Year 8: Quadrilateral Linkages
Lesson 2: Scissor Lift

Australian Curriculum: Mathematics (Year 8)

ACMMG201: Develop the conditions for congruence of triangles.
- Solving problems using the properties of congruent figures.

ACMMG202: Establish properties of quadrilaterals using congruent triangles and angle properties.
- Establishing the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites.
- Identifying properties related to side lengths, parallel sides, angles, diagonals and symmetry.

Lesson abstract

Students investigate the design and operation of a scissor lift to see how the geometric properties of a rhombus contribute to how the scissor lift works the way it does. They look at images, and make and operate a physical model and a computer simulation to explore the geometric properties on which a scissor lift depends. They then use congruent triangles to prove the key property: that the diagonals of the rhombus are perpendicular.

Mathematical purpose (for students)

A rhombus has geometric properties that make it useful in the design of a scissor lift.

Mathematical purpose (for teachers)

This lesson about the scissor lift provides an engaging context for students to explore rhombus properties. Students will recognise that the purpose of the scissor lift is to safely lift the work platform in a vertical direction while its base is on horizontal ground. Observation of images of extended and closed scissor lift, construction of a physical model and access to a computer simulation demonstrate the importance of the perpendicular diagonals of the rhombus in ensuring vertical movement. Starting with the definition of a rhombus, congruent triangles formed by the rhombus diagonals are the starting point for reasoning deductively about side, angle and diagonal properties.

The Car Jack and Scissor Lift lessons could be combined because the geometry is similar. The *Rhombus properties* deductive reasoning section would be undertaken after students have constructed both models and accessed both computer simulations.

Lesson Length 45 minutes approximately

Vocabulary Encountered
- rhombus
- diagonal
- congruent

Lesson Materials
- Geo Strips or similar (8 identical strips per pair of students, or print these strips onto card)
- Paper fasteners (10 per pair of students)
- Student Sheet 1 – Scissor Lift (1 per student)
- GeoGebra file *ST1_Yr8_2b_ScissorLift.ggb*
- OR GeoGebra Tube link https://ggbm.at/kMGAsdTq
- Slide show: *ST1_Yr8_2a_ScissorLiftImages.pptx*
- Slide show: *ST1_Yr8_1c_RhombusProperties.ppt*

We value your feedback after these lessons via https://www.surveymonkey.com/r/2JH6Z82

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Introducing the Scissor Lift

Teacher Notes

Numerous inventions incorporate rhombus linkages, for example, bathroom mirror brackets, scissor lifts, car jacks, and expanding trellises. The usefulness of a rhombus linkage relates to its ability to be extended and closed compactly while its diagonals remain perpendicular to each other. Investigating rhombus linkages enables students to understand the widespread applications of rhombuses in terms of their geometric properties.

Images courtesy of JLG Industries (Australia).

Key features of a scissor lift

- Show students images of a scissor lift in the slide show *ST1_Yr8_2a_ScissorLiftImages.pptx*. Scissor lifts are commonly seen around building sites and shopping centres. Ask students to recount where they have seen one in action.
- Discuss what the scissor lift is for and how it works. What are its key features?

Expected Student Responses

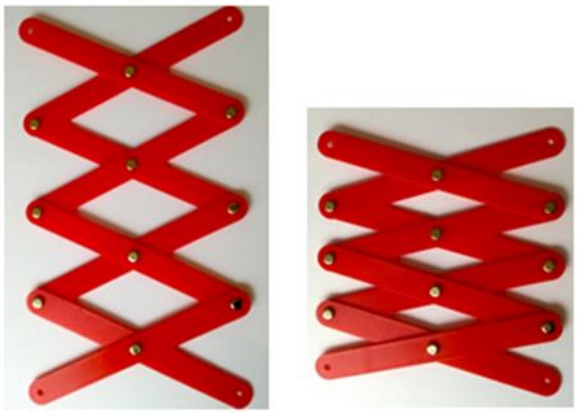
- The scissor lift is designed to move vertically. This is important for safety to ensure that the work platform stays horizontal and the scissor lift does not tip sideways.
- The scissor lift has a stable base so that it doesn't slip. The base must stay horizontal.
- The scissor lift folds up neatly for storage.

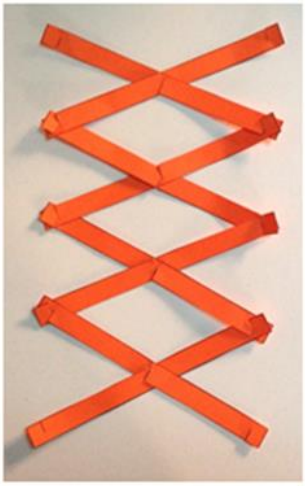
Models of the Scissor Lift

Making a physical model

- Hand out Student Sheet 1 – Scissor Lift
- Show students your own model or the images of the model from the slide show *ST1_Yr8_2a_ScissorLiftImages.pptx*
- Students construct a model of the scissor lift using 8 identical Geo Strips connected with 10 paper fasteners or strips cut from card.
    - The Geo Strip model is easy to assemble and students enjoy the tactile feel of opening and closing the scissor linkage.
    - The card model is more difficult to assemble as some connections are likely to come undone while others are being connected. It is best assembled and moved on a horizontal surface, whereas the more rigid Geo Strip model can be held vertically or placed horizontally.
- As students move the finished model, encourage them to observe the sides, angles and general shape: what things change and what things stay the same? Encourage geometric language. Manipulating the model allows students to see how the rhombus properties govern the scissor lift's operation.

Scissor Lift model made from Geo Strips (2 positions)





Scissor Lift model made from card strips (2 positions)

Expected Student Responses

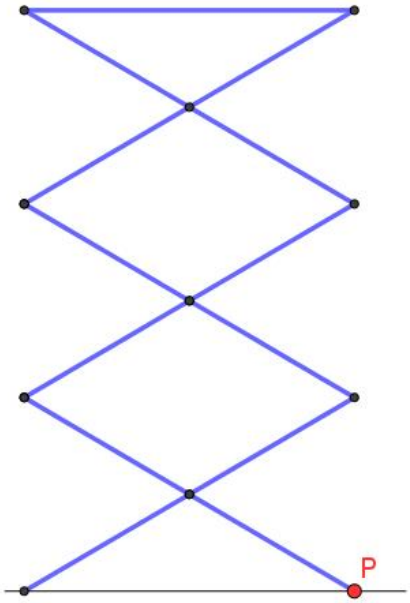
- The scissor lift has two sets of bars and the bars in each set stay parallel.
- The bars are hinged in the middle and form rhombuses.
- The side length of the rhombus is half the length of the bar.
- All the rhombuses stay the same shape as each other.
- There are many equal angles. Some of these are vertically opposite angles.
- As one pair of opposite angles increases in size, the other pair decreases in size.
- The hinged points in the centre of each bar all stay in a vertical line. This is why the lift moves vertically up and down.

Using a dynamic geometry simulation

- Students now use a simulation of the scissor lift, in the GeoGebra file *ST1_Yr8_2b_ScissorLift.ggb*, or use the simulation directly in the browser link https://ggbm.at/kMGAsdTq
- First students should check how the image and the movement relates to real scissors lifts.
- Then, in pairs or in a teacher-led discussion, they explore which properties of the rhombus makes it useful in the scissor lift. Measurement on the screen can be used to check conjectures. The intention here is to make the discussion more geometric than with the physical model.

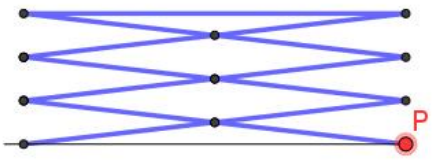
- The scissor lift uses the following geometric properties:
    - diagonals of each rhombus are perpendicular — the work platform moves vertically
    - both pairs of opposite sides are equal — neat, compact storage
    - vertically opposite angles are equal — all the rhombuses always have the same shape
- In pairs, students carefully explain to each other how perpendicular diagonals of each rhombus make the lift move vertically. Explanations are then shared.

Which properties of the rhombus make it useful in the scissor lift? Explain.



Drag point P to the left and right.

Which properties of the rhombus make it useful in the scissor lift? Explain.



Drag point P to the left and right.

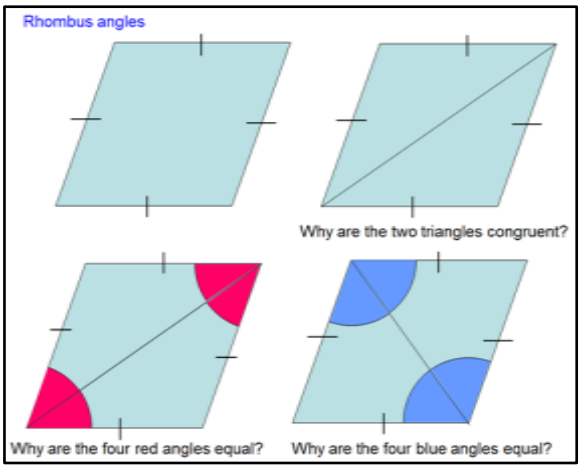
Proving the Diagonals are Perpendicular

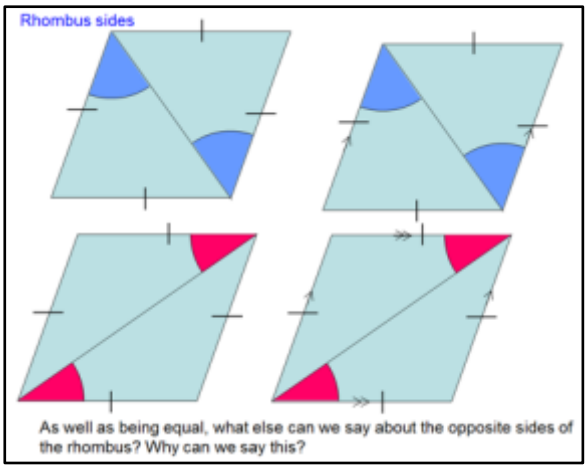
A challenge for students is to prove that the diagonals are indeed perpendicular rather than just appearing to be perpendicular.

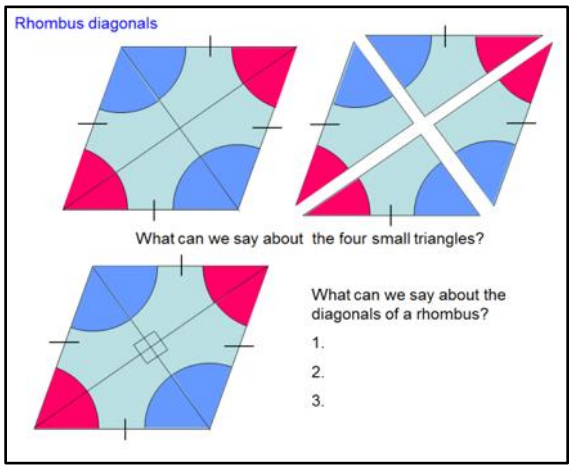
The slide show *ST1_Yr8_1c_RhombusProperties.ppt* leads students visually through the steps in reasoning that the diagonals of a rhombus are perpendicular, as well as other rhombus properties. Starting with the rhombus property that the four sides are equal, students can see that each diagonal divides the rhombus into two congruent triangles. Alternate angles are therefore equal and hence the opposite sides are parallel. Also, the two diagonals divide the rhombus into four congruent triangles. Hence the four angles at the centre must be equal and therefore must be right angles.

For some students, the reasoning could be left here. For other more advanced students, the reasoning could lead to a written proof that the diagonals of a rhombus intersect at right angles. Students may also observe that the diagonals bisect the angles of the rhombus and that the diagonals bisect each other, properties that are not shared by all parallelograms.

Teacher Sheet 1 – Rhombus Properties and Deductive Reasoning provides written solutions. (Note: student slideshow does not have answers.)







Conclusion

The scissor lift design makes use of two key properties of a rhombus:

- The equal sides allow the scissor lift to close compactly.
- The perpendicular diagonals ensure that the work platform moves vertically if the scissor lift is standing on horizontal ground.
- As part of a class discussion, students make a summary of rhombus properties, under three headings:
    - Sides: all sides are equal; both pairs of opposite sides are parallel.
    - Angles: both pairs of opposite angles are equal; adjacent angles add to 180°.
    - Diagonals: diagonals bisect each other at 90°; both diagonals bisect the rhombus angles.

Further Investigation: Other Designs for a Lift

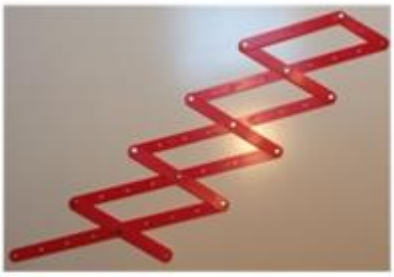
Student independent investigation:

- Could any quadrilateral rather than the rhombus be used for the design of the scissor lift?
  - For example, could a kite be used instead of a rhombus?

Teacher Notes

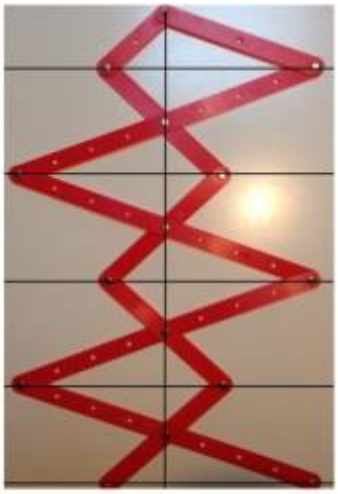
The images below show that a parallelogram linkage would not move up vertically, so this could not be used. However, it moves well, expands in a straight line, and folds down compactly.

The kite linkage would move vertically in the required way due to the perpendicular diagonals of the kite. It would close compactly if assembled as shown below. However, there would seem to be no point in this design for a lift, because it would require longer bars, and the rhombus works perfectly well.



A "lift" made from parallelograms folds well, but does not go up vertically.

A "lift" made from kites folds well and goes up vertically.



Diagonals of a kite intersect at right angles.

Scissor Lift

Name: _____

The scissor lift has a work platform which can be raised to give workers access to heights on construction sites and buildings.

Many inventions are based on four hinged bars that form a rhombus. Before you start this activity, make sure you know what a rhombus is!

- Write down the definition of a rhombus? Draw three examples with different size and shape.

Making a Physical Model

- Make a model of the scissor lift from 8 identical strips (use 10 paper fasteners if required).
- Move the scissor lift linkage to change its shape and observe the sides, angles and general shape. Observe the movement, and write down what things change and what things stay the same.

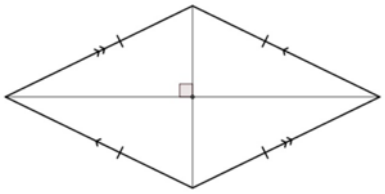
Using a Software Model

- Open the GeoGebra file *Scissor Lift*. This shows a simulation of an actual scissor lift like the one in the picture. Operate the lift, and write down some of the geometric properties that make it work. Check your conjectures by measuring on the simulation.

Rhombus Properties

- Now watch the slide show *Rhombus Properties*.
- How does each diagonal divide the rhombus?
- How do the diagonals intersect each other?
- What other properties of a rhombus are seen in this demonstration?

Summary of Rhombus Properties



List the properties of a rhombus under three headings:

- Sides
- Angles
- Diagonals

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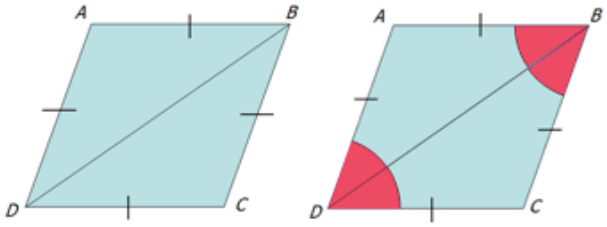
Teacher Sheet 1 - Rhombus Properties and Deductive Reasoning

From definition to proof of properties of a rhombus

A rhombus is defined to be a quadrilateral with four equal sides. Other properties may seem obvious to students, who may question the need for proof. Students may say, for example, that the opposite sides are parallel simply because they look parallel. However, the diagrams in the slide show *ST1_Yr8_1c_RhombusProperties.ppt* lead students through a process of reasoning to explain *why* properties such as the parallel opposite sides are always true.

The reasoning can be based on the visual demonstration in the slides, or can be made more formal by introducing letters for each vertex.

Opposite Sides of a Rhombus are Parallel



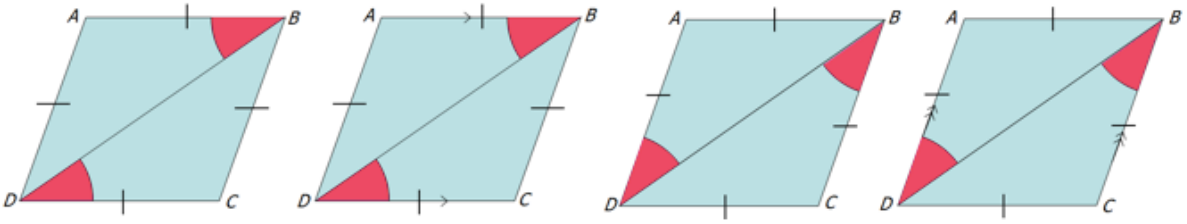
Diagonal BD divides the rhombus into two congruent isosceles triangles, ABD and CDB:

In ΔABD and ΔCDB

$AB = CD = AD = BC$

BD is common to both triangles

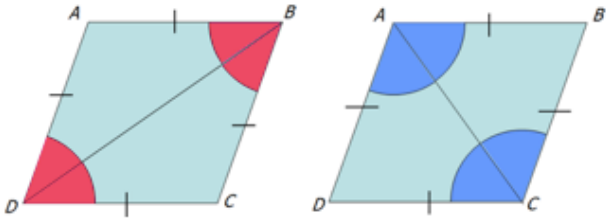
So $\Delta ABD \equiv \Delta CDB$ (S.S.S.)



$AB \parallel DC$ (alternate angles are equal)

$AD \parallel BC$ (alternate angles are equal)

Opposite Angles of a Rhombus are Equal



The four red angles are equal (isosceles congruent triangles)

So $\angle ABD + \angle CBD = \angle ADB + \angle CDB$

So $\angle ABC = \angle CDA$

Similarly, the four blue angles are equal (isosceles congruent triangles)

So $\angle BAD = \angle DCB$

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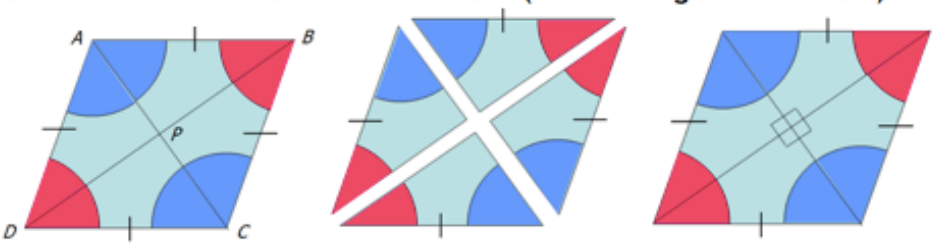
The Diagonals of a Rhombus are Perpendicular

The two diagonals divide the rhombus into four triangles.

Each of the triangles has two angles and the included side equal to two angles and the included side of the other three triangles, so all four triangles are congruent.

So $\angle APD = \angle BPC = \angle CPD = \angle DPA = 90°$ (the four angles add to 360°)



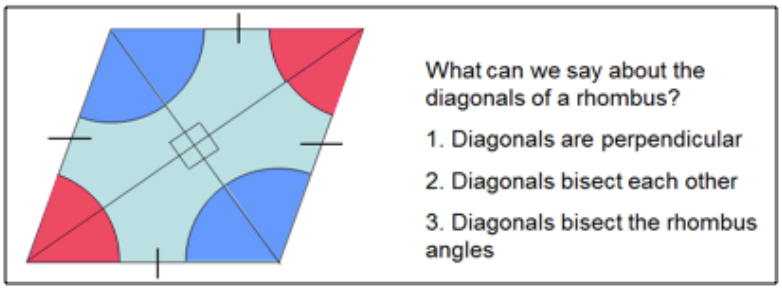
The diagonals of a rhombus bisect each other:

$AP = CP$ and $BP = DP$ (congruent triangles)

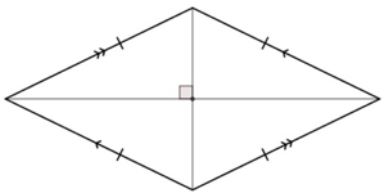
The Diagonals of a Rhombus Bisect the Angles of the Rhombus

$\angle ABD = \angle CBD = \angle ADB = \angle CDB$

Similarly,
$\angle DAC = \angle BAC = \angle BCA = \angle DCA$



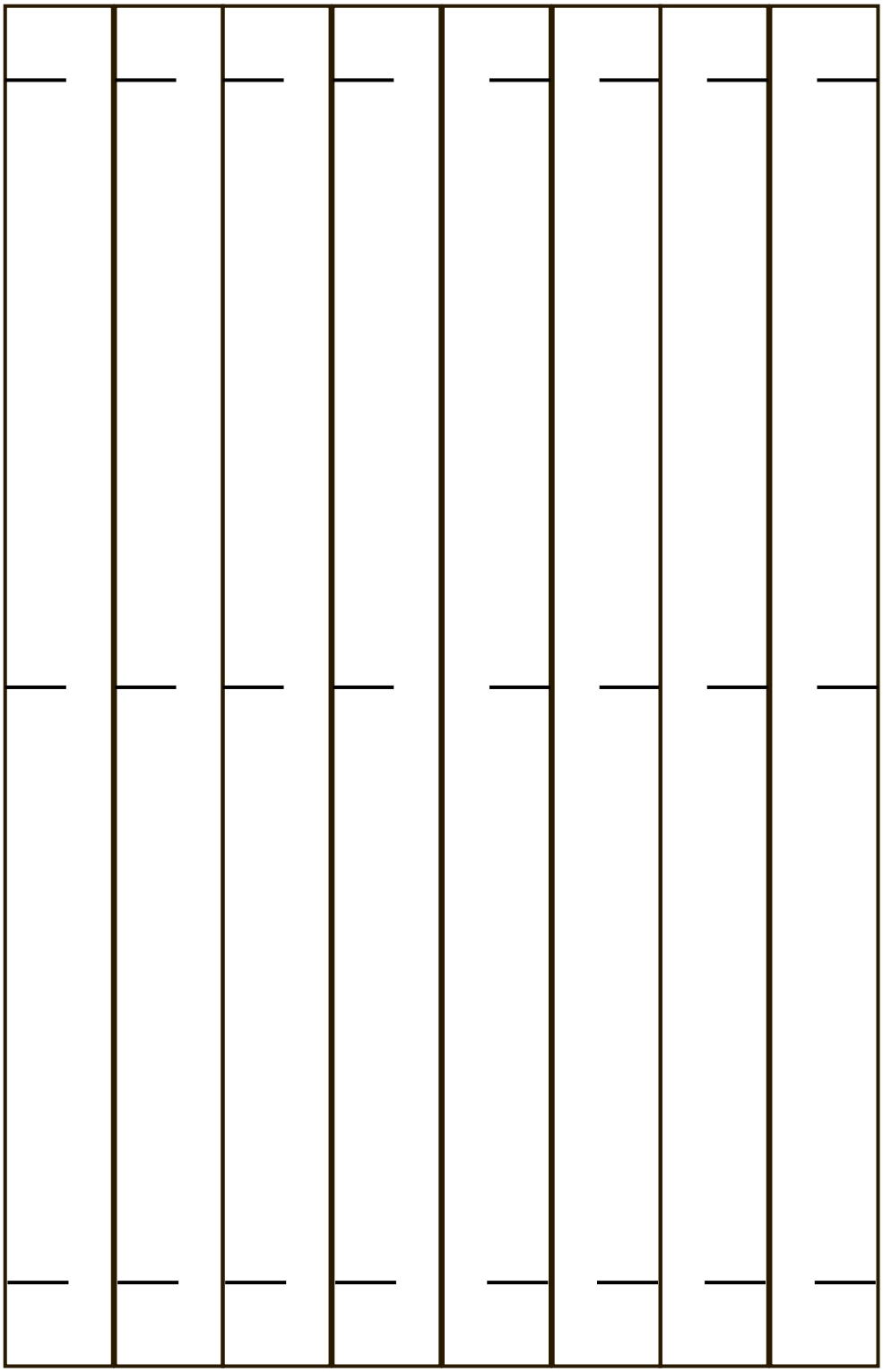
Summary of Rhombus Properties



- Sides — Four equal sides; opposite sides are parallel
- Angles — Opposite angles are equal; adjacent angles are supplementary
- Diagonals — Intersect at right angles; bisect each other; bisect the angles of the rhombus

Scissor lift strips template

Copy onto card, e.g. 140 gsm.

The template here is for one set of scissor lift links.

Cut out around the large rectangle shape, then cut down all the vertical lines to make 8 identical strips. Carefully cut along the short slit lines.