INTRODUCTION

re(Solve) MATHS BY INQUIRY

Right Angles

Y6

Summary of learning goals

- This sequence builds students' ability to create and identify right angles. They are challenged to reason mathematically and form generalisations.

Australian Curriculum: Mathematics (Year 6)

ACMMG141: Investigate, with and without technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles.

Summary of lessons

Who is this sequence for?

- Students who have been introduced to right angles and can identify and name polygons with up to eight sides. Students should be familiar with identifying and describing patterns in data that result from multiplication.

Lesson 1: How Many Right Angles?

Students create right angles using two ice-cream sticks. They discover they can make one, two or four right angles, but they cannot make three. They explore the number of right angles that can be made with any number of sticks.

Lesson 2: Six Internal Right Angles

Students are asked to construct a polygon with exactly six internal right angles. They see that it is possible to make an octagon with six right angles and two angles of 270°. Students sort the octagons based on the number of right angles that separate the 270° angles.

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Reflection on this sequence

Rationale

Although simply identifying and constructing right angles might not pose a significant challenge to some students, there is opportunity to build students' mathematical reasoning using the context of right angles. As such, these lessons concentrate on links to algebraic and geometric thinking and moving students to forming generalisations based on their findings.

reSolve mathematics is purposeful

- Students identify and construct right angles and develop their skills in forming generalisations.

reSolve tasks are inclusive and challenging

- While the activities in the lessons are accessible, challenge is provided through the reasoning and forming of generalisations.

reSolve classrooms have a knowledge-building culture

- The tasks require the students to look for patterns as they create right angles and shapes. The collective findings of the class are used to form generalisations.
- Students are encouraged to explain why patterns occur, creating the opportunity for rich mathematical discussions.

Acknowledgements

Lesson 2: Six Internal Right Angles has been used and adapted with permission from the *EPMC project: Encouraging Persistence, Maintaining Challenge*.

LESSON 1

re(Solve) MATHS BY INQUIRY

Right Angles

How Many Right Angles?

Y6

About this lesson

Students create right angles using two ice-cream sticks. They discover they can make one, two or four right angles, but they cannot make three. They explore the number of right angles that can be made with any number of sticks.

Australian Curriculum: Mathematics (Year 6)

ACMMG141: Investigate, with and without technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles.

Mathematical purpose

- To build students' ability to create and identify right angles. They also learn to generalise and reason algebraically to find the number of right angles that can be made with any number of ice-cream sticks.

Learning intention

- To find how many right angles can be made using a given number of ice-cream sticks.

Time

A lesson of approximately 1 hour.

Resources

- ice-cream sticks

Vocabulary

- right angle

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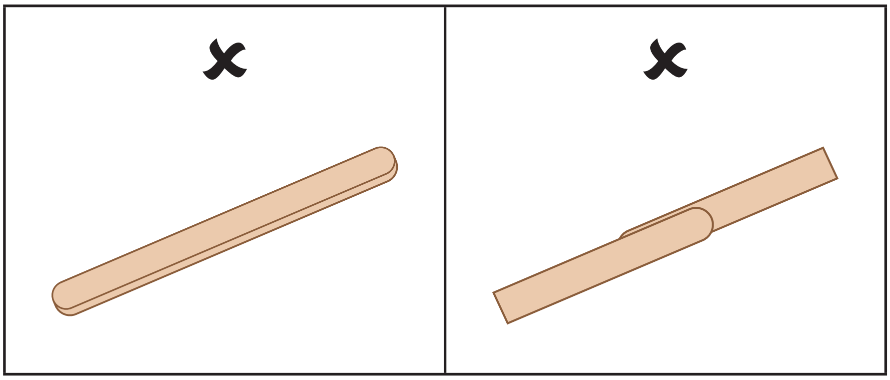
Making right angles

Identify examples of right angles around the classroom. Discuss the ways in which right angles create a ‘square corner’.

Resources: Provide each student with two ice-cream sticks.

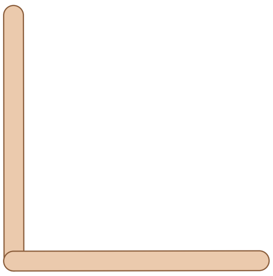
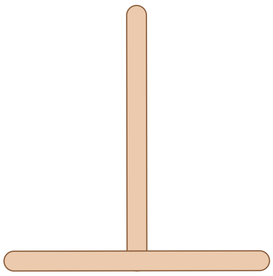
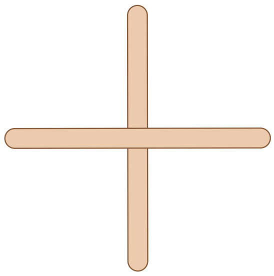
Pose the question: *How many right angles can you make with two ice-cream sticks?*

Explain that the ice-cream sticks cannot be stacked or extended, as shown below.



Possible student response:

- Students will be able to use two ice-cream sticks to make one, two or four right angles, but they will not be able to make three.

One right angle	Two right angles	Four right angles
		

Share the different solutions discovered and have students explain why it is not possible to make three right angles with two sticks.

Teacher note:

- Students count the right angles made as a 2D representation, not 3D. This means that two ice-cream sticks can make a maximum of four right angles, not eight.

Maximum number of right angles

Pose the question: *How many right angles can you make with three ice-cream sticks?*

Again, allow students plenty of time to explore the problem. They will find that they can create two, three, four, five, six and eight right angles, but they cannot make one or seven.

Explain that mathematicians collect and record data in an organised way to help them look for patterns. Introduce a table to record their data.

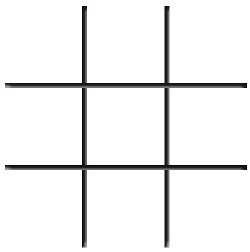
Number of sticks	Number of right angles that could not be made	Maximum number
2	3	4
3	1, 7	8
...	...	...

Pose the questions:
- *What do you predict we will find if we use four ice-cream sticks?*
- *What about using five or six ice-cream sticks?*
- *What about using any number of ice-cream sticks?*

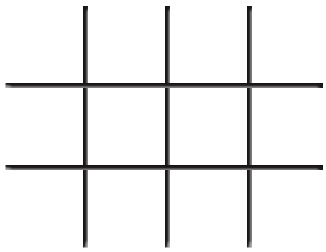
Allow students time to explore with different numbers of ice-cream sticks and to record their results in their own table. A class table could also be compiled and, as students find new possibilities, they can add their data to the class table.

Possible student response:

- Students will find that the method required to create the maximum number of right angles is to produce as many intersections as possible, using the ice-cream sticks like a grid. For example:



Four ice-cream sticks (16 right angles)



Five ice-cream sticks (24 right angles)

Reflection

Look at the table and discuss any patterns found. Can students derive a mathematical formula or relationships for the maximum number of right angles?

Possible student response:

Number of sticks	Number of right angles that could not be made	Maximum number
2	3	4
3	1, 7	8
4	1, 2, 15	16
5		24
6		36

- For an *even* number of ice-cream sticks, the maximum number of right angles is the number of ice-cream sticks squared.
- For an *odd* number of ice-cream sticks, the maximum number of right angles is the number of ice-cream sticks squared *minus one*.

Teacher note:

- There is potential here for a much higher level of mathematics to be explored regarding this generalisation, but for Year 6 students collecting the data and looking for patterns is complex enough.

Where to next?

In Lesson 2: Six Internal Right Angles students are asked to construct a polygon with exactly six internal right angles. They compare and sort the polygons created.

LESSON 2

re(Solve) MATHS BY INQUIRY

Right Angles

Six Internal Right Angles

Y6

About this lesson

Students are asked to construct a polygon with exactly six internal right angles. They see that it is possible to make an octagon with six right angles and two angles of 270°. Students sort the octagons based on the number of right angles that separate the 270° angles.

Australian Curriculum: Mathematics (Year 6)

ACMMG141: Investigate, with and without technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles.

Mathematical purpose

- Students build their familiarity with right angles and see that three right angles meeting at a point form an angle of 270°. Students also name irregular polygons based on the number of sides.

Learning intention

- To explore and categorise irregular polygons, using our understanding of right angles.

Time

A lesson of approximately 1 hour.

Vocabulary

- polygon (defined as a simple, closed shape with straight sides)
- right angle

Resources

- sticky notes or rectangular cards
- grid paper

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Drawing polygons

Pose the challenge: Draw some polygons that have exactly six internal right angles.

Resources: Provide students with a small sticky note or a rectangular piece of card.

Introduce the idea that the corner of this can be used to measure right angles. The advantage of using a post-it note is that students can stick it to their page, so it will not move.

Discuss the fact that a right angle is equal to 90°. Explain that degrees are units used to measure an angle. As with centimetres and other measurement units, each degree is a constant size.

Allow students time to explore and create shapes with exactly six internal right angles.

Enabling prompt:

- *Draw some closed shapes that have exactly four internal angles that are all right angles.*
- *In what ways could you change these shapes to create a shape with six internal right angles?*
- *Could you join some together to create a shape with exactly six internal right angles?*

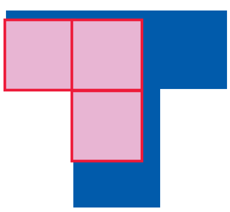
Possible student response:

- Most students will create polygons with six right angles and two reflex angles of 270°, as shown.

Categorising polygons

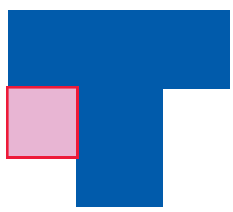
Ask students:

- *What are the names of these shapes?*
  - ◊ Each of the shapes has eight sides, so they are all octagons.
- *What do you notice about the size of the **other** internal angles in the shape?*
  - ◊ Students will note that all the other angles are *greater* in size than a right angle. They may also see that these angles are equal in size to the sum of three right angles. As each right angle is 90°, the size of these angles is $90° + 90° + 90° = 270°$. The external angle at this point is a right angle. Students can use the sticky note or card (used to measure right angles) to test this. Students may also see that a full rotation is equal to the sum of four right angles, which is equivalent to $90° + 90° + 90° + 90° = 360°$.



The internal angle is equal to three right angles.

$90° + 90° + 90° = 270°$



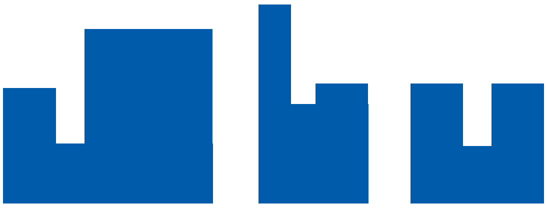
The external angle is equal to a right angle or 90°.

Provide students with grid paper. Ask them to use the grid to help them draw as many different octagons with six right angles and two angles of 270° as they can.

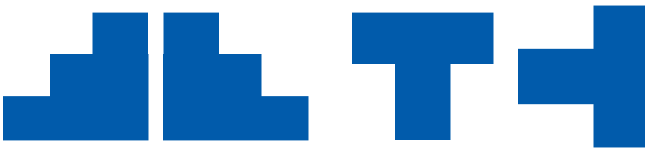
Ask students to sort the different shapes that they have made. It may be helpful for them to cut out their shapes and physically sort them into groups. Students can name their groups and ask a friend to work out the system they have used to sort.

Possible student response:

- There are four categories of octagons that can be created with these properties. These categories are based on the position of the right angles and the 270° angles.

These shapes are all shaped like the letter U. The 270° angles are next to each other. 	These shapes are all shaped like a staircase. The two 270° angles are separated by one right angle (or five right angles in the opposite direction). 
These shapes are all shaped like the letter T. The two 270° angles are separated by two right angles (or four right angles in the opposite direction). 	These shapes are all shaped like the digit 8. The two 270° angles are separated by three right angles. 

Teacher notes:



These shapes are the *same*. They have simply been reflected or rotated. It is important that students recognise them as congruent.

Ask students to look at the placement of the right angles and 270° angles. Record the angles in a linear sequence, starting from the first 270° angle. Have them sort the shapes into groups, based on whether the...

- ...270° angles are next to each other: 270, 270, 90, 90, 90, 90, 90, 90.
- ...270° angles are separated by one right angle: 270, 90, 270, 90, 90, 90, 90, 90.
- ...270° angles are separated by two right angles: 270, 90, 90, 270, 90, 90, 90, 90.
- ...270° angles are separated by three right angles: 270, 90, 90, 90, 270, 90, 90, 90.

If students do not have any shapes in one of these groups, ask them to try and make one.

Reflection

Ask students to look at the way someone else in the class has sorted their shapes.

Ask: *Do you agree that these shapes have been correctly sorted based on the position of the angles?*

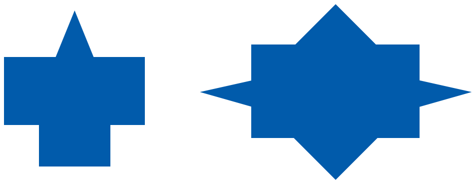
Discuss the ways in which the shapes in each category are similar and the ways in which they are different.

Further activities

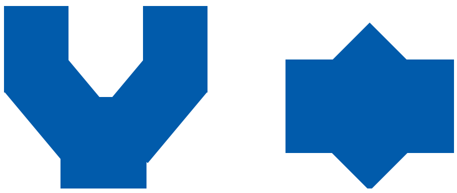
Activity 1

Investigate polygons with six internal right angles where the other angles are *not* 270°.

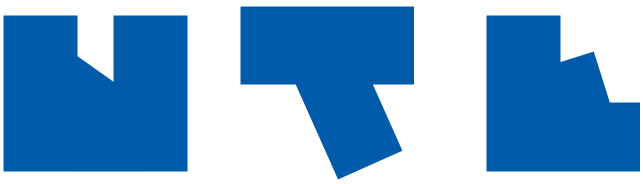
- *Can you draw some shapes that have exactly six internal right angles and at least one angle that is smaller than a right angle?*
  Examples:



- *Can you draw some shapes that have at least one angle that is not 270° and is greater than a right angle?*
  Examples:



- *Can you draw an octagon with six internal right angles and where the other two angles are not 270°?*
  Modifying the octagons created previously can generate answers to this question, such as:



- *Can you draw exactly six internal right angles in a ...hexagon? ...heptagon? ...octagon? ...nonagon? ...decagon? etc.*
  - ◊ It is not possible to draw a hexagon or heptagon with six internal right angles. This can be shown using the angle sum of a polygon.
  - ◊ There are four octagons that can be made. These are the ones shown in the lesson. It is possible to make variations of these octagons by changing the length of some sides.
  - ◊ It is possible to make nonagons, decagons, and polygons with more than 10 sides.

Activity 2

Have students use simple coding software (e.g. Scratch, Pro-Bots) to construct a program that creates polygons with six internal right angles.