INTRODUCTION

re(Solve) MATHS BY INQUIRY

Prime Factorisation

Y7

Summary of learning goals

- This sequence of four lessons explores prime factorisation and how it can be used to solve problems related to the properties of numbers. The activities demonstrate why prime numbers are important. Proficiency goals are to reason mathematically about properties of numbers, and to carry out a mathematical investigation related to prime factorisation.

Australian Curriculum: Mathematics (Year 7)

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

Summary of lessons

Who is this sequence for?

- Students undertaking this sequence need reasonable fluency with multiplication facts, although the sequence provides further practice. Multiplication and division are required to find factors of a number.
- Students also need to know the definition of prime and composite numbers and be able to give examples.

Lesson 1: Factor Strings

Students search a number grid for factor strings and then justify that the longest possible factor string for a number is made up of primes. This introduces the fundamental theorem of arithmetic: all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way.

Lesson 2: Prime Dice

Students play a game using dice labelled with prime numbers and learn how to use prime factorisations of a number to determine the properties of a number.

Lesson 3: Factors and Multiples

Students place numbers into a grid according to their properties. First, they do this with just the numbers, then they look at how prime factorisation can be used as an efficient way to determine number properties.

Lesson 4: HCF and LCM

This lesson explores how prime factorisation can efficiently determine the highest common factor (HCF) and lowest common multiple (LCM) of two numbers.

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INTRODUCTION

Reflection on this sequence

Rationale

Prime numbers form the building blocks of numbers. Every whole number can be represented as the product of prime numbers. Factorising numbers provides students with a deeper understanding of number, particularly the properties of these numbers. Students are provided with opportunities to participate in mathematical investigations, revealing prime factorisations as a powerful and efficient tool for identifying common factors and common multiples, and that prime factorisation has connections to many areas of mathematics, particularly work with integers and fractions.

Prime factorisation is an interesting study in and of itself, as this concept plays an ever-increasing role in our world today. The prime factorisation of numbers is used to encrypt data and protect personal details.

reSolve mathematics is purposeful

- This sequence highlights mathematics as a creative human endeavour while inviting students to participate in a deep exploration of prime numbers and prime factorisations.
- The ability to represent numbers as a product of primes provides students with a powerful tool to solve problems.

reSolve tasks are inclusive and challenging

- Students engage in experiences that provide a common point of access, and enabling and extending prompts are included to address the needs of students' different abilities.

reSolve classrooms have a knowledge-building culture

- Open-ended investigations allow students the opportunity to discuss their own solution methods and to develop more efficient ways to solve problems.
- Consolidation exercises are included to reinforce knowledge developed during the open-ended investigations.

LESSON 1

re(Solve) MATHS BY INQUIRY

Prime Factorisation

Factor Strings

Y7

About this lesson

Students search a number grid for factor strings and then justify that the longest possible factor string for a number is made up of primes. This introduces the fundamental theorem of arithmetic: all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way.

Australian Curriculum: Mathematics (Year 7)

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

Mathematical purpose

- Students learn that all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way.

Learning intention

- To find strings of factors that make up a composite number and investigate how different strings are related to each other.

Time

A lesson of approximately 1 hour.

Resources

- Student Sheet 1 – Factor String Puzzle

Vocabulary

- composite
- factor
- prime

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Factor string puzzle

15	84	2	6	92	3	80	51
5	32	20	24	2	180	2	4
96	2	9	2	4	60	5	120
12	32	2	44	12	40	5	2
8	5	6	3	8	48	15	16
48	29	80	5	10	5	2	3
3	16	3	4	2	2	8	2
4	204	10	4	16	10	51	2

Factor strings

Resources: Give students Student Sheet 1 – Factor String Puzzle.

Explain that factor 'strings' are created by connecting numbers horizontally, vertically and diagonally, and then multiplying the numbers in the string. Allow students time to explore the puzzle. This can be facilitated by asking questions, such as:

- *What is the largest number you can make using a string of two factors?*
- *What is the smallest number you can make using a string of two factors?*
- *What are the smallest and largest numbers you can make using a string of three or four factors?*
- *Can you find a string of numbers that makes 48?*
- *Can you find a string of numbers for 60? What about 120?*

Factor strings for 480

Ask students to search for strings of numbers that multiply to give 480. Encourage them to consider an appropriate way to collect, organise and record their strings.

Enabling prompt:

- Ask students to look for factor strings for 240, 120, 60 or 48. As these are factors of 480, the prime factor string for all these numbers can be found in the prime factor string for 480. Ask students: *How do the factor strings for these numbers help in finding factor strings for 480?*

Prompting student thinking

- *Is there a relationship between the strings for 480 that are three-factors long and the strings that are four-factors long? Can you use a string of three factors to make a string of five factors for 480?*
  - ◊ Factorising numbers already in a string increases the length of the string. For example, a string that is $48 \times 5 \times 2$ can be made into a four-factor string using $48 = 6 \times 8$.
- *Could we use a string of four factors to make a string of three factors?*
  - ◊ Reversing the process above will decrease the length of the string.
- *What is interesting about the numbers that make up the longer strings?*
  - ◊ Students will notice that the numbers get smaller and are often repeated (e.g. there are multiple 2s). The number of prime numbers in the string increases as the string gets longer.
- *What numbers in the grid cannot be used in a factor string for 480? Why not?*
  - ◊ Numbers that are not factors cannot be used.

Recording factor strings

As a class, collect the different strings found and group them based on the number of factors in each string. This could be done in a table (see Teacher Sheet – Factor Strings for 480).

Challenge students to find the longest string that they can. Have students add new strings to the table as they find them.

It may be that strings are repeated with numbers in different orders. This is an opportunity to discuss the commutative property of multiplication.

Teacher note:

- The prime factor string for 480 is $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$, which can be written as $2^5 \times 3 \times 5$.
  This string can be found in several places in the puzzle. The numbers are in a different order but they are the same factorisation.

15	84	2	6	92	3	80	46
5	32	20	24	2	180	2	4
96	2	9	2	4	60	5	120
12	32	2	44	12	40	5	2
8	5	6	3	8	48	15	16
48	102	80	5	10	5	2	3
3	16	3	4	2	2	8	2
4	204	10	4	16	10	18	2

Reflection

Discuss: *Is it possible to find a longer factor string than the one we have found? Why or why not?*
Is it possible to find another string of length equal to this one? Why or why not?

This introduces the fundamental theorem of arithmetic: all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way.
For example, $480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$ or $25 \times 3 \times 5$.

This explains why 1 is not considered prime. A whole number can be identified uniquely by its prime factorisation. If 1 were a prime number, this would not be true. Any string of primes could be extended with an unlimited number of 1s.

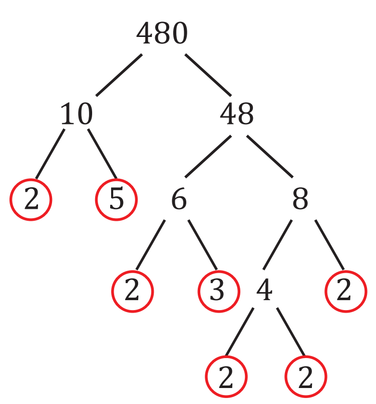
Extending prompt:

- *What are all the factor pairs for 480?*
- *What are all the factor strings that have three factors?*
- *How do you know you have found them all?*
- *What about factor strings that are four, five or six numbers long?*

Teacher note:

- Two methods for finding prime factors of a number are shown.

Factor tree



Factor ladder

2	480	$480 \div 2 = 240$
3	240	$240 \div 3 = 80$
2	80	$80 \div 2 = 40$
2	40	$40 \div 2 = 20$
2	20	$20 \div 2 = 10$
2	10	$10 \div 2 = 5$
5	5	
	1	

The prime factor string of 480 is 2, 3 and 5.
$480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$,
$480 = 2^5 \times 3 \times 5$

Further activities

Activity 1

Ask students to make a factor string puzzle that contains factor strings for a number of their choice. The number must contain several factors. Encourage students to choose a number that has larger prime factors, such as 7, 31 or 113. The puzzles can then be solved by other members of the class.

Activity 2

Prime factorisation is used today in the encrypting of data, including our financial information held by banks.

A possible way to look at this as a powerful tool is to have a competition in the class as to who finds the solution first. One half of the class is asked to multiply a secret pair of two-digit primes, such as $23 \times 89 = 2047$. At the same time, the other half of the class is given the product (e.g. 2047) and is asked to find the prime factorisation. This illustrates that it is much quicker to multiply primes than it is to find the prime factorisation for a number that contains two or more large prime numbers.

ABC Education provides a clip that describes this method for encrypting confidential information; see http://education.abc.net.au/home#!/media/154992/prime-number-keys.

Teacher Sheet – Factor Strings for 480

One factor	Two factors	Three factors	Four factors	Five factors	Six factors	Seven factors
480	2 × 240	***2 × 2 × 120***	***2 × 2 × 2 × 60***	2 × 2 × 2 × 2 × 30	2 × 2 × 2 × 2 × 2 × 15	***2 × 2 × 2 × 2 × 2 × 3 × 5***
	3 × 160	***2 × 3 × 80***	2 × 2 × 3 × 40	***2 × 2 × 2 × 3 × 20***	***2 × 2 × 2 × 2 × 3 × 10***	
	4 × 120	***2 × 4 × 60***	2 × 2 × 4 × 30	***2 × 2 × 2 × 4 × 15***	***2 × 2 × 2 × 2 × 5 × 6***	
	5 × 96	***2 × 5 × 48***	***2 × 2 × 5 × 24***	***2 × 2 × 2 × 5 × 12***	***2 × 2 × 2 × 3 × 4 × 5***	
	6 × 80	2 × 6 × 40	***2 × 2 × 6 × 20***	2 × 2 × 2 × 6 × 10		
	8 × 60	2 × 8 × 30	***2 × 2 × 8 × 15***	***2 × 2 × 3 × 4 × 10***		
	10 × 48	2 × 10 × 24	2 × 2 × 10 × 12	***2 × 2 × 3 × 5 × 8***		
	12 × 40	***2 × 12 × 20***	2 × 3 × 4 × 20	***2 × 2 × 4 × 5 × 6***		
	15 × 32	***2 × 15 × 16***	***2 × 3 × 5 × 16***			
	16 × 30	3 × 4 × 40	***2 × 3 × 8 × 10***			
	20 × 24	3 × 5 × 32	***2 × 4 × 4 × 16***			
		3 × 8 × 20	2 × 4 × 5 × 12			
		3 × 10 × 16	2 × 4 × 6 × 10			
		4 × 4 × 30	***2 × 6 × 8 × 5***			
		4 × 5 × 24	3 × 4 × 2 × 20			
		4 × 6 × 20	***3 × 4 × 4 × 10***			
		4 × 8 × 15	***3 × 4 × 5 × 8***			
		4 × 10 × 12	***4 × 4 × 5 × 6***			
		5 × 6 × 16				
		5 × 8 × 12				
		6 × 8 × 10				

Not all these strings appear in the puzzle; only those in bold italics.

The number 1 does not appear, as it could appear as many times as you like in any of the strings. This is the reason why 1 is not called a prime number or a composite number. It is useful to be able to say how many prime factors a number has.

Factor String Puzzle

Name: ______________________________

15	84	2	6	92	3	80	51
5	32	20	24	2	180	2	4
96	2	9	2	4	60	5	120
12	32	2	44	12	40	5	2
8	5	6	3	8	48	15	16
48	29	80	5	10	5	2	3
3	16	3	4	2	2	8	2
4	204	10	4	16	10	51	2

Search the puzzle above for strings of factors that multiply to give 480. The strings can connect numbers horizontally, vertically and diagonally. Write them here. Two have been done for you.

$15 \times 32 = 480$

$5 \times 2 \times 16 \times 3 = 480$

LESSON 2

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Prime Factorisation

Prime Dice

Y7

About this lesson

Students play a game using dice labelled with prime numbers and learn how to use prime factorisations of a number to determine the properties of a number.

Australian Curriculum: Mathematics (Year 7)

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

Mathematical purpose

- Students identify properties of a number based on its prime factorisation.

Learning intention

- To use the prime factorisation of a number to determine some of its properties.

Time

A lesson of approximately 1 hour.

Vocabulary

- 4th power
- number properties
- prime factorisation
- square number

Resources

- reSolve PowerPoint *2a Prime Dice Game*
- Four prime dice per group of two to four students. (Create prime number dice by writing the numbers 2, 2, 3, 3, 5 and 7 onto blank dice or foam cubes, in any arrangement.)
- Student Sheet 1 – Prime Dice Game
- Student Sheet 2 – Prime Dice Scorecard

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Playing prime dice

Resources: Show students reSolve PowerPoint *2a Prime Dice Game*.

Explain that the game Prime Dice is similar to the game Yahtzee; you get multiple rolls to try to score points for different categories. Discuss the rules of the game and look at the sample moves made by Amy and Nelson.

Resources: A copy of the rules is also provided in Student Sheet 1 – Prime Dice Game.

Teacher note:

- The rules do not tell students how to use prime factorisation to determine the properties of a number. Deciding how the prime factorisation can be used is the central inquiry.

Have students play Prime Dice in groups of two to four with a set of four prime dice.

Resources: Students record results on Student Sheet 2 – Prime Dice Scorecard.

As students play the game, ask questions such as:

- *How are you working out where to place your score on the scorecard?*
- *What prime factors and/or combination of prime factors are needed for each category on the scorecard?*

A number to the 4th power	Four of a kind
A number ending in double zeros	There is only one option: 2, 2, 5, 5
A number ending in one zero	You need prime factors of 2 and 5
Square	This can be four of a kind or two different pairs
Odd	Does not have 2 as a prime factor
Even	Must have 2 as a prime factor

Discuss the strategies that students are using to determine which dice to roll again and where to place their scores.

Extending prompts:

- *What is the smallest number that is possible to roll for each category? What is the largest number that is possible to roll for each category?*
- *What is the largest possible score in a complete game? What is the smallest possible score if you complete every category?*

The largest and smallest scores in each category are:

Category	Largest score	Smallest score (except for zero)
A number to the 4th power	$7^4 = 2401$	$2^4 = 16$
A number ending in double zeros	500	500
A number ending in one zero	$7^2 \times 2 \times 5 = 490$	$3^2 \times 2 \times 5 = 90$
Square	$7^4 = 2401$	$2^4 = 16$
Odd	$7^4 = 2401$	$3^4 = 81$
Even	$7^3 \times 2 = 686$	$2^4 = 16$
Total for the game	8879	719

Further activities

Ask the students to roll six prime factor dice. Using the prime factors, have them list as many number properties about their number as they can, using the knowledge they gained from the previous activity.

Prime Dice Game

Name: ________________________________

Aim

In this game, you will be rolling prime dice and trying to get the biggest product possible in each of six categories.

Category	Score for the category
A number to the 4th power	The product of the prime numbers showing on the dice
A number ending in double zeros	This roll scores 500 points
A number ending in one zero	The product of the prime numbers showing on the dice
Square	The product of the prime numbers showing on the dice
Odd	The product of the prime numbers showing on the dice
Even	The product of the prime numbers showing on the dice

How to play

1. Roll the prime dice and think about the product of the numbers you have rolled.
   - ◊ What categories will the product be in?
   - ◊ Which category will give you the biggest score?
   - ◊ If you roll one of the dice again, could you get a different category or a bigger score?
2. If you decide to try to improve your score, you may roll any or all of the dice again.
3. Decide on a category and work out your score.
4. The game ends when all the players have filled the six sections on the scorecard.

Rules

- You can record only one number when it is your turn.
- You can score only once in each category.
- You can roll the dice again only once during each turn.
- If your product doesn't fit any of the remaining categories, you must choose a remaining category and assign it a score of zero.

Prime Dice Scorecard

Name: ________________________________

	Names of players							
Category	**Numbers rolled**	**Score**	**Numbers rolled**	**Score**	**Numbers rolled**	**Score**	**Numbers rolled**	**Score**
A number to the 4th power								
A number ending in double zeros (500 pts)								
A number ending in one zero								
Square								
Odd								
Even								
Total score								

	Names of players							
Category	**Numbers rolled**	**Score**	**Numbers rolled**	**Score**	**Numbers rolled**	**Score**	**Numbers rolled**	**Score**
A number to the 4th power								
A number ending in double zeros (500 pts)								
A number ending in one zero								
Square								
Odd								
Even								
Total score								

LESSON 3

reSolve MATHS BY INQUIRY

Prime Factorisation

Factors and Multiples

Y7

About this lesson

Students place numbers into a grid according to their properties. First, they do this with just the numbers, then they look at how prime factorisation can be used as an efficient way to determine number properties.

Australian Curriculum: Mathematics (Year 7)

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

Mathematical purpose

- Students learn how the prime factorisation of a number can be used to determine the properties of a number.

Learning intention

- To identify the properties of a number.

Time

A lesson of approximately 1 hour.

Resources

- Student Sheet 1 – Factor and Multiple Grid

Vocabulary

- factors
- prime factorisation

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Factor and multiple grid

Resources: Provide students with Student Sheet 1 – Factor and Multiple Grid.

Explain that students must use the numbers at the bottom of the page to fill in the grid. Students should initially work without calculators.

Teacher note:

- The numbers are intentionally large and difficult to work with. Allow the students time to struggle so that they recognise the need for an easier way to complete the task and to appreciate the efficiency of using the prime factorisation of numbers to help solve the problem.

Enabling prompt:

- Allow students to use calculators.

Solution

	Square	Multiple of 8	Multiple of 6	Odd
Multiple of 15	225	360	180	135
Factor of 144	144	48	72	9
Multiple of 7	196	112	294	91
Cube	64	512	216	27

Discuss the strategies that students used to solve the puzzle.

Using prime factorisation

Pose the question: *Could prime factorisation be used to solve the problem?*

Allow students time to explore the prime factorisation of the numbers and how it can be used to determine the properties of numbers. Encourage them to draw on their learning from the game Prime Dice in the previous lesson.

	Square Prime factors can all be grouped into identical pairs	**Multiple of 8** $8 = 2^3$	**Multiple of 6** $6 = 2 \times 3$	**Odd** 2 is not a prime factor
Multiple of 15 $15 = 3 \times 5$	$3^2 \times 5^2$	$2^3 \times 3^2 \times 5$	$2^2 \times 3^2 \times 5$	$3^3 \times 5$
Factor of 144 $144 = 2^4 \times 3^2$	$2^2 \times 3^2$	$2^4 \times 3$	$2^3 \times 3^2$	3^2
Multiple of 7 $7 = 7$	$2^2 \times 7^2$	$2^4 \times 7$	$2 \times 3 \times 7^2$	7×13
Cube Prime factors can all be grouped into identical threes	2^6	2^9	$2^3 \times 3^3$	3^3

Prompting students' thinking

- *What similarities and differences can you see in each row and column?*
  - ◊ For example:
    - ■ All the multiples of 8 contain at least 2^3.
    - ■ The prime factorisations for numbers that are factors of 144 will contain only prime factors that make up the prime factorisation for 144.
- *What other numbers could fit in each cell of the grid?*
  - ◊ Students can use prime factorisation to determine other numbers that could go in each cell.

Reflection

Discuss findings.

Ask: *What generalisations can be made about how prime factorisation can be used to identify:*

- *factors?*
- *multiples?*
- *squares and cubes?*
- *odds and evens?*

Have students record these as generalised statements.

Factor and Multiple Grid

Name: ________________________________

Use the numbers below to fill in each space in the grid.

	Square	Multiple of 8	Multiple of 6	Odd
Multiple of 15				
Factor of 144				
Multiple of 7				
Cube				

196	48	216	360	512	112	64	91
225	72	27	9	180	294	144	135

LESSON 4

re(Solve) MATHS BY INQUIRY

Prime Factorisation

HCF and LCM

Y7

About this lesson

This lesson explores how prime factorisation can efficiently determine the highest common factor (HCF) and lowest common multiple (LCM) of two numbers.

Australian Curriculum: Mathematics (Year 7)

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

Mathematical purpose

- Students learn how prime factorisation can be used to efficiently find the HCF and LCM of numbers. They will see why $\text{HCF}(a, b) \times \text{LCM}(a, b) = ab$.

Learning intention

- To use efficient strategies to find the HCF and LCM of numbers.

Time

Two lessons of approximately 1 hour each.

Vocabulary

- highest common factor (HCF)
- lowest common multiple (LCM)

Resources

- Student Sheet 1 – Finding Numbers Using HCF & LCM
- Student Sheet 2 – HCF & LCM Reflection
- Prime dice from Lesson 2 (optional)

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Finding numbers using HCF & LCM

Two numbers have a HCF of 6. What might the numbers be?

Two numbers have a LCM of 120. What might the numbers be?

Two numbers have a HCF of 6 and a LCM of 120. What might the numbers be?

Find all pairs of numbers that have a HCF of 6 and a LCM of 120. What do you notice?

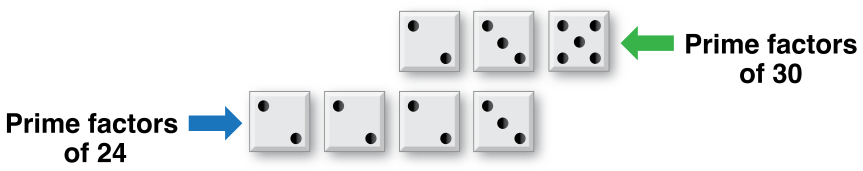
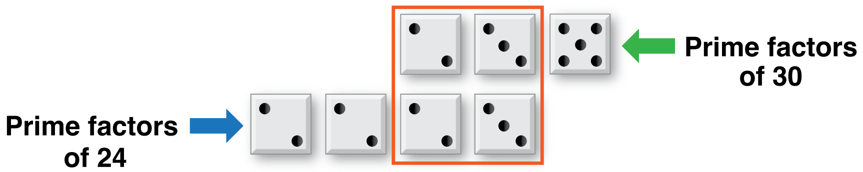
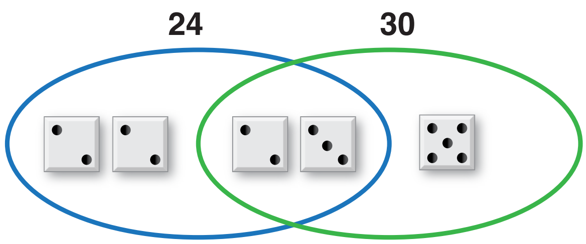
Resources: Provide students with Student Sheet 1 – Finding Numbers Using HCF & LCM.

Allow students time to explore the problem. Encourage students to find some rules to help answer the questions. For example:

- Two numbers with a HCF of 6: If one number is 6, then the other number can be any multiple of 6. Or, if one number is 12, then the other number can be any multiple of 6 that is not divisible by 4.

After the students have had time to explore the problems, introduce prime factorisation as an efficient way to find HCF and LCM. The factor dice from Lesson 2 can be used as a tool to explore the similarities in the prime factorisations for different numbers.

For example, to demonstrate the HCF and LCM of two numbers:

First, find the prime factorisation of 30 and 24.	
2 and 3 are the only prime factors that these numbers have in common. This means that the HCF of 24 and 30 is 6.	
This can be represented in a Venn diagram. Multiplying the HCF by the numbers in the outer segments gives the LCM, which is 120.	 $2 \times 2 \times 2 \times 3 \times 5 = 120$

The only pairs of numbers that have a HCF of 6 and a LCM of 120 are 6 and 120, and 24 and 30. The product of each of these pairs of numbers is 720.

HCF and LCM reflection

Resources: Pose the questions below verbally or hand out Student Sheet 2 – HCF & LCM Reflection.

Give students time to consider each problem, then as a class discuss responses and reasons thoroughly.

1. *Explain why we never ask for the lowest common factor or the highest common multiple of two numbers.*	The lowest common factor of any two numbers is always 1. Common multiples can be as large as we like.
2. *Can you find two numbers whose LCM equals the product of the two numbers?*	These numbers will not share any of the same prime factors.
3. *Can you find any two numbers whose LCM is smaller than the product of the two numbers?*	Any two numbers that share a common factor will have a LCM that is smaller than their product.
4. *Can you find any two numbers whose LCM is larger than the product of the two numbers?*	No, the product of the two numbers is itself a common multiple, so the LCM cannot be larger.
5. a. *Explain how you can use prime factorisation to find the HCF and LCM of two numbers.* b. *Use prime factorisation to find the HCF and LCM of 24 and 60.*	First, find the longest factor string common to both numbers. This is the HCF. $24 = 2 \times 2 \times 2 \times 3 \qquad 60 = 2 \times 2 \times 3 \times 5$ The HCF of 24 and 60 is $2 \times 2 \times 3$, which is 12. The LCM is the shortest factor string that contains the prime factorisation of both numbers. $2 \times 2 \times 3$ appears in both factor strings — we need to use this string only once. This means that the shortest string will be: $2 \times 2 \times 2 \times 3 \times 5 = 120$ The LCM is 120.

6. *Why is the HCF multiplied by the LCM always equal to the product of the two numbers?*

Looking at 24 and 60:

$24 = 2 \times (2 \times 2 \times 3) = 2 \times \text{HCF}$

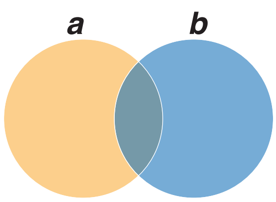
$60 = (2 \times 2 \times 3) \times 5 = 5 \times \text{HCF}$

$\text{HCF} = 2 \times 2 \times 3$

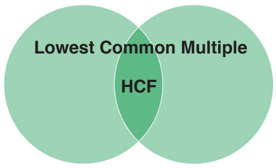
$\text{LCM} = 120 = 2 \times (2 \times 2 \times 3) \times 5$

$$\begin{aligned} 24 \times 60 &= [2 \times (2 \times 2 \times 3)] \times [(2 \times 2 \times 3) \times 5] \\ &= (2 \times \text{HCF}) \times (\text{HCF} \times 5) \\ &= 2 \times \text{HCF}^2 \times 5 \end{aligned}$$

$$\begin{aligned} \text{HCF} \times \text{LCM} &= (2 \times 2 \times 3) \times [2 \times (2 \times 2 \times 3) \times 5] \\ &= \text{HCF} \times 2 \times \text{HCF} \times 5 \\ &= \text{HCF}^2 \times 2 \times 5 \end{aligned}$$



a × *b* multiplies all the factors of *a* and all the factors of *b*



LCM = two complete circles (including the intersection)
HCF = factors in the intersection of the circle
LCM × HCF = the factors in both complete circles by the factors in the intersection of the circles

In both cases the product includes the factors in the intersection twice each.

Further activities

Make all the different three-digit numbers you can, using the digits 4, 6 and 8 once each. Find the HCF and LCM of this whole set of numbers.

Solution

There are six possible three-digit numbers: 468, 486, 648, 684, 846 and 864.

We know that each is divisible by 18, as each is even and the digits sum to a multiple of 9.

Dividing each by 18 (or 2×3^2), we obtain: 26, 27, 36, 38, 47 and 48.

The prime factors of these numbers are:

$26 = 2 \times 13$	$27 = 3^3$	$36 = 2^2 \times 3^2$
$38 = 2 \times 19$	$47 = 47$	$48 = 2^4 \times 3$

As these have no common factors, the HCF of the original set of six three-digit numbers must be 18.

The LCM must contain the maximum number of occurrences of each prime factor: 2^4, 3^3, 13, 17 and 47.

So, the LCM is $(2 \times 3^2) \times 2^4 \times 3^3 \times 13 \times 17 \times 47 = 2^5 \times 3^5 \times 13 \times 17 \times 47 = 80\,769\,312$.

Finding Numbers Using HCF & LCM

Name: ___________________________________

1. Two numbers have a HCF of 6. What might the numbers be?

2. Two numbers have a LCM of 120. What might the numbers be?

3. Two numbers have a HCF of 6 and a LCM of 120. What might the numbers be?

4. Find all pairs of numbers whose HCF is 6 and LCM is 120. What do you notice?

HCF & LCM Reflection

Name: ________________________________

1. Explain why we never ask for the lowest common factor or the highest common multiple of two numbers.

2. Can you find two numbers whose LCM equals the product of the two numbers?

3. Can you find any two numbers whose LCM is smaller than the product of the two numbers?

4. Can you find any two numbers whose LCM is larger than the product of the two numbers?

5. a. Explain how you can use prime factorisation to find the HCF and LCM of two numbers.

   b. Use prime factorisation to find the HCF and LCM of 24 and 60.

6. Why is the HCF multiplied by the LCM always equal to the product of the two numbers?