

# Introduction to Bar Models

## Lesson 5: Subtraction of Fractions

### Australian Curriculum: Mathematics (Years 4, 5, 6)

**ACMNA077:** Investigating equivalent fractions used in contexts (Year 4).

**ACMNA103:** Investigating strategies to solve problems involving subtraction of fractions with the same denominator (Year 5).

**ACMNA126:** Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6).

### Lesson abstract

Students learn how the part-whole bar models can represent subtraction contexts involving fractions. Students study examples and practice with further tasks, individually or in groups. The bar models provide support for intuitive methods of solving these problems, which supports the development of fraction concepts and skills.

### Mathematical purpose (for students)

Bar models can help us to visualise problems involving fractions

### Mathematical purpose (for teachers)

Students learn to use the part-whole bar model with problems involving subtraction of fractions. The lessons provide a variety of types of situations where subtraction arises, although comparison situations are not treated until Lesson 7 of this unit. In some problems, students need to express a quantity as part of two wholes (e.g. a fraction of a full box, and a fraction of a remaining box). In all these cases, the bar models support students using intuitive methods, so it can assist in developing both concepts and skills. Symbolic mathematical statements are given, thereby making a triad of reality, diagrams, and abstract mathematics. Drawing some diagrams requires students intuitively working out common denominators, in order to represent fractions with different denominators simultaneously.

Lesson Length            60 minutes approximately

#### Vocabulary Encountered

- Part-Whole Model
- Mathematical Statements

#### Lesson Materials

- Slide show *ST4\_BarModelIntro\_5a\_SubFract.pptx*
- [Student Sheet 1 - Bar Model Examples 5A](#) (1 per student)
- [Student Sheet 2 - Bar Model Examples 5B](#) (1 per student)
- Calculators as required

We value your feedback after these lessons via <https://www.surveymonkey.com/r/G6VGPZ8>



# Constructing Bar Models with Fractions

The emphasis in the lesson is intended to be on how the bar model helps students in visualising the relationships between fractional quantities in word problems.

The examples (with solutions) and some of the consolidating tasks are contained in the animated slide show *ST4\_BarModelIntro\_5a\_SubFract.pptx*, which can be used during initial instruction and class discussion. The slides demonstrate how students can build up the model step by step.

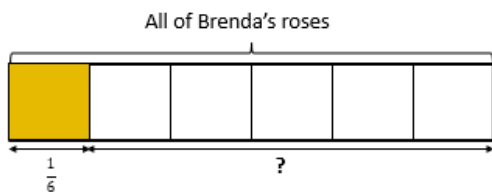
Hand out [Student Sheet 1 - Bar Model Examples 5A](#). Students should write the solutions to these examples, for future reference.

## Example 1

Read Example 1 with the class and discuss how to draw and label the model. Slide show *ST4\_BarModelIntro\_5a\_SubFract.pptx* can be used to show each stage of building the bar model.

Brenda had some roses.  
She gave Lily  $\frac{1}{6}$  of her roses.  
What fraction of the roses did Brenda have left?

### Sample Solution



$$1 - \frac{1}{6} = \frac{5}{6}$$

**Brenda had  $\frac{5}{6}$  of her roses left.**

### Enabling Prompts

Ask students to consider:

- Do we know the number of roses Brenda had? (ANS: no)
- What do we need to find? (ANS: the fraction of roses Brenda have left)
- How do we represent the fraction using the bar model? (ANS: draw one bar representing all of Brenda's roses; divide this whole into 6 equal units)

Note: Highlight how the bar model can help students come up with the mathematical statements. Notice how the full bar has been named to emphasise that it represents one whole.

### Extending Prompts

- What if Brenda gave more of her roses to Lily? How do we represent this on the bar? (ANS: by shading more sixths if the fraction is still in sixths - otherwise we will need to subdivide the bar in a second way)

## Example 2

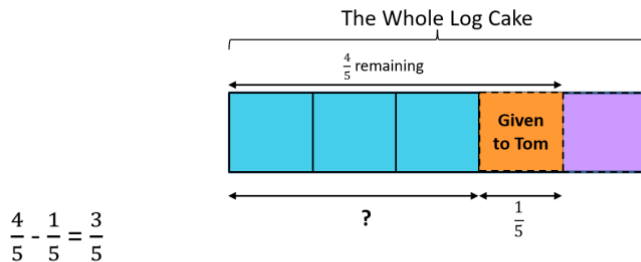
Read Example 2 with the class and discuss how to draw and label the bar model. Use the slideshow to demonstrate how to build the model. Students might attempt this question before the solution is shown.

Abby has  $\frac{4}{5}$  of a log cake remaining from a party.

The next day, she gave Tom  $\frac{1}{5}$  of the original amount of cake.

What fraction of the cake did she then have left?

### Sample Solution



**Abby had  $\frac{3}{5}$  of the log cake left.**

### Enabling Prompts

Ask students to consider:

- What happened to the log cake in the story? (ANS: Some was eaten at the party and then Abby gave some away.)
- What do we need to find? (ANS: the fraction of the cake Abby had left after she gave some to Tom)
- How do we represent this situation using the bar model? (ANS: one bar representing one whole log cake; sub-divided into 5 equal units; because of fifths)

### Extending Prompts

- What if Abby gave away more of her log cake to others? How do we represent this on the bar model we drew? (ANS: by shading, if the amount of log cake given away is in fifths; need to sub-divide in tenths or multiples of 5 if there are other fractional parts involved - see next example)

## Example 3

Read Example 3 with the class and discuss as needed. Students may attempt this question before the solution is shown. Notice how the full bar has been named to emphasise that it represents one whole.

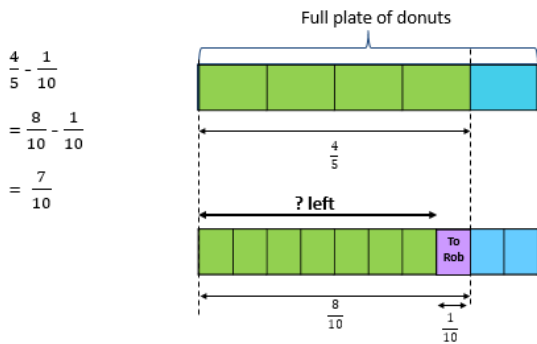
Patricia made a plate of donuts.

She had  $\frac{4}{5}$  of the donuts left after tea.

She gave Robert  $\frac{1}{10}$  of the plate of donuts.

What fraction of the plate of donuts did she then have left?

## Sample Solution



Patricia had  $\frac{7}{10}$  of the plate of donuts left.

## Enabling Prompts

Ask students to consider:

- What is the story of the donuts? (ANS: Patricia made a full plate, some were eaten for tea and then some were given to Robert.)
- What do we need to find? (ANS: the fraction of the plate of donuts Patricia had left)
- How do we represent this situation using the bar model? (ANS: draw one bar representing one full plate, sub-divided into 5 first, then 10 equal units. Alternatively, choose tenths at the start if you can look ahead; highlight that one fifth is two tenths.)

## Extending Prompts

What if Patricia gave away more of her plate of donuts? How could we show this on the bar model? (ANS: by shading more tenths, if the amount given away can be expressed in tenths out of one whole. If not, the bar will need to be divided again.)

## Example 4

This example shows a slightly different, more ‘dynamic’, way of drawing the bar model. Various parts of the bar are taken away as the problem progresses. This contrasts with the more static approach in the previous examples. Bar models are intended to be used flexibly - for most problems, there will be several good ways.

Read Example 4 with the class and discuss how to draw and label the model. Slide show *ST4\_BarModelIntro\_5a\_SubFract.pptx* can be to show how to build the model.

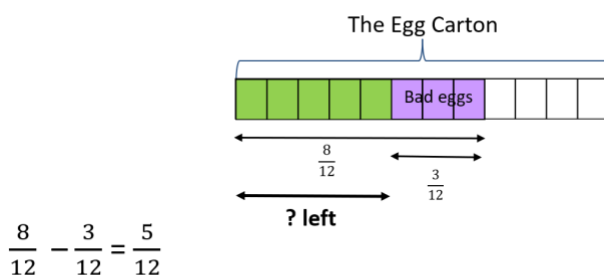
On Sunday, Vanessa had  $\frac{8}{12}$  of a carton of eggs in the pantry.

$\frac{3}{12}$  of the carton of eggs turned bad by Wednesday because the weather was hot.

Vanessa removed the rotten eggs.

What fraction of the carton of eggs did Vanessa have left?

## Sample Solution



Vanessa had  $\frac{5}{12}$  of the carton of eggs left.

## Enabling Prompts

Ask students to consider:

- What fraction of the carton of eggs did Vanessa have in the pantry at first? (ANS:  $\frac{8}{12}$  carton of eggs)
- What fraction of the carton of eggs turned bad? (ANS:  $\frac{3}{12}$  of the carton of eggs)
- What do we need to find? (ANS: the fraction of the carton of eggs Vanessa had left)
- How do we represent this situation using the bar model? (ANS: draw one bar representing one whole carton of eggs, sub-divided into 12 equal units because the fraction of eggs was expressed as twelfths.)

## Extending Prompt

- How would we do this problem if the fractions of the carton were expressed differently - two thirds instead of 8 twelfths and one quarter instead of 3 twelfths?

## Example 5

This example demonstrates how the bar model can be used, when the divisor changes for different amounts being considered from the whole.

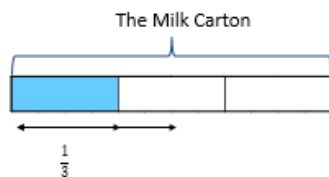
Read through this example with students, and discuss how to represent and label the problem in a part-whole bar model.

Vanessa had half a carton of milk left. She used  $\frac{1}{3}$  of this milk to make scrambled eggs.

- a. What fraction of the carton of milk does Vanessa then have left?

## Sample Solution

$$\begin{aligned}\frac{3}{6} - \frac{1}{6} \\&= \frac{2}{6} \\&= \frac{1}{3}\end{aligned}$$



Vanessa had  $\frac{1}{3}$  of the whole carton of eggs left.

## Enabling Prompts

Ask students to consider:

- When Vanessa makes scrambled eggs, what is the fraction  $\frac{1}{3}$  a fraction of? (ANS: The milk she has left, not the full carton).
- How can we show this on a bar model? (ANS: Divide the remaining milk into thirds, which is the same as dividing the whole carton into sixths).

# Consolidating and Concluding

## Further Practice

Students work through selected tasks on [Student Sheet 2 - Bar Model Examples 5B](#), either individually, in pairs or in groups. Discuss solutions as time permits.

Worked solutions are provided in [Teacher Sheet - Bar Model Solutions 5B](#). Animated solutions to Tasks 1 and 2 are included in the slideshow (*ST4\_BarModelIntro\_5a\_SubFract.pptx*).

- Task 1 (a) is similar to Example 1.
- Task 1 (b) has a new twist, also seen in Example 5. Ethan ate a quarter “of the *remaining* lollies”. This is different from a quarter “of the original bag of lollies”. The bar model can show this difference well. Finding a quarter of four fifths as required in (b) can be done by fraction multiplication, but the bar model supports students to do this intuitively.
- Task 1 (b) also requires the students to find the fraction “of the *original* bag of lollies” Ethan had after the movie. With all fractions, it is essential to consider what ‘whole’ is being referenced. In this case, there is a change of referent whole.
- Task 2 (a) is similar to Example 4.
- Task 2 (b) is similar to Task 1, but students should note that  $\frac{1}{4}$  “of the remaining eggs on the tray” is different from  $\frac{1}{4}$  “of the eggs on the tray”. The question asks for the fraction of the tray of eggs left. This can be deduced from a correctly drawn model. The bar model shows that  $\frac{1}{4}$  of the remaining eggs is equivalent to  $\frac{3}{24}$  or  $\frac{1}{8}$  of the tray of eggs. This is because  $\frac{1}{12}$  in the first bar is sub-divided into 2 equal parts; each part represents  $\frac{1}{24}$  of the tray of eggs. Hence, Abby’s mum was left with  $\frac{9}{24}$  or  $\frac{3}{8}$  of the tray of eggs.
- Task 3 (c) is similar to those in lesson 4 of this unit where the part-whole model is used in a multiplicative way.

## Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- Part-whole bar models can involve the use of many parts (more than two parts) within the whole; some of the parts can be equal in quantity.
- Either one part out of many or the whole can be unknown.
- In the examples discussed, we could find the unknown with the help of the bar model, because the problem situations provided information about the whole and we know the relationship between the parts and the whole.

## Example 1

Brenda had some roses.

She gave Lily  $\frac{1}{6}$  of her roses.

What fraction of the roses did Brenda have left?

## Example 2

Abby has  $\frac{4}{5}$  of a log cake remaining from a party.

The next day, she gave Tom  $\frac{1}{5}$  of the original amount of cake.

What fraction of the cake did she then have left?

## Example 3

Patricia made a plate of donuts.

She had  $\frac{4}{5}$  of the donuts left after tea.

She gave Robert  $\frac{1}{10}$  of the plate of donuts.

What fraction of the plate of donuts did she then have left?

## Example 4

On Sunday, Vanessa had  $\frac{8}{12}$  carton of eggs in the pantry.

$\frac{3}{12}$  of the carton of eggs turned bad by Wednesday because the weather was hot.

Vanessa removed the rotten eggs.

What fraction of the carton of eggs did Vanessa still have left?

## Example 5

Vanessa had half a carton of milk left. She used  $\frac{1}{3}$  of this milk to make scrambled eggs.

What fraction of the carton of milk does Vanessa then have left?

Draw bar models to represent the problem situations below and use them to help you solve the problems.

### Task 1

Ethan had a full bag of lollies. He ate  $\frac{1}{5}$  of the lollies from the bag.

(a) What fraction of the bag of lollies did Ethan then have?

While watching a movie, Ethan ate  $\frac{1}{4}$  of the remaining lollies.

(b) After the movie, what **fraction of the original bag of lollies** was Ethan finally left with?

### Task 2

Abby's mother loves to bake. She had  $\frac{10}{12}$  of a tray of eggs. She needed  $\frac{4}{12}$  of a tray of eggs to bake chocolate chip cookies.

(a) What fraction of the tray of eggs was Abby's mum left with after she baked the chocolate chip cookies?

Abby used a quarter of **the remaining eggs on the tray** to make some scones for breakfast the next day.

(b) What **fraction of the full tray of eggs** was she left with?

(c) If Abby's mum had 9 eggs left on the tray after making scones, how many eggs were there in the tray at first?

### Task 3

May had a bag of lollies. She ate  $\frac{2}{8}$  of the lollies from the bag.

(a) What fraction of the bag of lollies did May have left?

(b) While chatting with her friend, May ate one third of **the remaining lollies**. What **fraction of the bag of lollies** was May left with then?



## Task 1

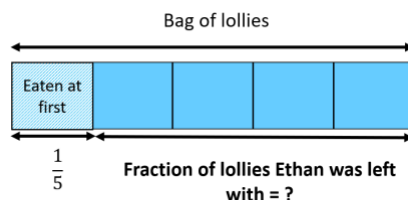
Ethan had a bag of lollies.

He ate  $\frac{1}{5}$  of the lollies from the bag.

- (a) What fraction of the bag of lollies was Ethan left with?

$$1 - \frac{1}{5} = \frac{4}{5}$$

Ethan was left with  $\frac{4}{5}$  of the bag of lollies

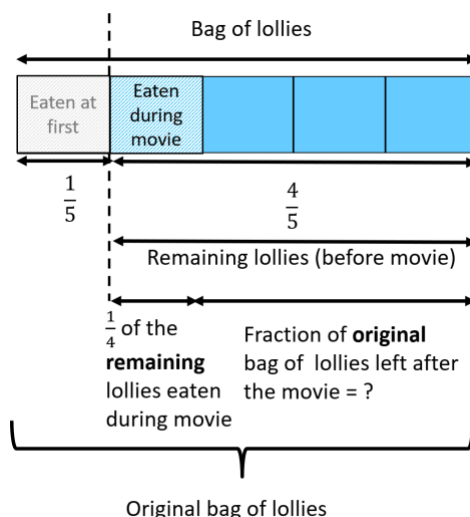


While watching a movie, Ethan ate  $\frac{1}{4}$  of the remaining lollies.

- (b) After the movie, what fraction of the original bag of lollies was Ethan finally left with?

$$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

Ethan was left with  $\frac{3}{5}$  of the original bag of lollies.

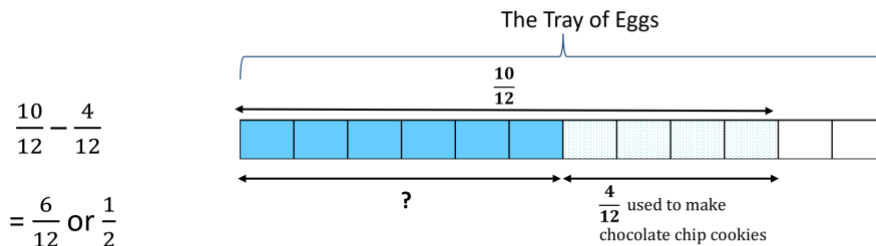


## Task 2

Abby's mother loves to bake. She had  $\frac{10}{12}$  of a tray of eggs.

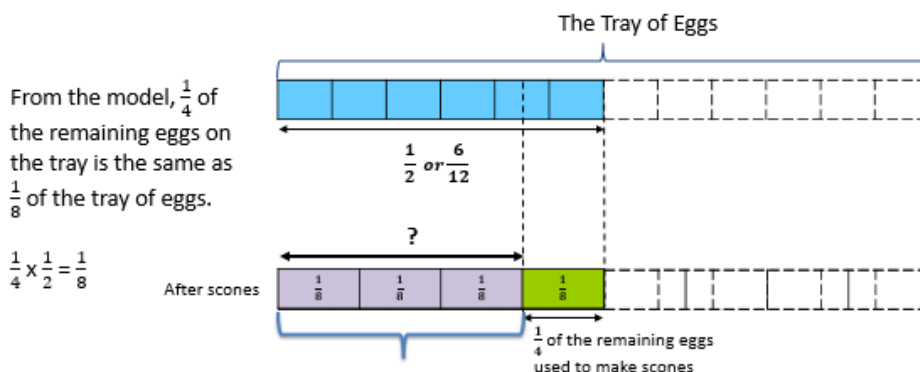
She needed  $\frac{4}{12}$  of a tray of eggs to bake chocolate chip cookies.

(a) What fraction of the tray of eggs was Abby's mum left with after she baked the chocolate chip cookies?



Abby's mum was left with  $\frac{6}{12}$  or  $\frac{1}{2}$  of the tray of eggs after making the chocolate cookies.

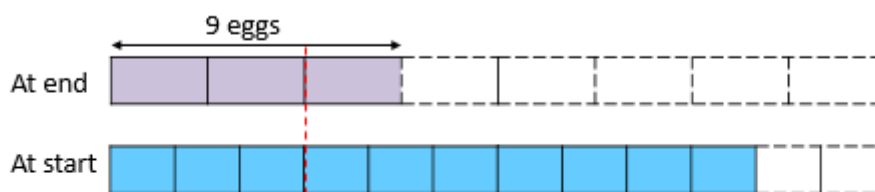
(b) Abby used  $\frac{1}{4}$  of the remaining eggs on the tray to make some scones for breakfast the next day. What fraction of the tray of eggs was she left with?



Abby's mum was left with  $\frac{3}{8}$  of the tray of eggs.

(c) If Abby's mum had 9 eggs left on the tray after making scones, how many eggs were there in the tray to begin with?

$\frac{3}{8}$  of the tray of eggs was left at the end (3 purple units)



3 purple units = 9 eggs  
 1 purple unit = 3 eggs  
 1 full tray = 8 purple units  
 = 24 eggs  
 1 blue unit = 2 eggs

To begin with, there were 10 blue units in the tray.  
 10 blue units = 10 x 2 eggs  
 = 20 eggs  
 There were 20 eggs in the tray to begin with.  
 (And 24 in the full tray before that!)

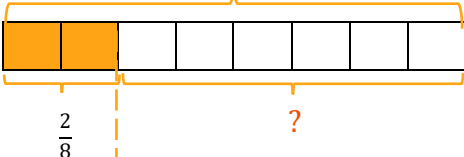
### Task 3

May had a bag of lollies.

She ate  $\frac{2}{8}$  of the lollies from the bag.

(a) What fraction of the bag of lollies did May have left?

(b) While chatting with her friend, May ate  $\frac{1}{3}$  of the remaining lollies. What fraction of the bag of lollies was May left with now?

<p>(a)</p> $1 - \frac{2}{8}$ $= \frac{6}{8}$ <p>May had <math>\frac{6}{8}</math> or <math>\frac{3}{4}</math> of the bag of lollies left.</p>	<p>Total Number of lollies in the Bag</p> 
<p>(b)</p> <p>From the bar model, there are six white units uneaten. May eats a third of these (2 units).</p> <p>May was left with <math>\frac{4}{8}</math> of the bag of lollies.</p> <p>May was left with <math>\frac{1}{2}</math> of the bag of lollies.</p>	<p>Total Number of lollies in the Bag</p> 