

Introduction to Bar Models

Lesson 1: Addition of Whole Numbers

Australian Curriculum: Mathematics

ACMNA015: Represent and solve simple addition problems using a range of strategies including counting on, partitioning and rearranging parts (Year 1).

ACMNA030: Solve simple addition problems using a range of efficient mental and written strategies (Year 2).

ACMNA291: Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (Year 5).

Lesson abstract

Students are introduced to the bar model method with the part-whole bar model through this lesson. Students learn how to construct the part-whole bar model and how it can help to represent addition contexts through worked examples with simple numbers and tasks with larger numbers. Three types of addition contexts are used: (1) one set of items made up of 2 parts; (2) the whole being formed from two distinct sets of items; and (3) another set of items “adding on” or “joining” to form a new whole.

Mathematical purpose (for students)

A bar model helpfully displays the information in word problems.

Mathematical purpose (for teachers)

This lesson introduces the use of bar models in different ‘part-whole’ problem contexts involving the use of whole numbers. Two types of bar models are used to give a diagrammatic representation of the mathematical relationships in word problems, to facilitate the students’ mathematical understanding. Straightforward examples are used to introduce the key points of constructing part-whole bar models, with larger whole numbers used in the tasks that students work on.

This lesson (and subsequent) could be used with much younger children. Teachers can provide concrete materials (e.g. multilink cubes), to help younger students relate to the pictorial bar model as required. The consolidating tasks would need to be simplified as appropriate.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- part-whole model
- mathematical statements

Lesson Materials

- Slide show *ST4_BarModelIntro_1a_Add.pptx*
- [Student Sheet 1 - Bar Model Examples 1A](#) (1 per student)
- [Student Sheet 2 - Bar Model Examples 1B](#) (1 per student)
- Multilink cubes (optional)
- Calculators (optional)

We value your feedback after these lessons via <https://www.surveymonkey.com/r/G6VGPZ8>



Bar Models For Part-Whole Problems

This lesson introduces bar models depicting the part-whole model in straightforward contexts involving the addition of whole numbers. The first examples are very simple, aiming to show how to draw the models.

The examples and animated solutions are contained in the slide show *ST4_BarModelIntro_1a_Add.pptx*, which can be used during initial instruction and class discussion.

Hand out [Student Sheet 1 - Bar Model Examples 1A](#). Students should write and draw the solutions for future reference.

Begin the lesson by explaining that the aim of these lessons is to introduce students to a method that will help them solve problems of many different types. Throughout the examples, emphasise how the part-whole models demonstrated in this lesson help students to visualise the relationships between the parts and the whole.

Example 1

Read Example 1 with the class and discusses how to draw and label the model, using the slide show *ST4_BarModelIntro_1a_Add.pptx* as a prompt.

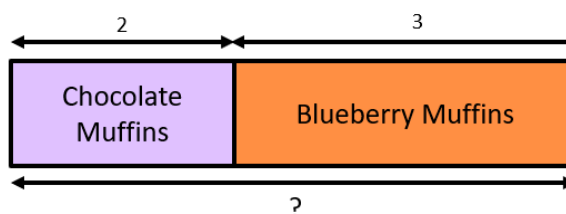
Sally has 2 chocolate muffins and 3 blueberry muffins.
How many muffins does she have altogether?

Students at lower year levels often use interlocking cubes to represent the muffins.

Sample Solution

$$2 + 3 = \underline{5}$$

Sally has 5 muffins altogether.



Example 2

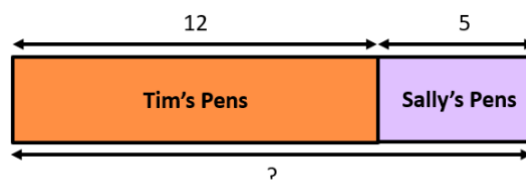
Students could try this simple example for themselves before the teacher discusses the solution. Use the slide show *ST4_BarModelIntro_1a_Add.pptx* as a prompt, and highlight how the bar model can help students come up with the mathematical statements.

Tim has 12 pens.
Sally has 5 pens.
How many pens do they have between them?

Sample Solution

$$12 + 5 = \underline{17}$$

Tim and Sally have 17 pens altogether.



Example 3

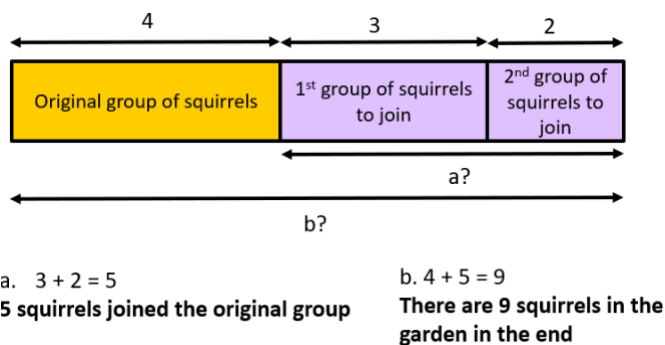
Students could try this example for themselves before the teacher discusses the solution with the group. Notice that different parts of the bar model answer the different parts of the question.

There are 4 squirrels in the garden.

3 squirrels joined them. After a while, 2 more squirrels joined the group.

- How many squirrels joined the original group of 4 squirrels?
- In the end, how many squirrels are there in the garden?

Sample Solution



Consolidating and Concluding

Further Practice

Hand out [Student Sheet 2 - Bar Model Examples 1B](#). Students work through selected tasks individually, in pairs or in groups.

The problems in the tasks use larger numbers than those in the examples, which will require students to use more complex addition strategies in their calculations. Some teachers may prefer to make the numbers smaller.

Discuss solutions as time permits. Worked solutions and some teaching notes are provided in [Teacher Sheet - Bar Model Solutions 1B](#). Solutions to Tasks 1 to 4 are included in the slide show *ST4_BarModelIntro_1a_Whole.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- Bar models are used as a way to help organise the information in a problem into a diagram, to help us see how to solve it.
- The length of the bars represent the size of the quantities in the problem, but they do not need to be drawn exactly in proportion.
- There is no exactly right way of drawing bars - the aim is to draw a model that helps you to solve the problem.
- The part-whole bar model can involve the use of many parts (more than two parts) within the whole; some of the parts can be equal in quantity.
- In all the problems in this lesson, a 'whole' is the unknown to be found
- In the examples discussed, we used addition to find the unknown with the help of the part-whole model, because we could see the additive relationship between the various parts and the whole.
- In other word problems, the "parts" can be the unknowns to be found.
- The bar model can be drawn horizontally or vertically.
- The bar models can be extended or changed as the question or solution moves along.

Example 1

Sally has 2 chocolate muffins and 3 blueberry muffins.

How many muffins does she have altogether?

Example 2

Tim has 12 pens.

Sally has 5 pens.

How many pens do they have between them?

Example 3

There are 4 squirrels in the garden.

3 squirrels join them. After a while, 2 more squirrels joined the group.

- How many squirrels joined the original group of 4 squirrels?
- In the end, how many squirrels are there in the garden?

Draw bar models to represent the situations below and use them to solve the problems.

Task 1

John works as a miner. Out of his salary for April, John spent \$645 on food, \$308 on transport, \$467 on insurance and saved \$1224.

- How much did John spend on food and insurance in April?
- What was John's salary in April?
- John's salary in May was increased because he received a bonus of \$369 for his hard work. What was his salary in May?

Task 2

The manager of a hardware store is doing a stock take.

He found there are 1276 screws, 3258 bolts, 7352 nails and 2359 rivets in his store.

- How many bolts and rivets are there?
- How many screws and nails are there?
- How many fasteners are there in stock altogether?

Task 3

20 634 adults and 2158 children were waiting for a music festival to start at Melbourne Stadium. Just before the event started, 285 children, 537 teenagers and 3439 adults joined them.

- How many adults were at the music festival?
- How many children were at the music festival?
- What was the total number of people attending the music festival?

Task 4

Some workers are doing landscaping in the botanical gardens.

6502 pots of *Lilies*, 3728 pots of *Anemones*, 5036 pots of *Bird of Paradise* and 9124 pots of *Banksia* are first planted.

Another 5269 pots of *Lilies* and 1184 pots of *Bird of Paradise* are added to improve the design.

- a. How many pots of *Lilies* are at the gardens?
- b. How many pots of *Bird of Paradise* are in the gardens?
- c. What is the total number of pots of *Lilies*, *Anemones*, *Bird of Paradise* and *Banksia* in the gardens?

Task 5

Jane works as a secretary. Out of her salary one month, she spent \$457 on grocery, \$1495 on her home loan, \$570 on transport and saved \$978.

- a. How much did Jane spend on groceries and transport?
- b. What was Jane's monthly salary?
- c. Jane was promoted and will be given an increment of \$800 her salary next month. What will be her salary next month?

Task 6

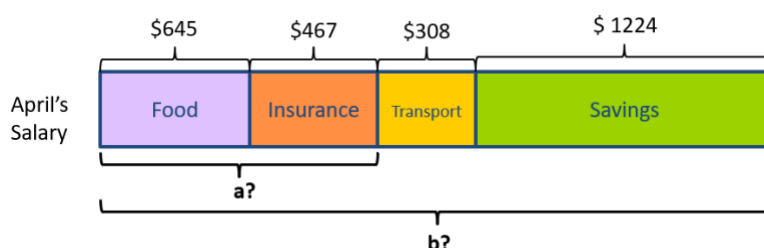
There are different types of books in the library. There are 7345 fiction books, 2450 non-fiction books, 5429 children's books and 109 magazines.

- a. How many fiction books and magazines are there?
- b. How many non-fiction and children's books are there?
- c. How many books are there in total?

Task 1

John works as a miner. Out of his salary for April, John spent \$645 on food, \$308 on transport, \$467 on insurance and saved \$1224.

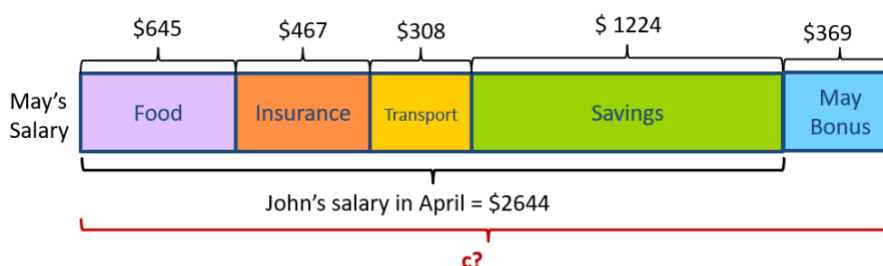
- How much did John spend on food and insurance in April?
- What was John's salary in April?
- John's salary in May was increased because he received a bonus of \$369 for his hard work. What was his salary in May?



[Note: The concept of “whole” which represents John’s salary in April is shown as the complex horizontal bar here. The parts are nested within the whole. This creates the imagery of many parts fitting together to form the whole.]

- $\$645 + \$467 = \$1112$
He spent \$1112 on food and insurance in April.

- $\$645 + \$467 + \$308 + \$1224 = \$2644$
John's salary in April was \$2644.



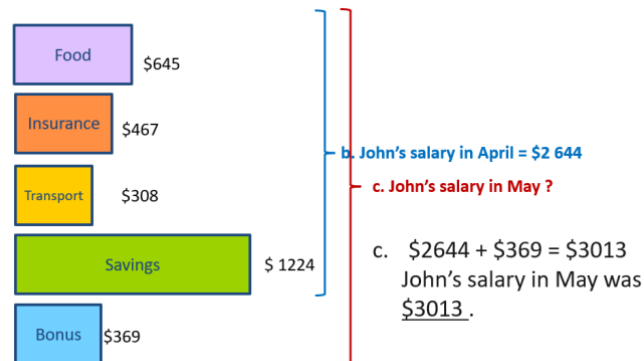
Note: Students may extend the bar from part (a) to include the bonus amount (label the bar as “May’s salary”) or simply do the addition without the model drawing. Bar models are intended to be used flexibly.

The part-whole model can be drawn in one **horizontal** complex bar with many parts inside like the one above. New parts will increase the total length of the whole bar. It is best if the bars are drawn very approximately in proportion e.g. bar showing \$369 is slightly longer than the bar showing \$308.

- $\$2644 + \$369 = \$3013$
John's salary in May was \$3013.

For Further Discussion about Task 1:

The part-whole model can also be presented **vertically** again making the bars roughly proportional in length. However, the effect of “fitting the parts together to make up the whole” is somewhat diminished compared to the horizontal complex bar above. If the part-whole model is presented vertically, the bars should be aligned to display the differences in length. This is especially useful for comparison models (see Lessons 6 & 7). In Singapore schools, vertical presentation is not encouraged for most part-whole models.



Task 2:

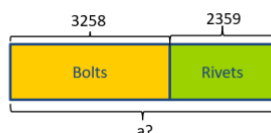
The manager of a hardware store is doing a stock take.

He found there are 1276 screws, 3258 bolts, 7352 nails and 2359 rivets in his store.

- How many bolts and rivets are there?
- How many screws and nails are there?
- How many fasteners are there in stock altogether?

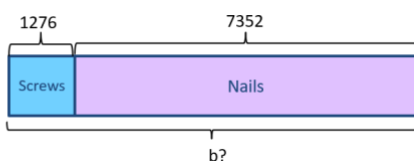
One complex horizontal bar model with four parts could be drawn from the start, or several smaller bar models might be drawn, as shown below.

a.



$$3258 + 2359 = 5617$$

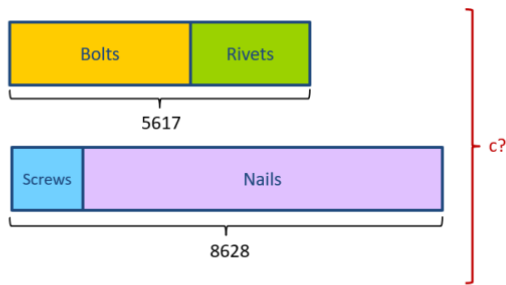
There are 5617 bolts and rivets in his store.



b.

$$1276 + 7352 = 8628$$

There are 8628 screws and nails in the store.



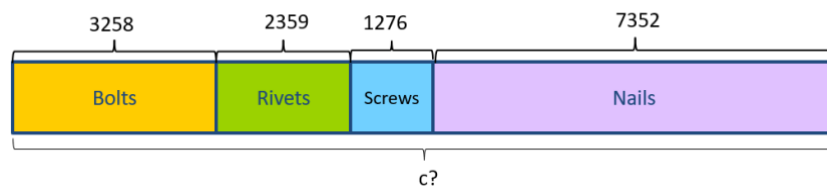
c. $5617 + 8628 = 14\ 245$

There are 14 245 fasteners in stock altogether.

For Further Discussion about Task 2:

The vertical representation is used above for part-whole model because “bolts” and “rivets” are not mentioned consecutively in the word problem. It is the same for “screws” and “nails”. This vertical representation can convey that different parts can be “shifted” to fit together to find partial sum.

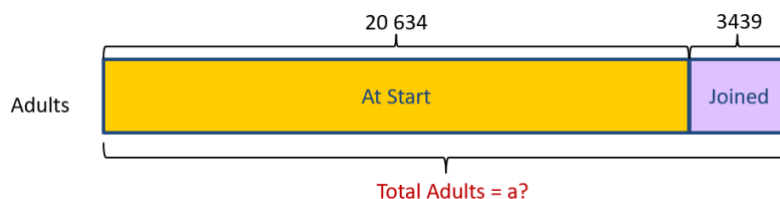
If a horizontal complex bar model is used, some students will want to place the parts representing “bolts” and “rivets” next to each other and the parts representing “screws” and “nails” together. Some students might redraw their original bar model in this new order. This is another aspect of flexibility. See below.



Task 3:

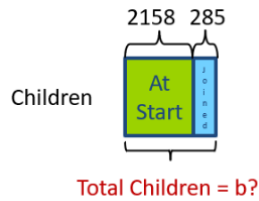
20 634 adults and 2158 children were waiting for a music festival to start at Melbourne Stadium. Just before the event started, 285 children, 537 teenagers and 3439 adults joined them.

- How many adults were at the music festival?
- How many children were at the music festival?
- What was the total number of people attending the music festival?



a. $20\ 634 + 3439 = 24\ 073$

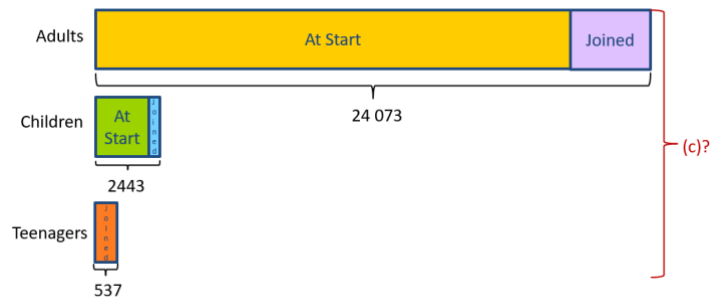
24 073 adults attended the music festival.



b. $2158 + 285 = 2443$

2443 children attended the music festival.

[Start filling in the answers from (a) and (b) in the bars above before working on (c). Align the bars so that it is easier to check that the bars drawn are reasonably proportional, based on the quantity each bar represents. The vertical arrangement to show the part-whole model is shown here. The horizontal complex bar can also be drawn for this.]



(b) $24\ 073 + 2443 + 537 = 27\ 053$

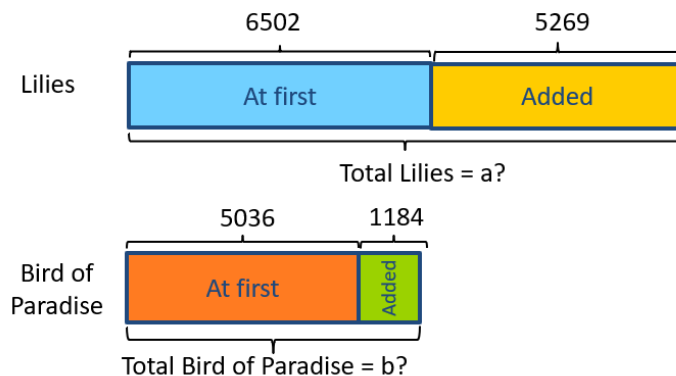
A total of 27 053 people attended the music festival.

Task 4:

The workers are doing landscaping in the botanical gardens.

6502 pots of Lilies, 3728 pots of Anemones, 5036 pots of Bird of Paradise and 9124 pots of Banksia are first planted. Another 5269 pots of Lilies and 1184 pots of Bird of Paradise are added to improve the design.

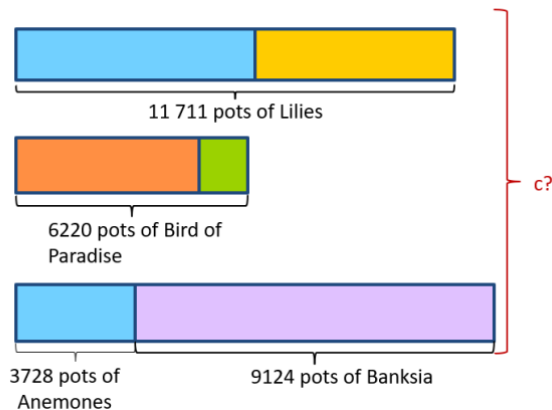
- How many pots of Lilies are at the gardens?
- How many pots of Bird of Paradise are in the gardens?
- What is the total number of pots of Lilies, Anemones, Bird of Paradise and Banksia in the gardens?



a. $6502 + 5269 = 11\ 771$

11 711 pots of Lilies are at the gardens.

- b. $5036 + 1184 = 6220$
6220 pots of Bird of Paradise are at the gardens.

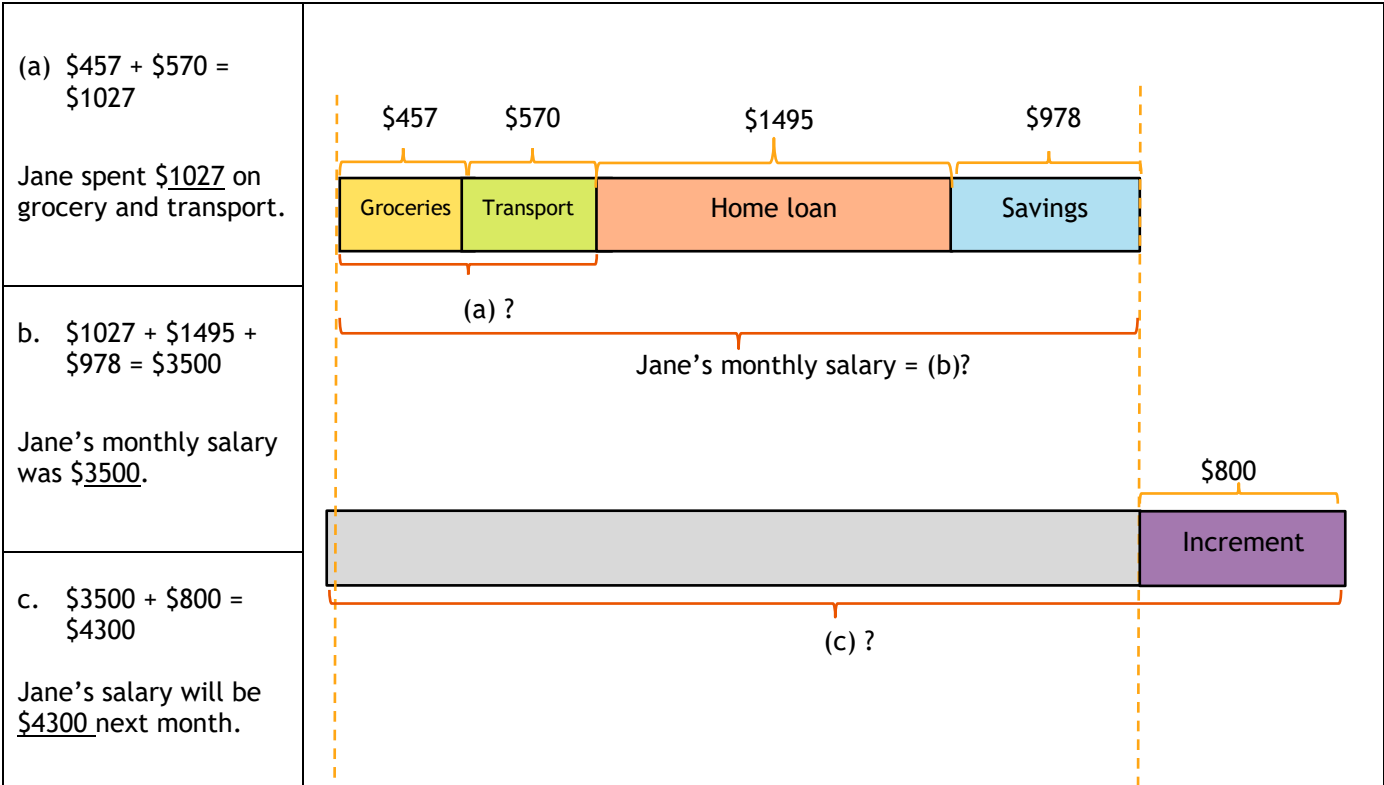


- c. $11\,711 + 6220 + 3728 + 9124 = 30\,783$
 A total of 30 783 pots of Lilies, Bird of Paradise, Anemones and Banksia are at the gardens.

Task 5

Jane works as a secretary. Out of her salary one month, she spent \$457 on groceries, \$1495 on her home loan, \$570 on transport and saved the rest, which was \$978.

- (a) How much did Jane spend on groceries and transport?
 (b) What was Jane’s monthly salary?
 (c) Jane was promoted and will be given an increment of \$800 in her salary next month. What will be her salary next month?



Task 6

There are different types of books in the library. There are 7345 fiction books, 2450 non-fiction books, 5429 children's books and 109 magazines.

- How many fiction books and magazines are there?
- How many non-fiction and children's books are there?
- How many books are there in total?

<p>a. $7345 + 109 = 7454$</p> <p>There are <u>7454</u> fiction books and magazines.</p>	
<p>b. $2450 + 5429 = 7879$</p> <p>There are <u>7879</u> non-fiction and children's books.</p>	
<p>c. $7454 + 7879 = 15\,333$</p> <p>There were <u>15 333</u> books in total.</p>	