Unit Overview: Exponential Functions

Summary of learning goals

As part of the Special Topic **Bringing the Real World into Algebra** this unit looks at exponential functions as seen in the real world, linking the representations of a function rule, a graph or a table of values.

Students will have the opportunity to consider real world objects and data that can be described using exponential functions. Through technology-assisted exploration these examples will become models for understanding transformations, reflections and stretches to graphs of exponential functions and how these are reflected in the parameters in the function rules. Students will also consider examples which do not show an exponential pattern or where the pattern changes, and so discuss models and their limitations.

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representation in relation to simple quadratics … using digital technology as appropriate

- Sketching graphs of exponentials
- Applying transformations to exponentials

Summary of lessons

Who is this unit for?

One or both of these lessons could be used at Year 10 (or 10A) to enhance a standard approach to teaching exponentials. The lessons could be to introduce the topic or at the culmination. The two lessons do no need to be used sequentially and either could be used for revision in Year 10 or Year 11. The lessons may introduce or consolidate students' conceptual links between various representations of exponential functions, using contexts other than the very common population growth or decay. The lessons use different algebraic forms of exponential functions, which may be compared if both are undertaken.

Basic familiarity with Geogebra software is assumed in these lessons. Students could build up the necessary skills by taking the first lesson from the Quadratic Functions unit of the Special Topic **Bringing the Real World into Algebra.**

The lessons can be used with other graphing and geometry software, but these materials only give instructions for Geogebra https://www.geogebra.org/. Lesson 1 requires more skill with Geogebra than Lesson 2.

Lesson 1: Exponentials and Ammonites

Students use geometry and graphing technology to explore the relationships in the spacing of curves in the spiral of an ammonite fossil. They measure from an image, create tables of values, plot points, and fit functions, in the process exploring the roles of parameters in exponential functions.

Lesson 2: Modelling World Energy Production

This lesson uses technology to model the trend in world energy data (oil and wind). Students can explore the fit (by eye) of exponential functions to a large part of the data series then see that a trend may not continue for ever. In this task students consider what is and what is not exponential growth. Students link graphic and algebraic representations and interpret the functions rules developed in terms of their meaning as models of real world energy production.

We value your feedback after these lessons via https://www.surveymonkey.com/r/RKRDYBW

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Reflection on this sequence

Rationale

In teaching or revising exponential functions by starting in the real world we aim to link visual, graphical, symbolic and numerical representations of functions in a way that will help students build strong and memorable conceptual schema. Using digital images to link to the real world can engage students, especially if the photos and data can have some meaning for them. Then, with the aid of technology, the tasks of exploring links between the parameters in a function rule and transformations of a graph can be made accessible to students across a wide range of mathematical ability.

If we can help our students understand and engage with the algebra and graphs of exponential functions through even a few real examples then it is a small step to help them see that the same mathematical principles might be applied to use algebra to model far more complicated real world shapes and paths.

Exponentials are typically taught with an abstract introduction and lead up to looking at applications related to population growth and radio-active decay. These lessons provide variety of context. The first lesson explores the identification of mathematical patterns in nature thus showing a relevance of mathematics to art and design. A second lesson demonstrates the role of mathematics in describing data that alerts us to problems and helps to inform forward planning. While working on real contexts students practice and gain experience with linking the numeric, graphic and algebraic representations of exponential function.

All units in the Special Topic **Bringing the Real World into Algebra** are guided by the principles of Realistic Mathematics Education (RME). As a consequence this unit:

- Uses realistic situations to develop mathematics,
- Places less emphasis on memorising and more on describing patterns and creating function rules,
- Place more emphasis on sense making,
- Uses 'guided reinvention' of mathematical ideas and concepts.

reSolve Mathematics is Purposeful

Students commonly wonder why the curriculum requires them to consider different representations of exponentials and what purpose this can have in the real world. These lessons focused on exponential functions aim to provide experiences for students that help them to recognise the value of graphs, tables and algebraic rules in providing useful information for describing real world phenomena.

While some aspect of the tasks are artificial, experience has shown that students enjoy the challenge of curve fitting and so are motivated to pay attention to the impact of varying each parameter of the exponential function rule. The modelling by-eye will be inaccurate but will introduce students to the value of inspecting data and recognising whether a linear, quadratic or exponential model will be helpful.

reSolve Tasks are Challenging Yet Accessible

reSolve contests a view that some students can 'do mathematics' and others cannot. By working with appropriate software all students are able to participate in the activities of measuring and modelling and learn through the experience of guided experimentation. Activities can be scaffolded to different degrees for different students.

Accessibility to the concepts encapsulated by exponential functions arises from the use of the real world situation to develop a firm mental model. Transition from relying fully on the real world situation, to using the somewhat more abstract graphs then algebra occurs at a pace set by the student. The use of several real world situations throughout the lessons provides an opportunity to reinforce the new ideas, as well as extend them.

reSolve Classrooms Have a Knowledge Building Culture

This unit builds knowledge through active exploration of different representations of exponentials and in particular requires students to work to achieve the best models they can by varying the parameters of the exponential function rule. The process of working through the process of strategic trial and error requires students to keep trying and to reflect, at each stage, on the impact each change they make to the algebraic rule has on the graphic representation. The use of software means that errors may be quickly corrected or even erased. There need be no embarrassment!

Further Reading

Mathematics education in the Netherlands: A guided tour