INTRODUCTION

re(Solve) MATHS BY INQUIRY

Number Chart Chess – The King

Y4

Summary of learning goals

- This sequence builds students' algebraic reasoning and understanding of number as they explore computation on the number chart.
- The key understandings of symmetry, equivalence and compensation are developed.

Australian Curriculum: Mathematics (Year 4)

ACMNA073: Apply place value to partition, and rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems.

ACMNA083: Find unknown quantities in number sentences involving addition and subtraction, and identify equivalent number sentences involving addition and subtraction.

Summary of lessons

Who is this sequence for?

- Students are required to add two-digit numbers and should be familiar with using the number chart as a tool for exploring patterns.
- The sequence applies students' general computation strategies and moves to explore some algebraic generalisations based on symmetry, compensation and equivalence.

Lesson 1: The King

In this task, the king has escaped from a regular chessboard onto a hundreds chart. The students explore the moves of the king and the ways in which the value of the numbers change as he moves. This builds into an algebraic exploration of equivalent values that can be found on the number chart.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Reflection on this sequence

Rationale

Our number system is based on 10. To highlight the base-10 structure of our number system, the hundreds chart is frequently used in the classroom. The chart allows students to explore patterns and gain a deeper understanding of the base-10 property of number. This task uses a 1–100 number chart to explore patterns and asks student to explain why these patterns occur. In doing so, the key developmental understandings of *compensation* and *equivalence* are addressed.

In this context, the different number charts are also used as a tool for algebraic reasoning. Students move from solving the problem for one number to exploring whether the patterns will exist for any number or any sized chart. This moves the students towards forming the generalisation that, regardless of the numbers or the organisation of the chart, the patterns remain.

reSolve mathematics is purposeful

- This task requires students to use algebraic reasoning to build a deeper understanding of number and the key understandings of compensation and equivalence.

reSolve tasks are inclusive and challenging

- Students can access the task by simply adding the numbers and looking at why the patterns occur.
- The high ceiling of the task is realised when students form generalisations and test these generalisations when exploring different charts.

reSolve classrooms have a knowledge-building culture

- This problem encourages students to look for patterns and to share their thinking with the class. Strategies are discussed, giving opportunity to question and evaluate the strengths of different solution methods.
- The teacher is an active participant in the lesson, questioning students to promote deeper inquiry.

LESSON 1

re(Solve) MATHS BY INQUIRY

Number Chart Chess – The King

The King

Y4

About this lesson

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ACMNA073: Apply place value to partition, and rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems.

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Mathematical purpose

- This task builds students' algebraic reasoning and understanding of number as they explore computation on the number chart.
- The key understandings of equivalence and compensation are focused on as the students solve problems.

Learning intention

- To use the moves of the king to explore patterns on the number chart.

Time

A lesson of approximately 1 hour.

Vocabulary

- compensation
- equivalence

Resources

- Number chart and counters (as needed for enabling prompt)
- reSolve PowerPoint *1a Number Chart Chess – The King*
- Chart provided in reSolve PDF *1b Number Chart*

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Australian Academy of Science

The king on the chessboard

Introduce students to the Number Chart Chess scenario: a group of chess pieces has escaped from the chess board and have been found on a regular 1–100 number chart. Explain that today they will be investigating the king.

Resources: Present the reSolve PowerPoint *Number Chart Chess – The King*, which provides a slideshow with the moves of the king and the associated questions.

Discuss the moves of the king. It can move one space in any direction.

Ask the students to visualise the number chart and to answer the questions:

- *What numbers on the chart can the king move to if it starts on 12? 94? 27? 50? 81?*
- *My king can move to 23 and 41. Where might my king be?*
  - ◊ 32 is the only possible answer.
- *My king can move to 47 and 46. Where might my king be?*
  - ◊ 36, 37, 56 and 57 are all possible answers.

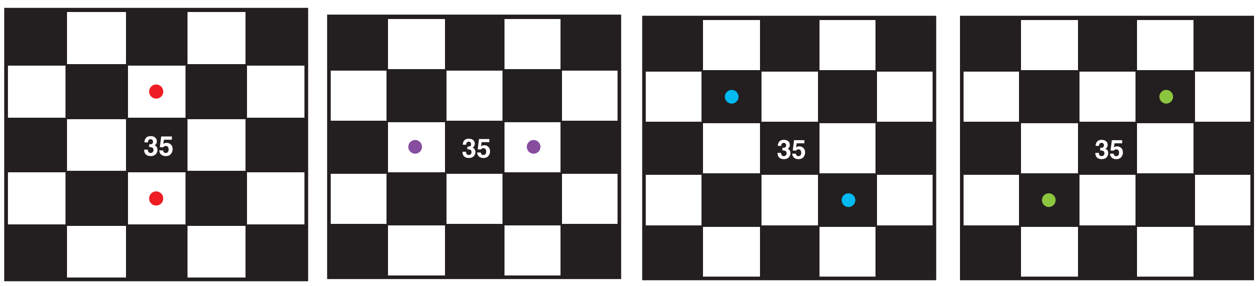
Enabling prompt:

- Provide a number chart and counter (or other representation of the king) rather than having students visualise.

Looking at equivalence

Again, ask the students to visualise the chart. The king is on 35. Ask:

- *Which two numbers can the king move to in the same column as 35? Add these numbers together.*
- *Which two numbers can the king move to in the same row as 35? Add these numbers together.*
- *Which numbers can the king move to diagonally? Add each pair of opposite diagonals together.*

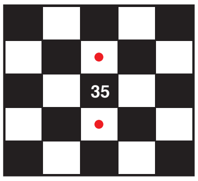
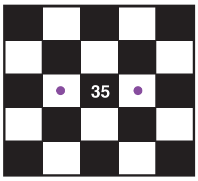
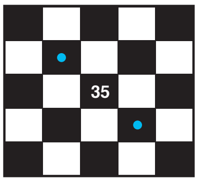
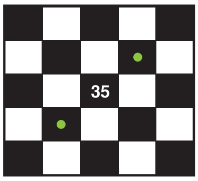


25 + 45 = 70 | 34 + 36 = 70 | 24 + 46 = 70 | 44 + 26 = 70

Pose the questions: *What do you notice? Why does this happen? Can you see a relationship to 35?*

Teacher note:

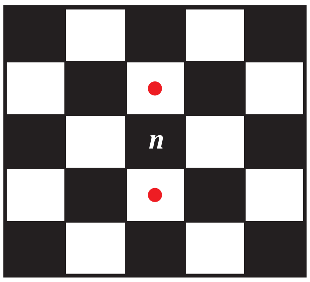
- For each of these number combinations you are moving the same distance from 3: one in a positive direction and the other negative. This means that the average of the numbers will be 35 and therefore their sum will be 35 doubled.

The number above is 10 less and the number below is 10 more. 10 more and 10 less balance each other out.  $25 + 45 = (35 - 10) + (35 + 10)$ $25 + 45 = (35 + 35) + (10 - 10)$ $25 + 45 = 35 + 35$	The number on the right is 1 more and the number on the left is 1 less. 1 more and 1 less balance each other out.  $34 + 36 = (35 - 1) + (35 + 1)$ $34 + 36 = (35 + 35) + (1 - 1)$ $34 + 36 = 35 + 35$
The number above is 11 less and the number below is 11 more. 11 more and 11 less balance each other out.  $24 + 46 = (35 - 11) + (35 + 11)$ $24 + 46 = (35 + 35) + (11 - 11)$ $24 + 46 = 35 + 35$	The number above is 9 less and the number below is 9 more. 9 more and 9 less balance each other out.  $26 + 44 = (35 - 9) + (35 + 9)$ $26 + 44 = (35 + 35) + (9 - 9)$ $26 + 44 = 35 + 35$

Any number on the chart

Pose the question: *Will the same thing happen if the king is on another number on the chart?*

This finding applies for any square on the board. To represent any number, a symbol (e.g. #), a word(s) (e.g. 'any number') or a letter (e.g. n) can be used. In this case, n represents any number and the example of $\pm$ 10 squares is illustrated.



The square above $= n - 10$

The square below $= n + 10$

So:

$(n - 10) + (n + 10)$

$= (n + n) + (10 - 10)$

$= n + n$

Extending prompts:

- *Will the same thing continue to happen if you move farther out? Will the sum of the squares with the red dots be equal to double n? Which other squares added together will give double n?*
  - ◊ Students will see that it doesn't matter how far you move from n, the sum of the two numbers will always be double n so long as the moves are the same distance in a positive and negative direction. In this case, the dots are -20 and $+20$ from n.

Reflection

The purpose of this reflection is to move students towards a more formalised and generalised understanding of this equivalence.

Select some students to share their working. Look at what the students have deduced about the king's moves on the hundreds chart.

With 35 in the centre, squares in the same…

- …row will be ± 1 away from 35
- …column will be ± 10 away from 35
- …diagonals will be ± 9 or ± 11 away from 35.

Generalising this understanding:

- With any number in the centre, squares in the same…
  - ◊ …row will be ± 1 away from this number
  - ◊ …column will be ± 10 away from this number
  - ◊ …diagonals will be ± 9 or ± 11 away from this number.

So long as you are always adding and then subtracting the same amount, it doesn't matter how far you move away from the centre square. The sum of these numbers will always be double the centre number. Exploring the king's moves in this manner is a clear illustration of the key understandings of **compensation** and **equivalence**.

Further activities

Activity 1

Ask the students to explore whether this understanding would apply to any chart. For example: *What if the king is moving on a calendar? What about a chart with just 1–8 on the top row? Would the same thing happen?*