student response:

- The depot should be located at **B**, the middle shop; that is, at (1, –1). The delivery distance is 14 units.

Show slide 12. Ask students to extend the problem by adding a fourth, then a fifth shop along the same east–west road (i.e. with y-ordinate –1). Where should the depot be located in each case?

Ask students to come up with a rule for locating the depot if there are two, three, four or five shops along the same road. Ask them to test their rule with shops located at (4, –3) and (4, 0), and then add more shops with x-ordinate 4.

Ask students to put together the rules for the two-, three-, four- and five-shop problems to make a 'super general' rule for any number of shops along the same road. Ask them to test the rule with a six- or seven-shop problem.

Possible student response:

- If there is an odd number of shops, then the depot should be located at the middle shop.
- If there is an even number of shops, then the depot can be located anywhere at or between the two middle shops.

Extending to two dimensions

Show slide 13 and ask students to look at the diagram on page 2 of Student Sheet 3 – Locating the Depot.

Pose the question: *Where should Iris and Cian locate the depot?*

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Possible student response:

- The depot can be located anywhere in a rectangle that has **A** and **B** at opposite corners; that is, at:
  - ◊ (–3, –1), (–2, –1), (–1, –1)
  - ◊ (–3, 0), (–2, 0), (–1, 0)
  - ◊ (–3, 1), (–2, 1), (–1, 1)
  - ◊ (–3, 2), (–2, 2), (–1, 2)
  - ◊ (–3, 3), (–2, 3), (–1, 3)
  - ◊ (–3, 4), (–2, 4), (–1, 4)
- The delivery distance for each of these locations is 14 units.

Show slide 14: Ask students to add a third shop at (1, 2) and locate the best position for the depot.

Possible student response:

- The depot should be located at (–1, 2). The delivery distance is 18 units. This is the middle *x*-ordinate and *y*-ordinate of the three shop locations, so is an extension of the one-dimensional rule for three shops.

Show slide 15: Ask students to add a fourth shop at (–2, 0) and *predict* and then *test* where the depot should be.

Possible student response:

- Extending the one-dimensional rule for four shops, we would predict the best location to be anywhere between and including the two middle *x*-ordinates and the two middle *y*-ordinates for the four shops; that is, anywhere in the rectangle bounded by (–2, 0), (–1, 0), (–2, 1), (–1, 1), (–2, 2) and (–1, 2).

Ask students to write down a 'super general' rule for a two-dimensional problem for any number of shops.

Possible student response:

- If there is an odd number of shops, then the depot should be located at the point that has the middle *x*- and *y*-ordinates of the shops. If there is an even number of shops, then the depot can be located anywhere on or inside a rectangle whose opposite vertices are the two middle *x*- and *y*-ordinates of the shops.

Returning to the original problem

Return to the original problem (see slide 16).

Pose the question: *Now that you have a 'super general' rule for locating a depot, can you use it to find the best location? Does this agree with your exploration?*

After solving a four-shop problem for another client, the best depot location is any of the points (–2, 0), (–1, 0), (0, 0), (–2, –1), (–1, –1) or (0, –1). *Where might the four shops be?*

Ask students to find several possibilities and check.

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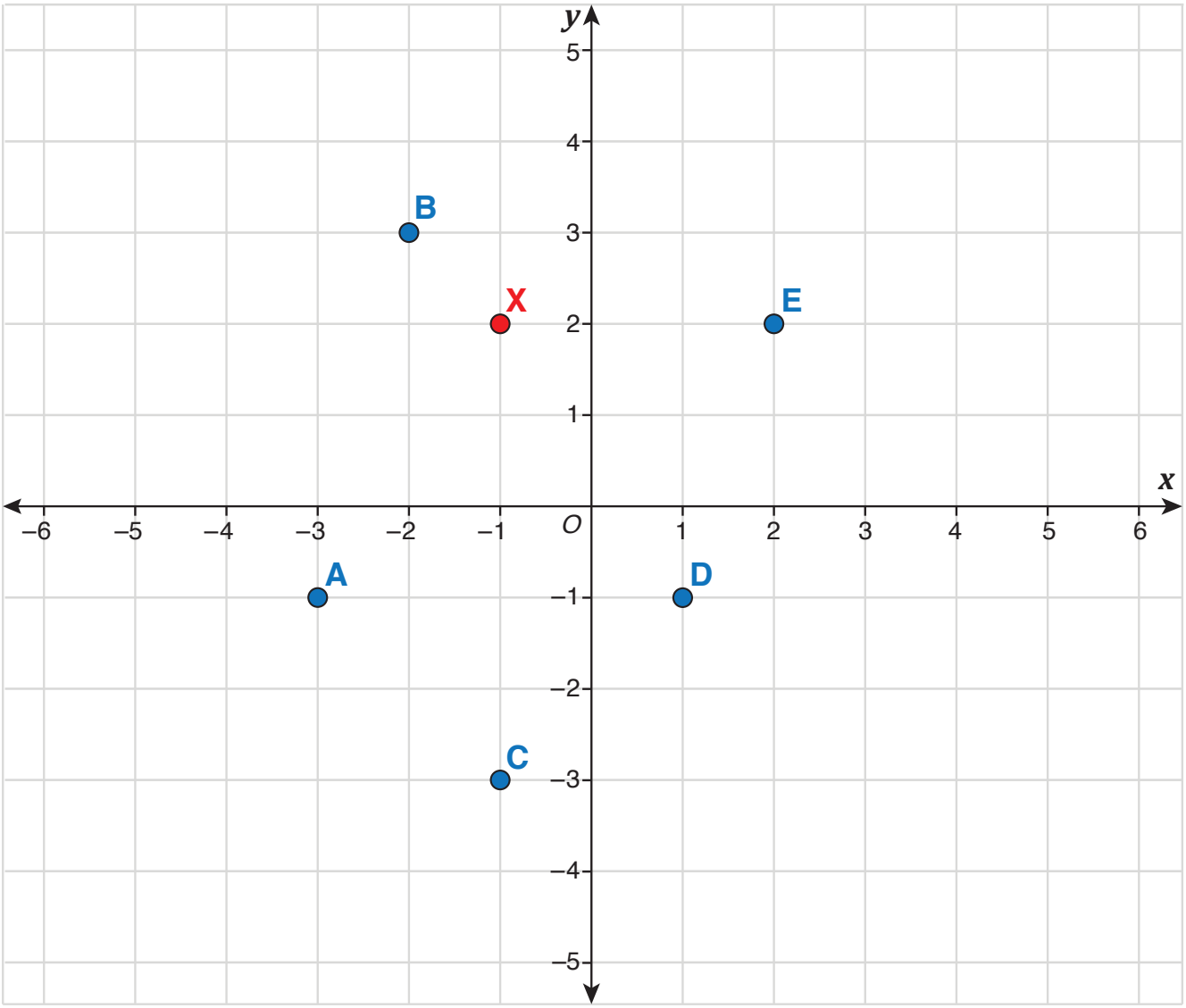
Locating the Depot

Name: _____



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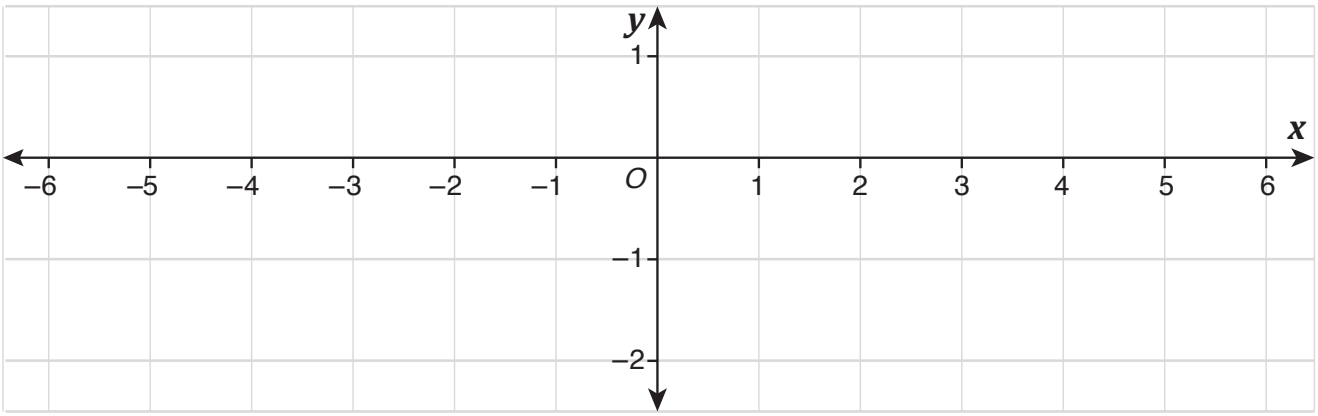

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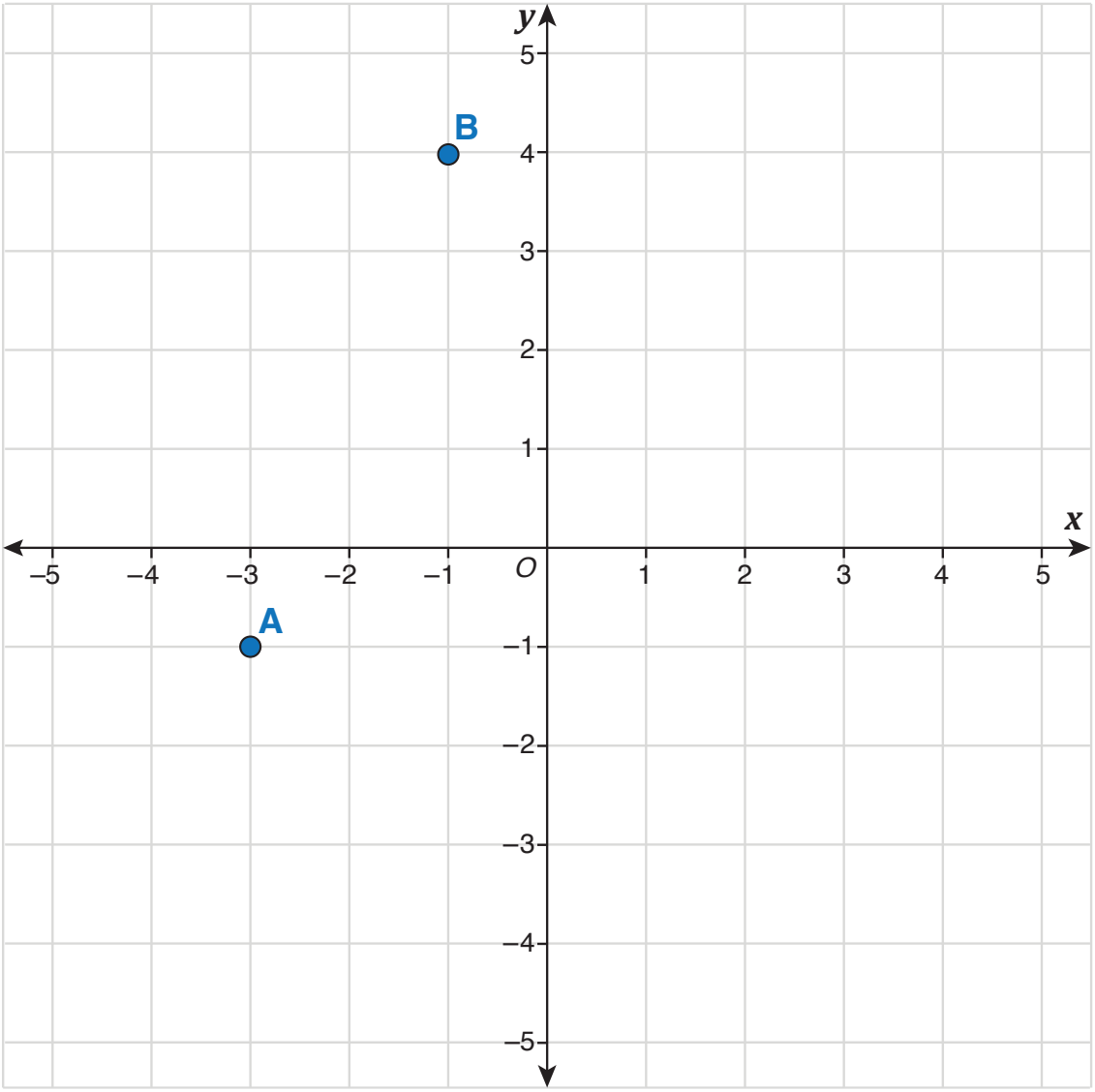

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