# Unit Overview: Quadrilateral Linkages

## Summary of learning goals

As part of the Special Topic **Mechanical Linkages and Deductive Geometry** this unit looks at geometry behind everyday tools and objects. Examples are folding umbrellas, tool boxes, car jacks, and scissor lifts. The lessons aim to show students that there is mathematics in the world around them. With the aid of physical models and computer simulation, the lessons move from a view of geometry as a study of static diagrams to encompass movement.

The properties of quadrilaterals, especially rhombuses and parallelograms, are involved in explaining how these tools and objects move. An important goal is to give students experience of deductive reasoning and a reason to engage in it.

### Australian Curriculum: Mathematics (Year 8)

**ACMMG201**: Develop the conditions for congruence of triangles.

- Solving problems using the properties of congruent figures.

**ACMMG202**: Establish properties of quadrilaterals using congruent triangles and angle properties.

- Establishing the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites.
- Identifying properties related to side lengths, parallel sides, angles, diagonals and symmetry.

## Summary of lessons

### Who is this unit for?

The unit is for students who are learning about the properties of quadrilaterals. It is hoped that these lessons may appeal to a wide range of students, building on their interest in these everyday tools. The lessons stand alone, and teachers may select one or many. *Car jack* and *Scissor lift* lessons involve the same rhombus geometry, so are alternatives, or can be combined. *Folding umbrella*, *Cherry picker* and *Toolbox* lessons also involve the same parallelogram geometry, so are alternatives, or can be combined. *Angle Bisectors* lesson includes rhombus and kite.

### Lesson 1: Car Jack

Students investigate the design and operation of a rhombus car jack (as supplied with many cars) to see how the geometric properties of a rhombus contribute to how it works. Students first examine a real jack or images, and make a physical model to observe how properties of sides and angles enable the operation of the jack. They then examine a dynamic geometry simulation, which reveals the importance of diagonal properties to safety. Students can then use congruent triangles to reason deductively about rhombus properties.

### Lesson 2: Scissor Lift

Students investigate the design and operation of a scissor lift to see how the geometric properties of a rhombus contribute to how the scissor lift works the way it does. They look at images, and make and operate a physical model and a computer simulation to explore the geometric properties on which a scissor lift depends. They then use congruent triangles to prove the key property: that the diagonals of the rhombus are perpendicular.

We value your feedback after these lessons via https://www.surveymonkey.com/r/2JH6Z82

***Mathematics by Inquiry*** is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

### Lesson 3: Folding Umbrella

Students investigate the design and operation of a folding umbrella to see how the geometric properties of a parallelogram contribute to neat folding. Students observe an actual umbrella and its ribs, make a physical model and operate a computer simulation, at each stage seeing the geometric features more fully. This linkage is then used as a stimulus for students to engage in deductive reasoning about the properties of quadrilaterals.

### Lesson 4: Toolbox

Students investigate the design and operation of a cantilever toolbox to see how the geometric properties of a parallelogram contribute to how the trays of the toolbox open and close. Students observe an actual toolbox, make a physical model and use a computer simulation to understand how the design based on parallelograms constructed from bars of equal length allows the toolbox to open and close with the trays remaining horizontal. There is an opportunity for deductive reasoning, supported by a slideshow.

### Lesson 5: Cherry Picker

Students investigate the design and operation of an aerial work platform commonly called a cherry picker to see how the geometric properties of parallelograms contribute to how the work platform is maintained in a horizontal position and how changing shapes of the parallelograms allow the work platform to be manouvered into different positions. They construct a physical model and use a computer simulation of the cherry picker. They also see how articulated parallelograms are used in robotic arms in certain machinery and design a robotic arm for cake icing.

### Lesson 6: Angle Bisectors

Students investigate the design and operation of two different angle bisector tools and use the geometry of rhombuses and kites to explain why they work. The tools are useful for carpenters when constructing mitred corners. The lesson begins by observing mitred joints. Students make physical models and observe computer simulations that highlight the geometry and give accurate measurements. They investigate the reasons why the tools work. This stimulates investigation of other angle properties supported diagrammatically. An extension examines a third type of angle bisector tool.

## Reflection on this sequence

### Rationale

When devising these tasks in deductive reasoning in geometry, the guiding principles have been:

- Providing rich visual imagery, both static and dynamic.
- Providing an opportunity for students to use the language of geometry.
- Providing meaningful context that can motivate argumentation and conjecturing.
- Highlighting the need for deductive reasoning as an answer to the question ‘why’.
- Providing links with other STEM subjects, in particular, engineering and technology.
- Providing links with history through historical inventions.

A strong feature of these lessons is that students are able to see and manipulate the linkages. Students operate the real linkage whenever possible, they make physical models out of plastic strips or light card, and they use pre-prepared dynamic geometry computer simulations.

The tactile experience of operating the actual tool or a physical model of the linkage provides an instant sense of satisfaction and gives insight into the way in which the linkage moves. The provided dynamic geometry computer simulations show the geometry more clearly, and enable accurate measurements to be made. Students can observe what stays the same and what varies as the dynamic geometry models are operated.

### reSolve Mathematics is Purposeful

Lessons in this unit show mathematics both as a way of modelling the real world and as an abstract discipline. The linkages are used in practical everyday tools, and analysis of the tools leads to conjecture, argumentation and proof. There are clear links to STEM, including engineering.

### reSolve Tasks are Challenging Yet Accessible

For many students, motivation for this unit will come from their own experience with the real tools (e.g. car jack). Using the physical and computer models assists students to visualise the motion and they can use them to develop and test conjectures. The expectations for the deductive reasoning can be moderated by the teacher, and structured approaches to support are offered. In many of the lessons, there is substantial opportunity for challenge and further investigations.

### reSolve Classrooms Have a Knowledge Building Culture

Students work together to build models, make and test conjectures and develop argumentation.