BRINGING THE REAL WORLD INTO ALGEBRA

Exponential Functions
Lesson 1: Exponentials and Ammonites

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and *exponentials* using digital technology as appropriate
Elaboration: sketching the graphs of exponential functions using transformations

Lesson abstract

Students use geometry and graphing technology to explore the relationships in the spacing of curves in the spiral of an ammonite fossil. They measure from an image, create tables of values, plot points, and fit functions, in the process exploring the roles of parameters in exponential functions.

Mathematical purpose (for students)

Exponential relationships exist in nature. Function rules can be used to describe the graph of such relationships.

Mathematical purpose (for teachers)

- Build conceptual schema linking visual, numeric, graphic and algebraic representations of an exponential function.
- Appreciate that exponential relationships exist in the real world.
- Identify the graphical shape of an exponential function.
- Describe the shape of the graph of an exponential function.
- Compare the graph of an exponential function with linear and quadratic functions.
- Explore the role of each parameter and variable in an exponential function rule in affecting the position (translation) steepness (dilation) and orientation (reflection) of its corresponding graph.

Lesson Length 50 minutes approximately

Vocabulary Encountered

- exponential
- ammonite
- parameters

Lesson Materials

- Student Sheet 1 - Exponentials and Ammonites (1 per 2 students).
- Ammonite image *ST2_Exponential_1a_Ammonite.jpg* (for all students)
- Other images (e.g. *ST2_Exponential_1b_Newgrange.jpg*) optional
- Access to Geogebra or equivalent software for all students.

We value your feedback after these lessons via https://www.surveymonkey.com/r/RKRDYBW

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Investigating Spirals

Hand out Student Sheet 1 - Exponentials and Ammonites. This leads students through the Geogebra steps to fit an exponential curve to data from a spiral on an ammonite. Students will:

- insert an image in a Geogebra file,
- measure successive widths of spiral sections and/or width of spiral from the spiral centre,
- create a table of values and plot points, by entering the coordinates of the points or linking to the spreadsheet,
- find values of parameters *a, b, c, d* in an exponential function of the form $y = a + be^{(cx+d)}$ or $f(x) = a + be^{(cx+d)}$ to fit the spiral,
- systematically explore variation in the parameters of an exponential function and record notes about the impact of the change in parameter on the shape, position or orientation of the graph,
- apply this knowledge to investigate other spirals (optional).

Teacher Notes

- Graphing technology other than Geogebra could be used for this activity, but the instructions provided are for Geogebra.
- Discuss with the students the fact that the units of the distance measurements do not actually matter in this example because we are considering the relative distances. Students can record the lengths as 'y' units.
- The table of values for the width of the spiral, in sections or from the centre could be entered into Geogebra's spreadsheet and linked to the graph window as a series of points. The sections should be numbered 1, 2, 3 etc from the centre out. These numbers form the *x* co-ordinates.
- Sliders for the parameters could be used to assist this exploration for a function to fit the curve formed by the points.

Images supplied

Ammonite
ST2_Exponential_1a_Ammonite.jpg
Spirals occurred naturally as these sea creatures grew.

Newgrange, Ireland
ST2_Exponential_1b_Newgrange.jpg
Spirals carved on stones in megalithic tomb.

Conclusion

- Class discussion on what has been learned about the impact on the graph of function of variation in each parameter of $f(x) = a + be^{(cx+d)}$
  - changing *a* moves graph up and down,
  - changing *b* and changing *c* both influence when the graph begins to rise very quickly, but in different ways,
  - changing *d* has a similar effect to changing *b* but to a different extent for a given amount of change.
- Compare exponential functions and quadratic functions. What is the same? What is different?

Extending prompt

Are all the parameters *a, b, c, d* necessary? (Ans: The laws of logarithms show that only one of *b* and *d* is required to give all exponential functions in this category. Why?)

More about Spirals

- This spiral is called a logarithmic spiral – ask students to find out why and report back.
  - Follow this up with discussion of images of various spirals. Which ones have features that could be modelled using exponential functions and which ones do not? How can you tell?
- Examine the spiral in a spider's web.
- Examine the spirals from the megalithic tomb at Newgrange, Ireland.
  - Photo supplied *ST2_Exponential_1b_Newgrange.jpg*.
- Students could search online for further images of spirals, which could be inserted into Geogebra and the properties of the spiral analysed. Online starting points for spirals give details about angles and polar coordinates however there are simple images and some descriptions of examples that could be discussed in a less sophisticated way:
  - Logarithmic spiral https://en.wikipedia.org/wiki/Logarithmic_spiral
  - Linear or Archimedean spirals https://en.wikipedia.org/wiki/Archimedean_spiral

Exponentials and Ammonites

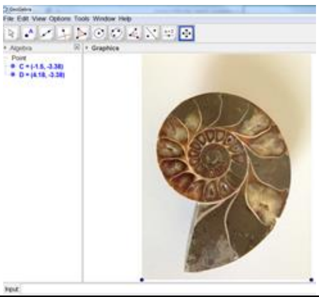
Name: ____________

This ammonite fossil is more than 60 million years old. These creatures lived in the sea and died out about the same time as dinosaurs.

When the fossilised shell was carefully split and then polished, beautiful spirals were revealed.

Your task is to use Geogebra to help you explore spatial properties of this spiral.

To make distances and measurements easier to see turn off the axes and grid in the graphics window. This may be done using the **View, Layout, Graphic-preferences** menu

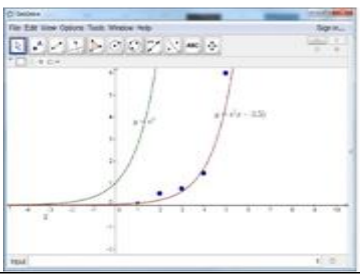


Insert the image of ammonite by Using Edit, Insert image and choosing the photo from the files supplied. Zoom in to make it easier to see the detail of the image.

- Place a line segment to the outer edge of the ammonite anywhere across the spiral.
- Place points along the line each time it crosses the spiral.
- Number your segments, starting at 1 in the centre
- Use Geogebra to measure the width of each segment.

Make a table of values

Segment number | Width



Open a new Geogebra graph window with the axis and grid showing.
Use the input line to enter each ordered pair as a point e.g. something like A=(1,0.2)

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Find the rule for an exponential function that describes the width attribute of the spiral by using trial and error to systematically change each of the parameters *a, b, c, d* in $y = a + be^{(cx+d)}$.

Keep a list of your findings about the role of *a, b, c, d* in shifting and shaping the graph of your exponential function.

Rule entered	**Parameter changed e.g. increased "a"**	**Result on orientation, position or shape of the graph**

Describe the resulting graph. These questions will help you:

- Is the new curve steeper or flatter than $y = e^x$?
- Is the new curve translated up or down relative to $y = e^x$?
- Is the new curve translated left or right (in the negative or positive direction) relative to $y = e^x$?
- Has the curve been reflected in the *x* or *y* axes?

Repeat the measurement process, this time measuring the length of the line from the centre to the edge of each rotation in the spiral.

- The table of values for the width of the spiral, in sections or from the centre could be entered into Geogebra's spreadsheet and linked to the graph window as a list of points. The sections should be numbered 1, 2, 3 etc from the centre out. These numbers form the *x* dimension of the coordinates.
- Sliders for each of *a, b, c, d* could be used to assist this exploration for a function to fit the curve formed by the points.
- Rules for exponential functions with a base other than *e* could also be explored.

You could place a different line across the spiral and repeat one of these measurement processes.