Year 8: Quadrilateral Linkages
Lesson 6: Angle Bisectors

Australian Curriculum: Mathematics (Year 8)

ACMMG201: Develop the conditions for congruence of triangles.

- Solving problems using the properties of congruent figures.

ACMMG202: Establish properties of quadrilaterals using congruent triangles and angle properties.

- Establishing the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites.
- Identifying properties related to side lengths, parallel sides, angles, diagonals and symmetry.

Lesson abstract

Students investigate the design and operation of two different angle bisector tools and use the geometry of rhombuses and kites to explain why they work. The tools are useful for carpenters when constructing mitred corners. The lesson begins by observing mitred joints. Students make physical models and observe computer simulations that highlight the geometry and give accurate measurements. They investigate the reasons why the tools work. This stimulates investigation of other angle properties supported diagrammatically. An extension examines a third type of angle bisector tool.

Mathematical purpose (for students)

A rhombus and a kite both have geometric properties that are used to make angle bisecting tools.

Mathematical purpose (for teachers)

The lesson provides a context for students to consider the angle properties of a rhombus and a kite. In both quadrilaterals, angles are bisected by a diagonal, a property that is exploited in angle bisecting tools. The *Rhombus properties* and *Kite properties* slideshows provide a visual stimulus for deductive reasoning, using knowledge of congruent triangles, isosceles triangles, and parallel lines.

Lesson Length 60 minutes approximately (with optional second lesson for extension)

Vocabulary Encountered

- bisect
- mitre
- diagonal
- congruent
- perpendicular

Lesson Materials

- Angle bisector tool (optional, rhombus version available as Angle divider from McJing Tools, NSW; 'Mitey Miter' from various websites)
- Geo Strips per pair of students: 4 short strips, 2 longer strips and 1 long strip, 4 paper fasteners
- Student Sheet 1 - Angle Bisectors (1 per student)
- GeoGebra file *ST1_Yr8_6b_Anglebisector1.ggb* OR GeoGebra Tube https://ggbm.at/t2kJA83n
- GeoGebra file *ST1_Yr8_6c_AngleBisector2.ggb* OR GeoGebra Tube https://ggbm.at/K3rXM7rb
- Slide show: *ST1_Yr8_6a_AngleBisectorImages.pptx*
- Slide show: *ST1_Yr8_6d_KiteProperties.pptx*
- Slide show: *ST1_Yr8_1c_RhombusProperties.ppt*

We value your feedback after these lessons via https://www.surveymonkey.com/r/2JH6Z82

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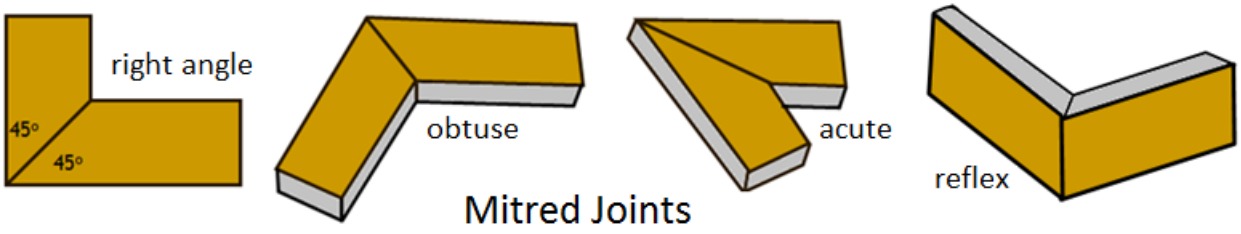
Introducing Angle Bisectors

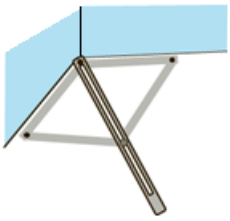
Teacher Notes

Angle bisector tools are used by carpenters when determining the required angles for making a mitred corner, for example in a skirting board, picture frame. Frequently, the corner angle is 90º so that both pieces of wood are mitred by being cut at an angle of 45º.



There are several different designs of angle bisecting tool. This lesson is focussed on the rhombus angle bisector and the kite angle bisector. An extension at the end of the lesson examines a third type.

The rhombus angle bisector tool is a rhombus linkage with a slotted diagonal. The shape of the rhombus can be changed so that it matches the angle, for example, between two walls. The screw is then tightened to fix the angle on the angle bisector. The half angle is marked by drawing along the slot in the diagonal arm of the angle bisector. The design of the kite angle bisector is essentially the same as the rhombus angle bisector except that its sides form a kite rather than a rhombus (which is in fact a special type of kite). It has the advantage that a reflex angle can be bisected (e.g. an outside angle of a decorative box).



Rhombus angle bisector

Kite angle bisector

Getting started

Hand out Student Sheet 1 – Angle Bisectors if desired.

Point out some mitred joints in the classroom (e.g. in a skirting board, a ceiling coving, a window frame).

Show students an angle bisector tool if you have one, and demonstrate its use, or use the first three slides of the slideshow *Angle bisectors.ppt*, making sure students understand what the tools are used for and that they understand the need for mitred joints.

Perhaps show a video clip of woodworkers making a mitred corner. The following links are to two videos showing the 'Mitey Miter' kite angle bisector in action. The first shows the bisection of an inside angle and the second shows the bisection of an outside (reflex) angle. The videos should be of particular interest to students who are doing woodwork.

https://youtu.be/yyLSpS6Y9w8 and https://youtu.be/UjKU0v9enQs

Key geometric features of the angle bisectors

- Using images of the rhombus angle bisector in the slide show, discuss the key features of the tool including:
  - The angle bisector is in the shape of a rhombus.
  - The rhombus is jointed (a linkage), so its angles can change but the sides remain the same length.
  - The slotted bar divides two of the rhombus angles in half.

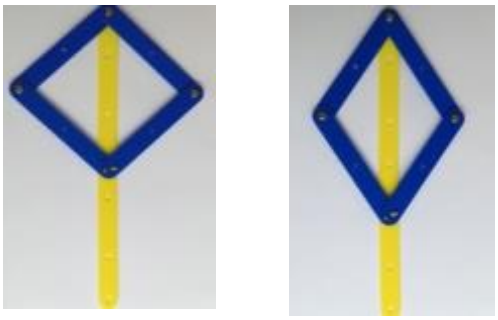
- Show students images of the kite angle bisector in the slide show *ST1_Yr8_6a_AngleBisectorImages.pptx* (slides 4,8,11). Discuss the key features of the tool (similar to above).
  - Note: All rhombuses are also kites, so really there is only one angle bisector tool. They are treated separately here so that students can revisit ideas.

Models of the Angle Bisector Tools

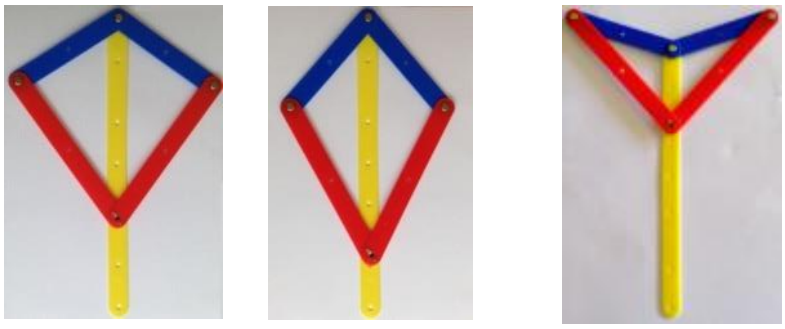
In this part of the lesson, students make physical models, giving them hands-on experience of how the tools move and how they are used. Then they use a computer simulation to measure angles easily and highlight the geometry.

Physical models

- Show students a model you have made or the images of the models from the slide show *ST1_Yr8_6a_AngleBisectorImages.pptx*
- Students construct a model of the rhombus angle bisector and the kite angle bisector using 5 Geo Strips connected with 4 paper fasteners for each model. The Geo Strip models are easy to assemble, but light card strips could be substituted if necessary.
- The long Geo Strip (the yellow diagonal) is connected to the upper vertex only. The lower vertex is not attached to the yellow strip and hence is free to slide up and down the diagonal. It slides more smoothly if the paper fastener is inserted with its cap downwards.
- As students move their finished models, they should observe and write down what things change and what things stay the same.



A rhombus angle bisector in two positions.



A kite angle bisector in three positions.

Expected Student Responses

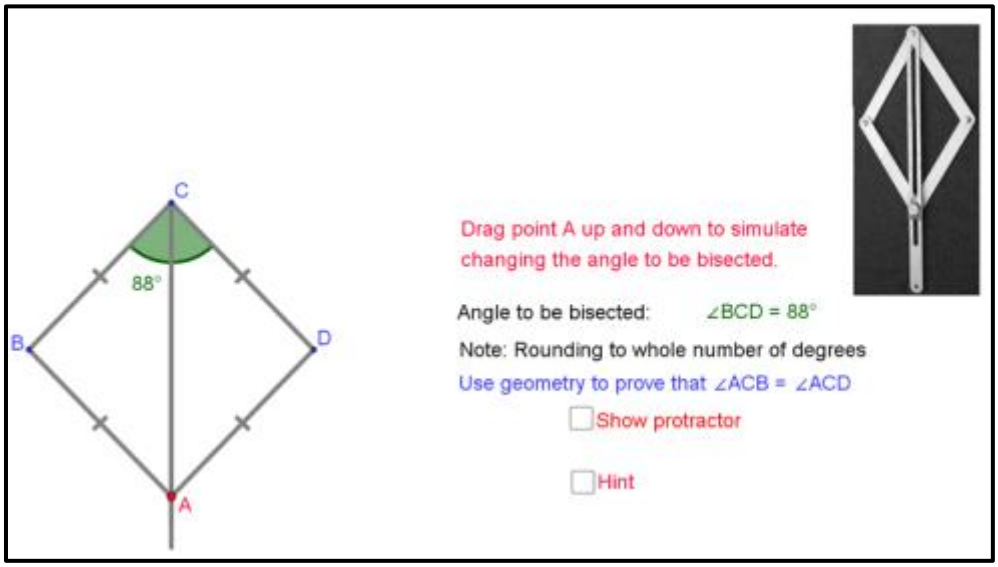
- Rhombus and kite angles change.
- Angles at the top and bottom of the rhombus are the same, but the kite angles are different.
- Angles on the right and left sides are equal.
- Length of the diagonal changes, when the angle changes.
- The diagonal strip always divides the rhombus or kite in half, and divides the angles in half.

Computer simulations

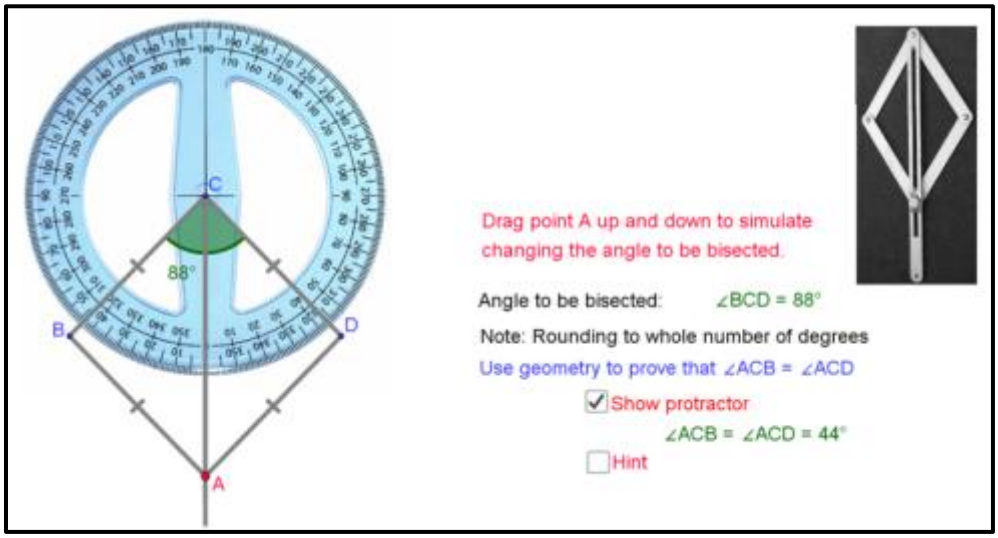
Students use a computer model of each tool. The models show again how the tools move, with angle measurements and equal sides marked. This brings the students to reasoning about why the tool works. The simulations can be accessed by using the Geogebra files (requiring Geogebra installed, also may be changed deliberately or accidentally) or directly in an internet browser from Geogebra Tube without installation.

Angle bisector 1: The rhombus angle bisector

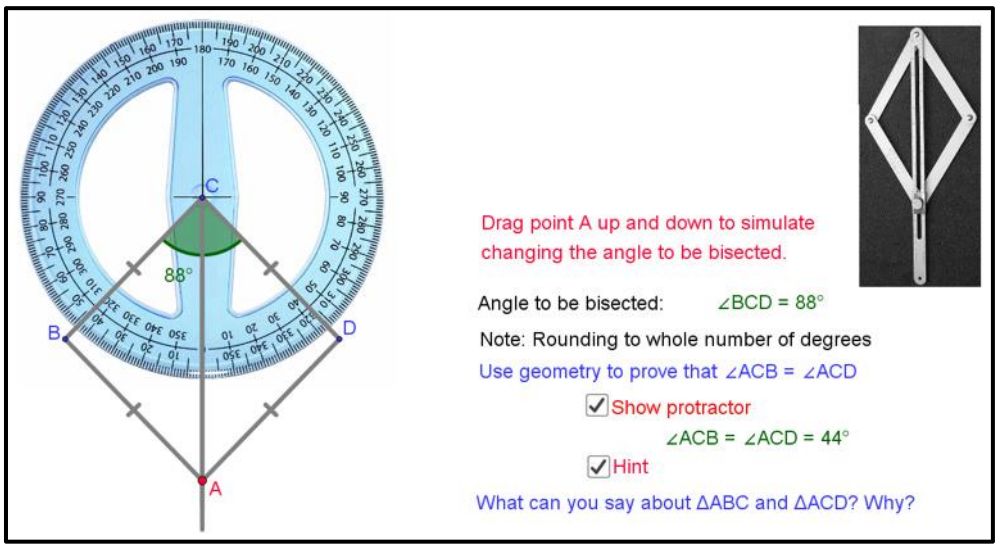
The GeoGebra file *ST1_Yr8_6b_Anglebisector1.ggb* (or GeoGebra Tube https://ggbm.at/t2kJA83n) is a simulation of the rhombus angle bisector.



- By checking the *Show protractor* checkbox, students can see the size of the angle to be bisected and can read the protractor to see that indeed the angle is bisected by the diagonal.



- The *Hint* checkbox prompts students to think about why the two triangles formed by the rhombus and its diagonal are congruent and hence why the diagonal bisects the rhombus angles.



- Students now work, perhaps in pairs, to make conjectures about the two triangles, and about why the tool does bisect the angle. This may be too difficult for many students to manage alone, so when necessary,

bring the class together to look at the computer simulation, and lead students to think about why the angle is bisected.

- The kite angle bisector involves very similar geometric reasoning, so the following section and also the kite properties slideshow will reinforce these ideas. Discuss the geometric reasons after considering the kite tool simulation, which involves very similar ideas.

Angle bisector 2: The kite angle bisector

- The GeoGebra file *ST1_Yr8_6c_AngleBisector2.ggb* (or GeoGebra Tube https://ggbm.at/K3rXM7rb) is a simulation of the kite angle bisector.
- Students use the simulation to observe the motion of the tool and observe what is constant and what changes.
- By selecting the *Show protractor for interior angle* checkbox, students can see the size of the inside angle (acute or obtuse angle) to be bisected and can read the protractor to see that the angle is bisected by the diagonal.
- By selecting the *Show protractor for exterior angle* checkbox, students can see the size of the outside angle (reflex angle) to be bisected and can read the protractor to see that the angle is bisected by the diagonal.
- The *Hint* checkbox prompts students to think about why the two triangles formed by the kite and its diagonal are congruent and hence why the diagonal bisects the kite angles.
- Discuss students' conjectures and arguments for why the tool always bisects the angle. Students can work in pairs to write a sound argument, or this section may need to be led by the teacher.
- These ideas are revisited again when students use the slide show *ST1_Yr8_1c_RhombusProperties.ppt*

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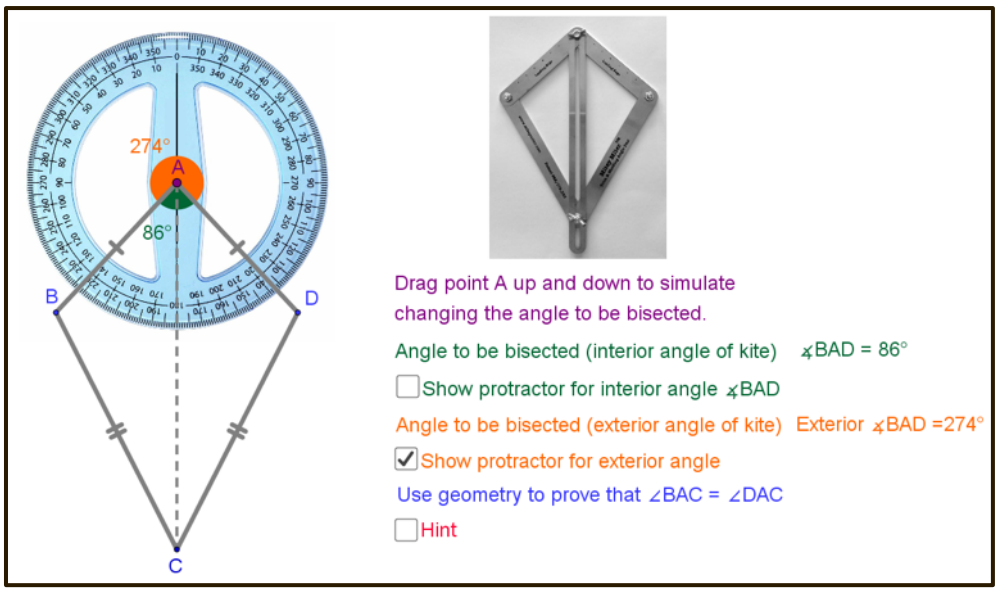


Expected Student Response

A good solution (perhaps attained only after class discussion) is:

- Three sides of triangle ABC are equal to three sides of the triangle ADC.
- So the diagonal divides the rhombus or kite into congruent triangles.
- Hence the angles of one triangle are equal to the angles of the other triangle, in particular angle BAC is equal to angle DAC.
- The diagonal always bisects the angle.

What is the advantage of the kite angle bisector?

- The ability of the kite to bisect a reflex angle gives it an advantage over the rhombus tool.
- Students could consider whether it is possible for a rhombus to have a reflex angle.

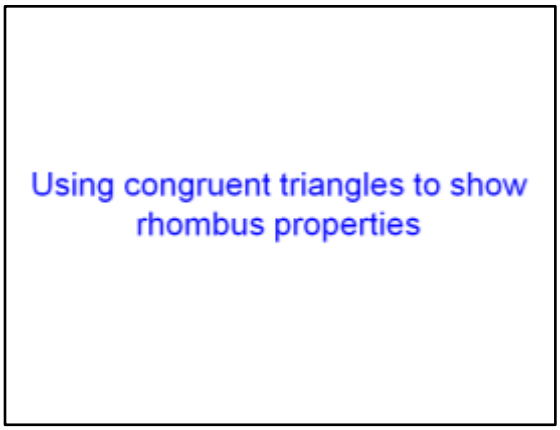
- By definition, reflex angles are greater than 180°. The rhombus must have opposite angles equal. There could not be two reflex angles as this would be greater than the angle sum of the rhombus (360°).

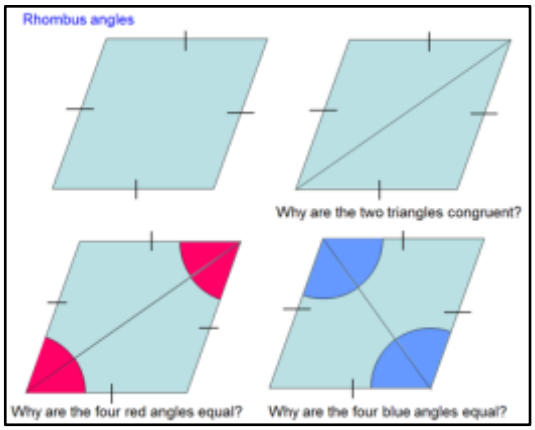
Showing that the diagonal bisects the rhombus and kite angles

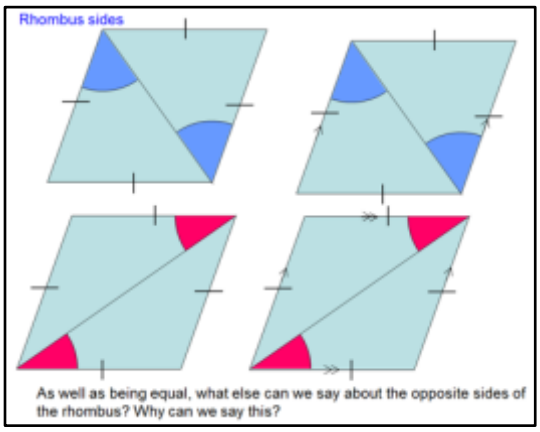
Rhombus angles and diagonals

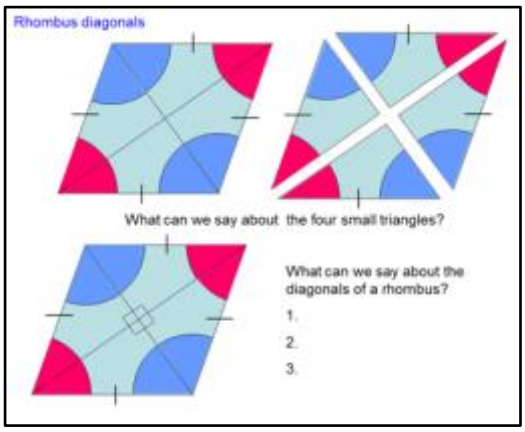
The slide show *ST1_Yr8_1c_RhombusProperties.ppt* leads students visually through the use of congruent triangles to explain rhombus properties. If the students have already watched it in the Scissor lift or Car jack lessons, slides 3 and 4 can be used for revision. In slide 5, students can see that each diagonal divides the rhombus into two congruent isosceles triangles and that together, the two diagonals divide the rhombus into four congruent triangles. Hence we can see that:

- The diagonals bisect the angles of the rhombus (the key property for angle bisector tools).
- The diagonals are perpendicular.
- The diagonals bisect each other.



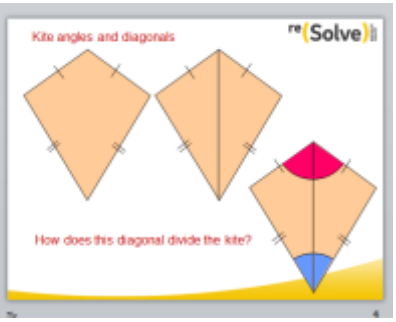


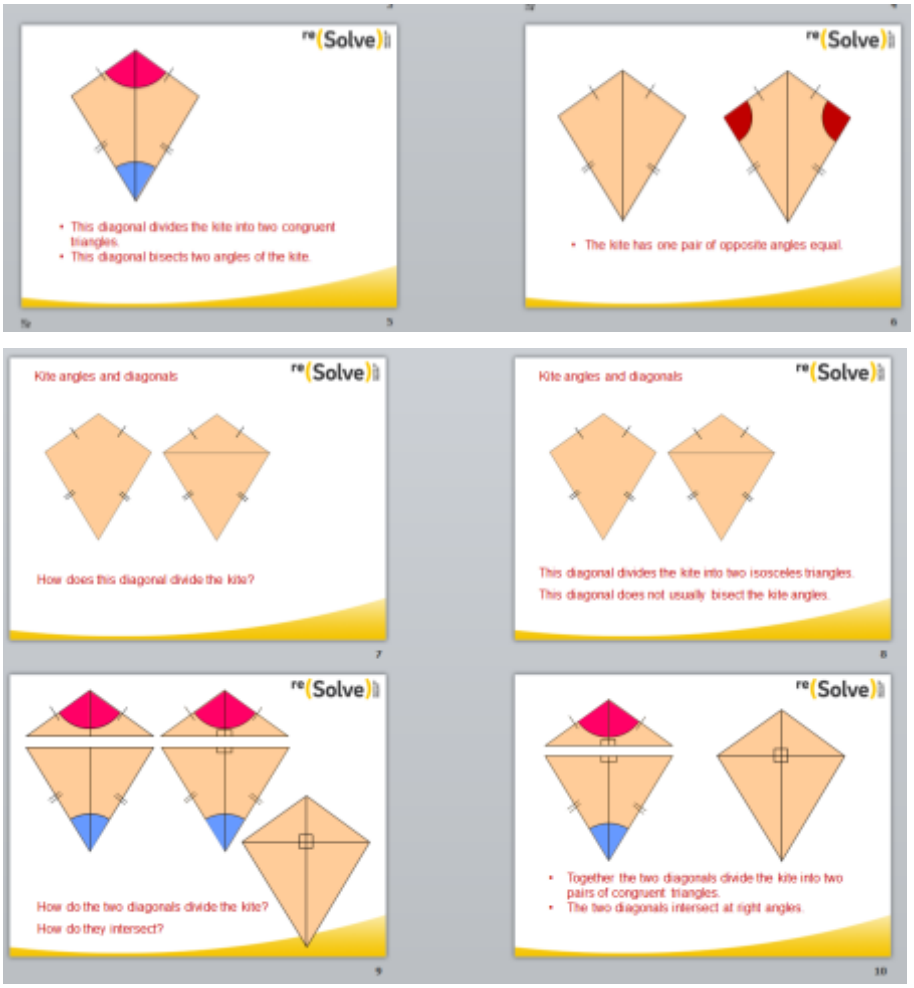




Kite angles and diagonals

Showing that one diagonal of a kite bisects the kite angle again proceeds by first showing that this diagonal divides the kite into two congruent triangles. The slide show *ST1_Yr8_6d_KiteProperties.pptx* can be used to prompt the class discussion: two sides in each triangle are equal by definition of a kite, third side, the diagonal, is shared by both triangles, so S.S.S.).





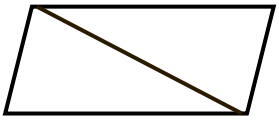
Conclusion

Ask student to share the most important things that they learned in the lesson. In addition to the practical use of the angle bisector tools and experiences of making the models, ensure that the geometry is discussed.

The central idea is that both the rhombus and kite are divided into two congruent triangles by a diagonal. This diagonal is used to bisect an angle.

Further investigations

- Could any parallelogram that is not also a rhombus be used for the design of an angle bisector? Explain using a model or drawings. Answer. No, because the diagonals of a parallelogram only bisect the parallelogram angles in the special parallelogram case of a rhombus.



- Do both diagonals of a kite bisect the angles? What quadrilaterals have this property?

Extension – Compass and Ruler Bisection

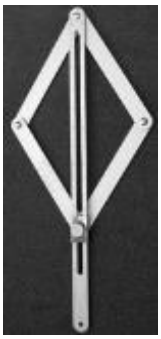
This video https://youtu.be/SqixXVIpHJA shows a third design of angle bisector tool which mimics the bisection of an angle with compass and ruler. Students can investigate why this tool works.

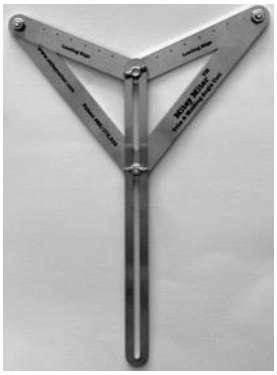
A solution is given on Teacher Sheet1: Compass and Ruler Bisection. There are useful images in the slide show *ST1_Yr8_6a_AngleBisectorImages.pptx*

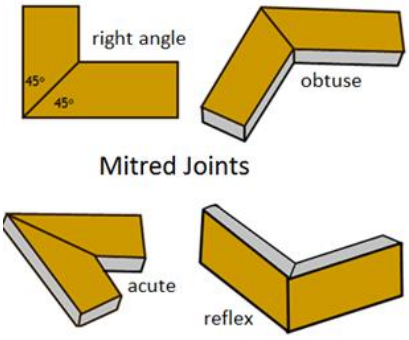
Angle Bisector Tools

Name: _____

Angle bisector tools are used by carpenters when working out the angles needed for making a mitred corner, for example in a skirting board or a picture frame or a







Make a Physical Model

- Make models of the rhombus and kite angle bisectors using Geo Strips and paper fasteners.
- As you operate your models, write down what things change and what things stay the same.

Using the Software Models

- Open the GeoGebra files *Angle Bisector 1* and later *Angle Bisector 2*. These show simulations of the two different angle bisectors. Measure angles to check if the tools work. Why do the angle bisector tools work? What geometric properties are involved?

Rhombus Properties

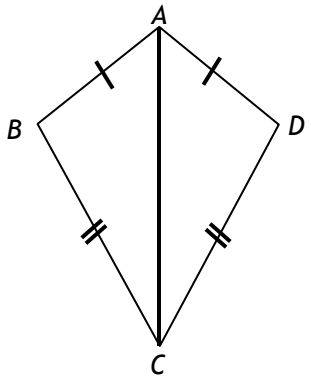
- Watch the slideshow *Rhombus properties*.
- How does each diagonal divide the rhombus?
- What other properties of a rhombus do we know from this slideshow?

Kite properties

- Watch the slideshow *Kite properties*.
- How does each diagonal divide the kite?
- What other properties of a kite do we know from this slideshow?

Explaining why the angle bisector tools work

Use this diagram of a kite to explain why triangles *ABC* and *ACD* are congruent and hence why the diagonal *AC* bisects angle *BAD*. Explain how this relates to bisecting an angle with the angle bisector tools.



Rhombus or kite?

- What is the advantage of the kite version of the angle bisector tool?

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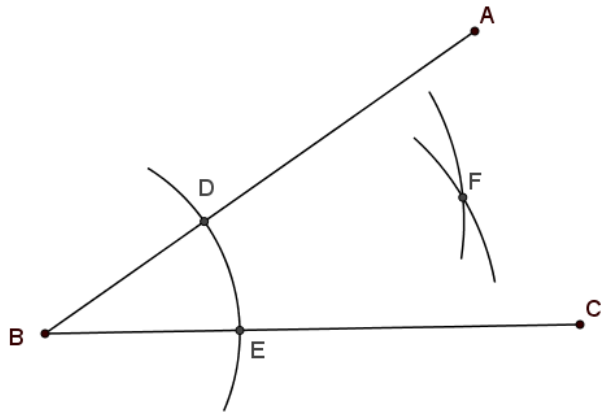
Teacher Sheet 1 - Compass and Ruler Bisection

Instructions for compass and ruler bisection of an angle

$\angle ABC$ is the angle to be bisected. With centre at point B, an arc is drawn to cut BA at D and BC at E. With centre D, an arc is drawn, then with same compass opening and centre E, a second arc is drawn. The two arcs intersect at F. The segment BF is the bisector of $\angle ABC$.



Why does the construction work?

In ΔBDF and ΔBEF

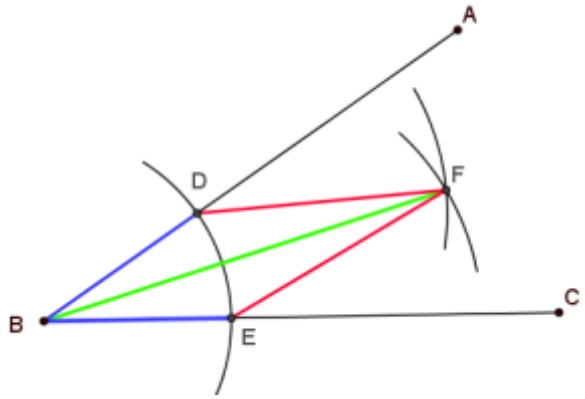
$BD = BE$ (same compass radius)

$DF = EF$ (same compass opening)

BF is common to both triangles.

So $\Delta BDF \equiv \Delta BEF$

So $\angle ABF = \angle CBF$



The link below is to an animation of a compass and ruler bisection of an angle.

http://www.mathopenref.com/constbisectangle.html

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