Year 7: Angles and Lines
Lesson 2: An Extended Protractor

Australian Curriculum: Mathematics (Year 7)

ACMMG164: Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning.

- Defining and identifying the relationships between alternate, corresponding and co-interior angles for a pair of parallel lines cut by a transversal.

ACMMG165: Classify triangles according to their side and angle properties and describe quadrilaterals.

- Describing squares, rectangles, rhombuses, parallelograms, kites and trapeziums.

Lesson abstract

Students explore the unique design of an extended protractor that has a rhombus linkage attached to it. This award-winning design (CCKL-Creator) enables easy measuring of inside and outside angles in both two-dimensional and three-dimensional situations. Students construct a physical model of the extended protractor and observe angle sizes in a computer simulation. They identify the relationships between angles formed by the parallel lines in the protractor, and use the equalities to explain how the extended protractor works.

Mathematical purpose (for students)

The extended protractor works because of the properties of angles associated with parallel lines.

Mathematical purpose (for teachers)

Prior to the lesson students would have used an ordinary protractor to measure and draw angles. Students will see how parallel lines are used in a real-world object for a particular purpose. Whereas some students may only identify the pairs of equal angles formed by the parallel bars, others can demonstrate using these properties why the protractor works. Angle properties of a rhombus can also be observed.

Lesson Length 40 minutes approximately

Vocabulary Encountered

- vertically opposite angles
- supplementary angles
- alternate angles
- corresponding angles
- co-interior angles
- rhombus

Lesson Materials

- CCKL-Creator or similar protractor (optional)
- Geostrips or similar (1 short blue & 3 longer blue per pair of students, or print these strips onto card)
- Protractor Image (printed on card, 1 per pair of students)
- Paper fasteners (4 per pair of students)
- GeoGebra file *ST1_Yr7_2b_Protractor.ggb*
- OR GeoGebra Tube link https://ggbm.at/JAuwyVHu
- Slide show: *ST1_Yr7_2a_Protractor.pptx*
- Student Sheet 1 – An Extended Protractor (1 per student)

We value your feedback after these lessons via https://www.surveymonkey.com/r/2JH6Z82

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Introducing the Extended Protractor

Teacher Notes

- The extended protractor shown below is a protractor with an attached rhombus linkage. It is used by woodworkers or people doing crafts when making or matching complex shapes.
- This protractor can measure inside and outside angles such as where two walls meet, or the angle between two inside edges of a box. The lower left hand corner of the rhombus (see image below) is placed in the corner, and the size of the angle is read from the scale.
- Two inner sets of numbers give the angles from 0° to 180° in reverse directions as on a normal school protractor. The innermost set of numbers gives angles from 0° to 90°. The rotating arm also includes a Vernier, allowing accurate measurement to the nearest degree.

The focus of this lesson is how the linkage contributes to easy angle measuring and drawing, and how the angles formed by the sides of the rhombus are related to each other.

Getting started

- If possible, show students one of the protractors (e.g. CCKL-Creator available from various internet sites). and demonstrate how it is used. If not available, use images of the protractor in the Slide show: *ST1_Yr7_2a_Protractor.pptx*
- Discuss how the protractor might make it easier for drawing and measuring angles.

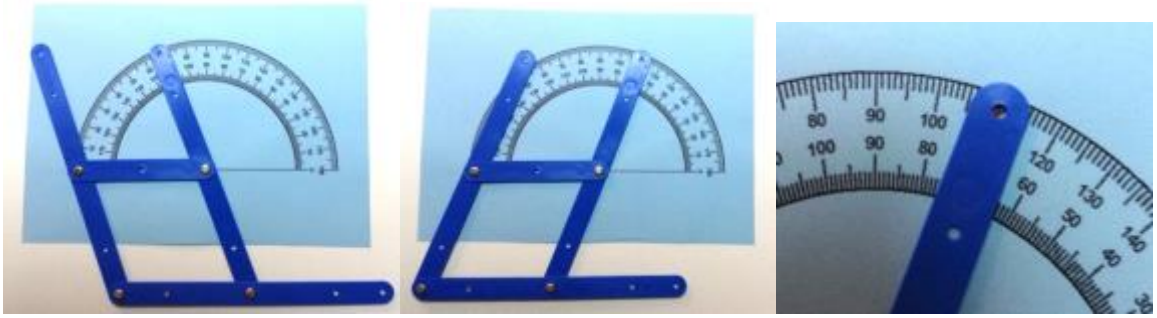
Expected Student Response

- It is easy to draw an angle of a particular size because the position on the paper where you draw the angle isn't covered by the protractor.
- It is easy to measure an angle of a particular size because the protractor does not need to be placed over the angle.
- This protractor can measure outside angles as well as inside angles, such as where two walls meet, or the angle between two inside edges of a box.

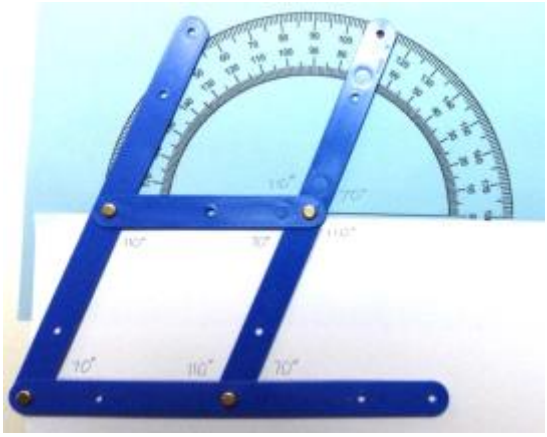
Models of the Protractor

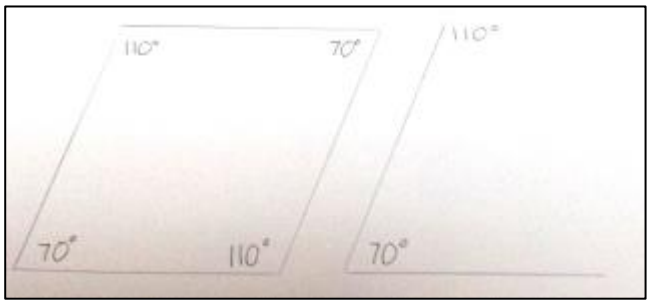
Making a physical model

- To each pair of students, give 4 paper fasteners, 1 short blue GeoStrip and 3 blue GeoStrips twice the length of the short one and a copy of the Protractor Image printed on card.
- Alternatively, instead of using GeoStrips, this set of Strips can be printed on light card, one set per pair of students. Small holes should be made in the card at the positions shown by the small circles. It is important that the distances match.
- Students then construct the rhombus linkage using the four strips and paper fasteners. Note that two of the paper fasteners must pass through the holes in the card protractor.
- Students then use their extended protractor to measure some angles, and observe how it works. Note that the actual angle mark is seen through the hole at the end of the rotating strip.
- Students then observe its geometric properties, especially the fact that the 'horizontal' strips always stay horizontal (and hence parallel) and the other two strips also stay parallel to each other. (The fact that the bars are parallel follows from the equal lengths, but this is just assumed in this lesson.)



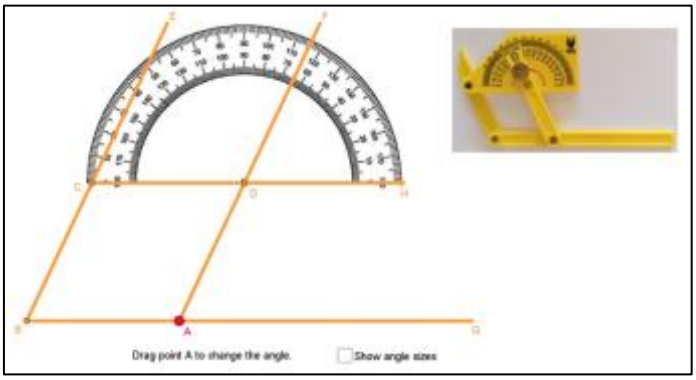
- Place a piece of A4 paper under the linkage as shown. Set the protractor to an angle, for example, 70º.Trace along the bars of the linkage.
- Students can use their knowledge of acute and obtuse angles, supplementary, vertically opposite, cointerior (allied) and alternate angles to record all the other angle sizes. This demonstrates clearly the angle properties of a rhombus.

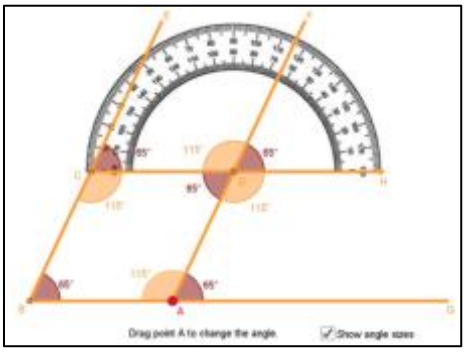




Using a computer simulation

- The GeoGebra simulation of the protractor enables students to see the sizes of all angles and the relationships between them.
- Take the opportunity for a class discussion to reinforce supplementary angles, as well as vertically opposite angles, alternate angles, corresponding angles and cointerior (allied) angles.
- The task can also highlight the angle properties of a rhombus/parallelogram.





Why Does the Protractor Work?

Drawing an angle of fixed size

Looking at the diagram of the computer simulation, suppose we wish to draw an angle of 65° with its vertex at A. We move the protractor arms so that arm DF passes through 65° on the protractor. Explain why the outside (exterior) angle at A will be 65°.

Ans. If ∠HDF = 65°, then ∠CDA = 65° (vertically opposite angles).

So ∠GAD = 65° (alternate angles between parallel sides of the rhombus CD and BA cut by transversal FA).

Students could also suggest using supplementary angles ∠HDF and ∠ADH and co-interior angles ∠ADH and ∠GAD.

Measuring an angle inside a box

Place point B in the angle to be measured. Move the arms of the protractor so that they fit along the sides of the box. Say, for example, that ∠HDF = 65°. We want to show that ∠ABC = 65°.

Ans. If ∠HDF = 65°, then ∠CDA = 65° (vertically opposite angles).

So ∠GAD = 65° (alternate angles between 'horizontal' parallel sides).

So ∠ABC = 65° (corresponding angles between 'sloping' parallel sides).

Further Investigation

Considering the design

- What properties of the rhombus make it useful in the design? (Ans. Parallel sides allow the angle selected on the protractor to be transferred to the angle formed by the sides of the rhombus; equal sides of the rhombus allow the protractor to fold compactly)
- Does the linkage need to be a rhombus or would any parallelogram work? Models could be constructed from strips and paper fasteners to find out. (Ans. Any parallelogram would work, but the rhombus is compact)

Angles of a rhombus

- What can we learn from the extended protractor about the angles of a rhombus? (Ans. Opposite angles are equal, adjacent angles are supplementary.)

Conclusion

- The parallel sides of the rhombus provide a useful way of transferring the size of an angle between the protractor and an angle on paper.
- The parallel sides of the rhombus allow an inside or outside angle to be transferred to the protractor to see its size.
- The opposite angles of a rhombus are equal.
- The adjacent angles in a rhombus add to 180°.

An Extended Protractor

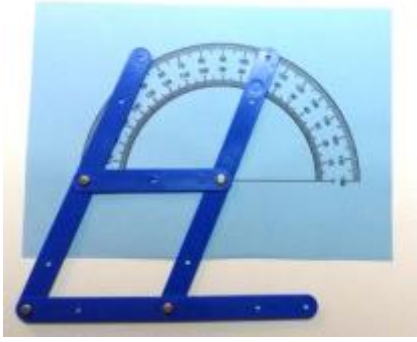
Name: _____

- The protractor shown below is unusual because it has four bars connected it.
- This enables it to measure angles in places where an ordinary protractor does not fit, such as inside a box.

Making a physical model

- Use 2 identical short strips and 2 strips twice as long. Connect the strips together with paper fasteners.
- Note that 2 of the paper fasteners must pass through the holes in the card protractor.

- The actual angle measure is seen through the hole at the end of the rotating strip, not the edge of the strip (compare this with the hole in the rotating bar of the actual protractor in the photo above).
- Place a mark on an angle of your choice and look through the hole to check if the rotating strip is in the correct position over the mark. Make sure you correctly read the outer or inner scale depending on whether your angle is obtuse or acute.



- Place a sheet of A4 paper between the card and the three free strips. Draw around the inside of the quadrilateral shape and draw along the Geostrips where they form an angle beside the quadrilateral.
- Label each of the angles with their sizes and with a different letter name, for example, a° .
- Using the letter names, name a pair of:
  - Vertically opposite angles
  - Alternate angles
  - Corresponding angles
  - Cointerior angles
  - Supplementary angles

Using a computer simulation

- Open the GeoGebra file *Protractor*. This shows a simulation of the protractor.
- Drag point A to change the angle. Observe which angles are equal to each other.
- Can you prove why the protractor works the way it does? Why is the angle indicated on the protractor the same size as the outside angle at point A? Use your knowledge of angle types to explain.
- What do you notice about the opposite angles of the rhombus?
- What do you notice about the pairs of adjacent angles of the rhombus, that is, the pairs of angles in the rhombus that are next to each other?

A design question

- Could any parallelogram be used for the protractor or does it have to be a rhombus?

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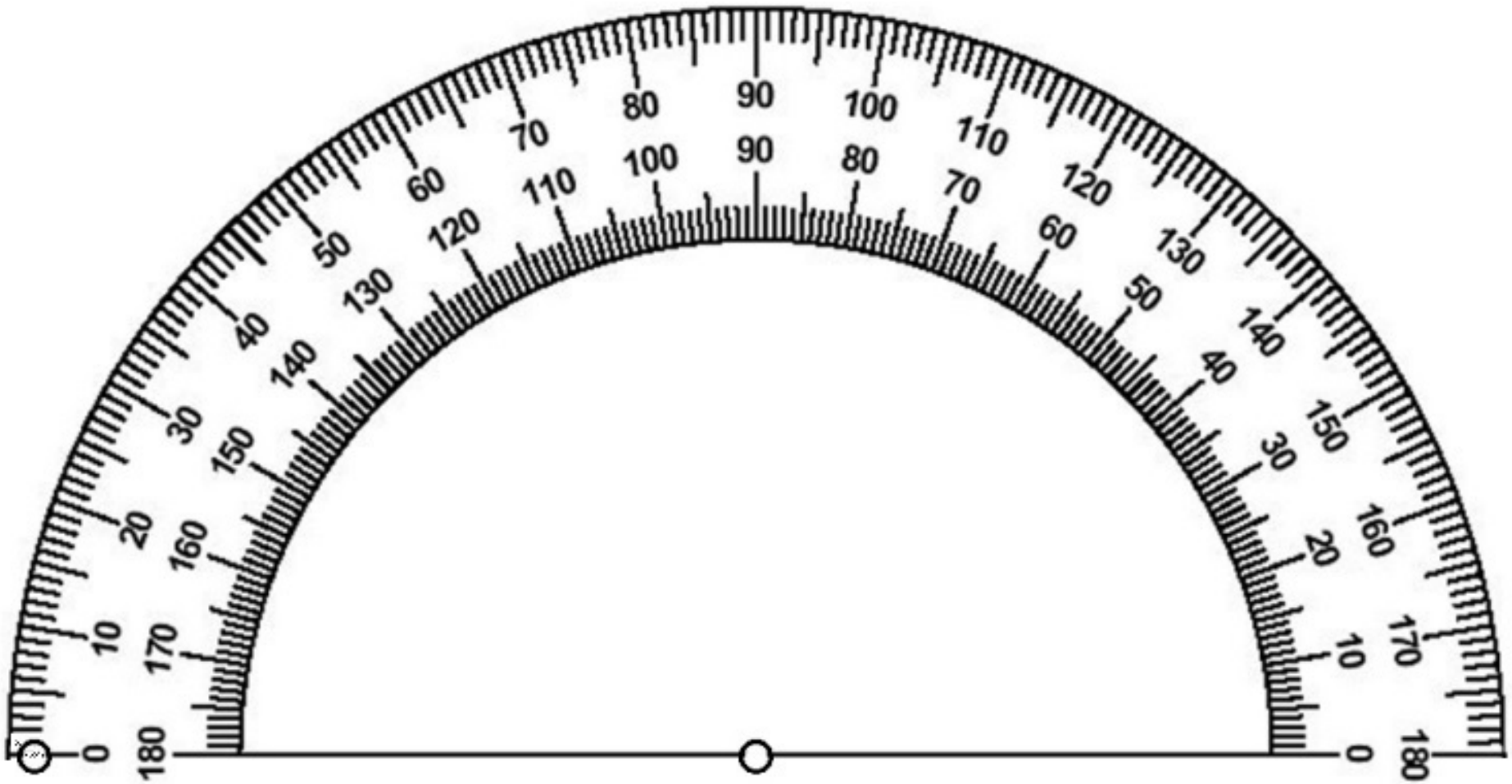
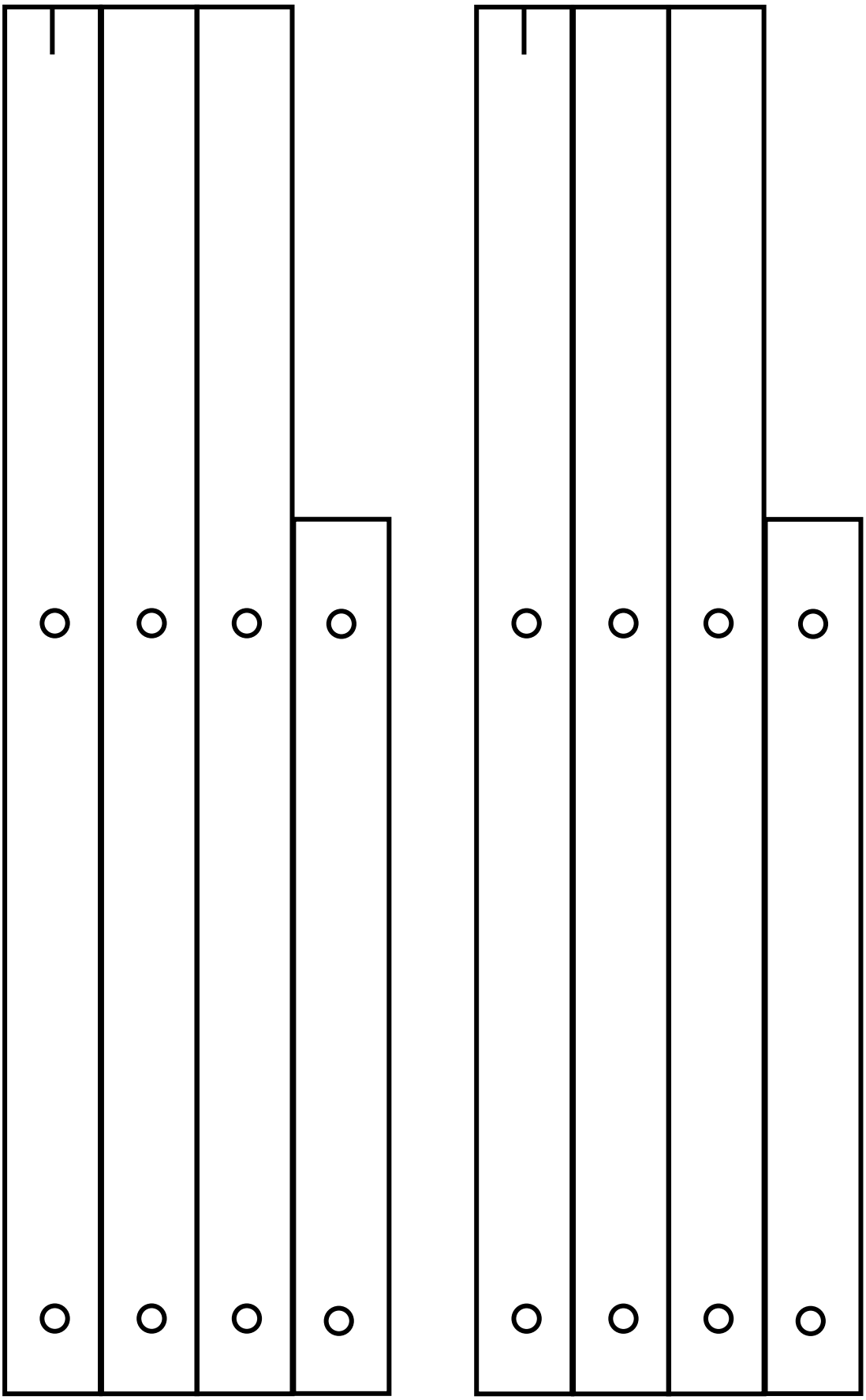

0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0

Protractor model strips

These strips should be printed on thin card, e.g. 140 gsm card, then cut out and holes made as shown. The marker at the end of the left hand strip is to point to the angle.