BRINGING THE REAL WORLD INTO ALGEBRA

Exponential Functions
Lesson 2: Modelling World Energy Production

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and *exponentials* using digital technology as appropriate

Elaboration: sketching the graphs of exponential functions using transformations

ACMSP252: Investigate and describe bivariate numerical data where the independent variable is time.

Lesson abstract

This lesson uses technology to model trends in world energy data (oil and wind). Students can explore the fit (by eye) of exponential functions to a large part of the data series then see that a trend may not continue forever. In this task students consider what is and what is not exponential growth. Students link graphic and algebraic representations and interpret the function rules developed in terms of their meaning as models of real world energy production.

Mathematical purpose (for students)

Exponential functions may model real world data. The pattern of data that could be well fitted by a particular mathematical model may change. Be cautious in prediction!

Mathematical purpose (for teachers)

- Build conceptual schema linking visual, numeric, graphic and algebraic representations of exponential functions.
- Appreciate that exponential relationships exist in the real world.
- Identify the graphical shape of an exponential function.
- Explore the role of each parameter and variable in an exponential function rule in affecting the position (translation) steepness (dilation) and orientation (reflection) of its corresponding graph.
- Recognise that a model may fit well to part of a data set but the pattern may change with time.

Lesson Length 50 minutes approximately

Vocabulary Encountered

- exponential
- parameter

Lesson Materials

- Student Sheet 1 - Modelling World Energy Data (1 per student)
- GeoGebra files: *ST2_Exponential_2a_Wind.ggb* and *ST2_Exponential_2b_Oil.ggb* (accessible to each pair of students)
- GeoGebra files: *ST2_Exponential_2c_Wind_teacher.ggb* and *ST2_Exponential_2d_Oil_teacher.ggb*

We value your feedback after these lessons via https://www.surveymonkey.com/r/RKRDYBW

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Modelling Wind Energy and Oil Production Data

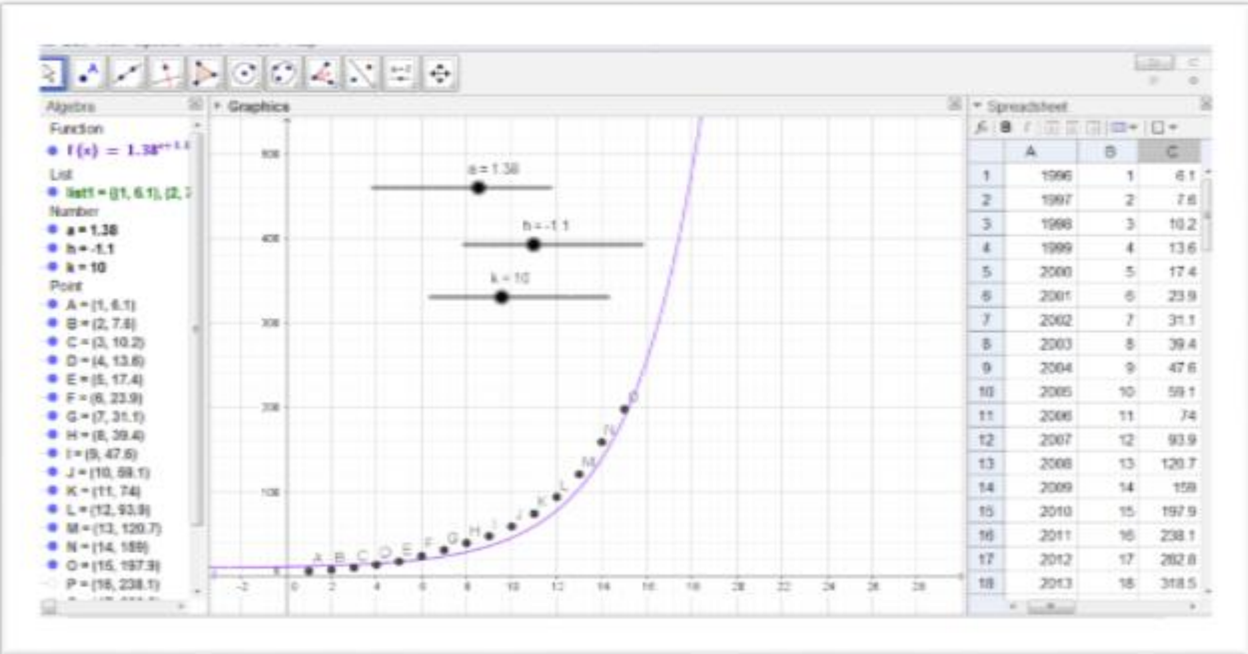
Hand out Student Sheet 1 - Modelling World Energy Data. Ensure that students (perhaps in pairs) have access to the GeoGebra files *ST2_Exponential_2a_Wind.ggb* for the first section and *ST2_Exponential_2b_Oil.ggb* for the second section.

This student sheet leads students to look at trends in the installed capacity of wind power generation around the world from 1996 to 2015, and then trends in oil production from 1880 to 1996.

- Students use GeoGebra to find the best exponential model that will 'fit' the data supplied. Best fit in this case is done "by eye" with an attempt to minimise the overall error. Some points will lie above the curve, some on the curve and some below the curve.
- The function $y=a^{(x-h)}+k$ has been entered into the GeoGebra file, and sliders have been created for each of the parameters *a*, *h* and *k*. Sliders are used to adjust the model and students should keep notes on the changes they make to each parameter and the impact that has on the curve.
- Once students have their 'best' attempt this should be tested by checking a couple of known points.
- Not all data points are visible in the first instance. The later values should be activated and the fit of the model to these more recent values discussed. Showing the points involves clicking in the open circle alongside the point names (P, Q, R, S, T) in the algebra window. When the point shows, the open circle is coloured in.

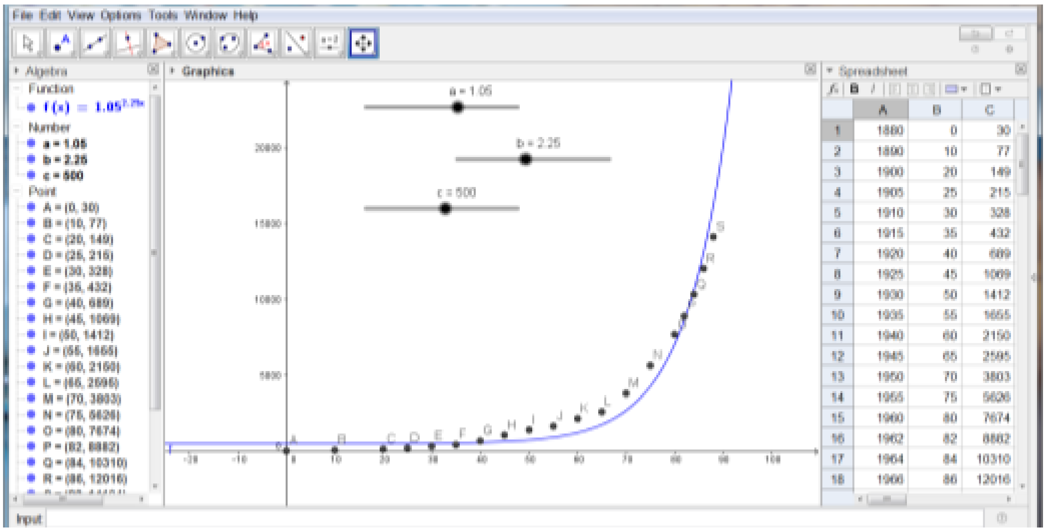
Wind Power Model

The GeoGebra file *ST2_Exponential_2c_Wind_teacher.ggb* has been supplied for teachers. We can see that an exponential function $f(x) = 1.38\text{^}(x + 1.1) + 10$ provides a reasonable model for 15 years of data but unhiding the rest of the data points (by clicking on them in the algebra window) shows that this pattern did not continue.



Oil Power Model

The GeoGebra file *ST2_Exponential_2d_Oil_teacher.ggb* has been supplied for teachers. The exponential function $f(x) = 1.05\text{^}(x - 2.25) + 500$ provides a reasonable (not perfect) model for 100 years of data but unhiding the rest of the data points (by clicking on them in the algebra window) shows that this pattern did not continue.



Conclusion

Share reflections on students work, focussing on

- The role of each parameter *f(x)* $=a^{(x-h)}+k$ and its impact on the graph of an exponential function.
- Mathematical models - their value and their limitations.
- The features of exponential growth contrasted with growth that is not exponential. An important feature of exponential growth is that a constant increase in the independent variable (e.g. time) results in a constant *percentage* increase in the dependent variable (e.g. oil production).

Extension

- Look at the algebra of logarithmic functions by comparing the function used in this lesson *f(x)* $=a^{(x-h)}+k$ and the function used in Lesson 2 of this unit $f(x) = a + be^{(cx+d)}$.
  - Is there any significance in writing the addition before or after the exponential (ANS: No)
  - One function has a constant multiplier b and the other does not. What difference does this make? (ANS: It makes no difference to the range of functions that are represented.)
  - One function has the fixed number *e* and the other has a variable parameter (number) in that place. Why? (ANS: *a* = *e*^c)
  - One function has a negative sign, the other a positive sign. What difference does that make?
- How can you tell what the constant percentage increase is from the algebra formula?

Data Sources

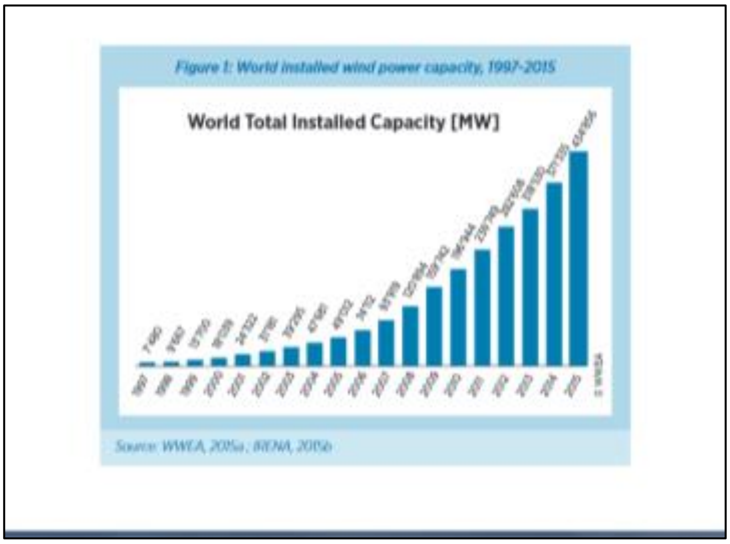
Data for global installed wind power capacity

Data and charts on world wind energy capacity were retrieved from

- https://www.gwec.net/wp-content/uploads/vip/GWEC-PRstats-2015_LR.pdf
- http://www.gwec.net/global-figures/graphs/

The chart used for the students data is from

http://www.irena.org/DocumentDownloads/Publications/IRENA-ETSAP_Tech_Brief_Wind_Power_E07.pdf



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Data for Oil Production

Data or charts on Annual world crude oil production were available from Data And Story Library (DASL):

- http://lib.stat.cmu.edu/DASL/Stories/Oilproduction.html
- http://peakoilbarrel.com/world-oil-yearly-production-charts/
- https://www.indexmundi.com/energy/?product=oil&graph=production

The data entered into Geogebra for the students to use came from:

Chatterjee, S. & Hadi, A.S. (2015). *Regression Analysis by Example*. P189 Online: John Wiley & Sons.

Table 6.19 Annual World Crude Oil Production in Millions of Barrels (1880–1988)

Year	OIL	Year	OIL	Year	OIL
1880	30	1940	2,150	1972	18,584
1890	77	1945	2,595	1974	20,389
1900	149	1950	3,803	1976	20,188
1905	215	1955	5,626	1978	21,922
1910	328	1960	7,674	1980	21,722
1915	432	1962	8,882	1982	19,411
1920	689	1964	10,310	1984	19,837
1925	1,069	1966	12,016	1986	20,246
1930	1,412	1968	14,104	1988	21,338
1935	1,655	1970	16,690		

re(Solve) Modelling World Energy Data

Name: ______________

Your task is to explore world oil production and world wind energy installed capacity using exponential functions to model some of the data and so make predictions. There are sophisticated statistical techniques to create models to fit data so that the overall difference between the model's predictions and the actual value at any data point are minimised. However, we will fit a model 'by eye', one that looks close to the points.

World oil production is measured in millions of barrels of oil produced per year. The installed wind energy capacity is measured in Gigawatts. A Gigawatt is a billion watts. A normal LED lamp consumes about 10 watts.

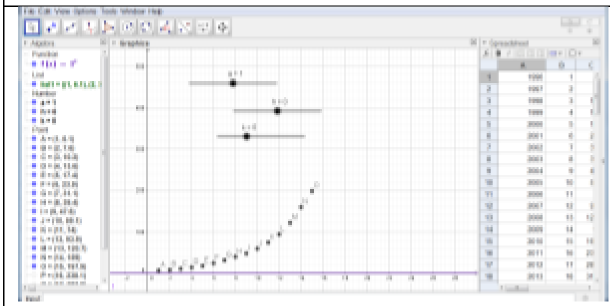
Wind Power

A Geogebra file has been provided for you. The data, taken from IRENA-ETSAP_Tech_Brief_Wind_Power_E07.pdf has been entered into the spread sheet. This data has been linked as a list of points to the Algebra window and plotted in the Graph window.

Open the file ST2_Exponential_2a_Wind_Power.ggb using Geogebra. This shows the total global wind power capacity from 1996 to 2015. Your teacher will tell you where to access this file.

A general exponential function rule $y=a^{(x-h)}+k$ has been entered via the input line and sliders have been created for each of the parameters a, h and k.

When you open the file you will see these sitting at 1, 0 and 0 ready for you to start investigating.



Use the sliders to systematically vary the parameters in the function rule $y=a^{(x-h)}+k$ and find a curve that is a good fit for the pattern shown by the points.

Note the effect that each change of parameter has on the graph.

Summarise your findings about the impact on the graph of changing each of the parameters, a, h and k.

Changing a

Changing h

Changing k

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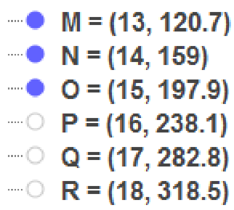
Check your model: Pick a year and note its number in the sequence (a number between 1 and 15). Calculate an estimate of installed wind capacity based on your exponential model (i.e. use your best values of *a*, *h* and *k*.

Compare your result with the data provided in the spreadsheet.

The data plotted goes up to number 15 (i.e. the year 2010). What does your model predict will be the values for 2012, 2015? Plot these points.

Check what did happen in those years by 'unhiding' the rest of the points entered in the algebra window by clicking on the open dot next to each one.



Is your exponential function a good fit for all of the data? Describe and sketch the growth from 2010-2015.

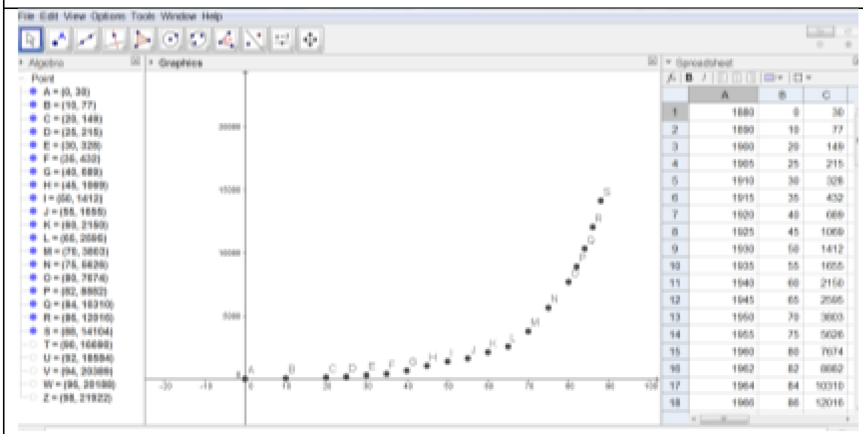
Oil Production

A Geogebra file has been provided for you. Data has been entered into the Geogebra spread sheet for you. This data is available from: Chatterjee, S. & Hadi, A.S. (2015). *Regression Analysis by Example*. P189 Online: John Wiley & Sons.

This data has been linked as a list of points to the Algebra window and plotted in the Graph window.

Open the file ST2_Exponential_2b_Oil_Production.ggb using Geogebra. This shows the annual world crude oil production in millions of barrels for the years 1880 to 1998.

A general exponential function rule $y=a^{(x-h)}+k$ has been entered via the input line and sliders have been created for each of parameters *a*, *h* and *k*. When you open the file you will see these sitting at 1, 0 and 0 ready for you to start investigating.



Use the sliders to systematically vary the parameters in the function rule $y=a^{(x-h)}+k$ and find a curve that is a good fit for the pattern shown by the points.

Note the effect that each change of parameter has on the graph.

Summarise you findings about the impact on the graph of this function of changing each of *a*, *h* and *k*.

Changing *a*

Changing *h*

Changing *k*

Check your model: Pick a year and note its number from the spreadsheet. Calculate an estimate of installed wind capacity based on your exponential model. Compare your result with the data provided in the spreadsheet.

The data plotted goes up to number 88 – that is the year 1968. What does your model predict will be the values for 1980, 1988? Plot these points.

Check what did happen in those years by 'unhiding' the rest of the points entered in the algebra window by clicking on the open dot next to each one.

Is your exponential function a "good fit" for all of the data? Describe and sketch the growth from 1970-1988.