

Year 4 Exemplar

Magic V

Australian Curriculum: Mathematics (Year 4)

ACMNA071: Investigate and use the properties of odd and even numbers.

- Using the four operations with pairs of odd or even numbers or one odd and one even number, then using the relationships established to check the accuracy of calculations.

Abstract

The Magic V task affords an opportunity to explain the reasons why a conjecture is true. Students begin by creating Magic Vs by trial and error, but come to see constant features. By manipulating numbers, analysing totals and recording their ideas, students will come to see why the properties of odd and even numbers are relevant. The main purpose is to develop students' capacity to analyse situations, to find reasons and develop logical arguments.

Mathematical purpose (for students)

To convince someone that a mathematical statement is true or not.

Mathematical purpose (for teachers)

Teachers support and challenge students to analyse a situation and unearth the underlying mathematical structure that justifies a conjecture by:

- Comparing and contrasting examples to analyse the situation.
- Testing a conjecture by using examples, then looking for reasons.
- Forming a logical argument using properties of odd and even numbers.

There are opportunities to generalise the findings to a variety of related situations, and to make increasingly general arguments.

Time needed 100 minutes approximately

Vocabulary Encountered

- compare
- convince me
- justify/explain why
- testing conjectures
- verifying ideas
- because
- if... then...

Materials

- [Student Sheet 1 - Magic V Template](#) (1 per pair, optional)
- [Number Cards](#) (1 set per pair, optional, for use with template)
- [Student Sheet 2 - What is a Magic V?](#) (1 per student, optional)
- [Student Sheet 3 - Magic V Record Sheet](#) (1 per pair)
- [Student Sheet 4 - Sam's Conjecture](#) (1 per student)
- Reasoning Prompt Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- [Assessment Sheet](#) (1 per student or group)

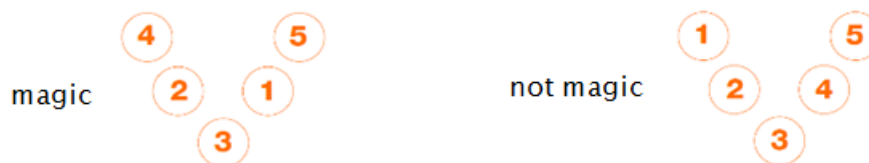
We value your feedback after these tasks via <https://www.surveymonkey.com/r/RJC6FPC>



Magic V: The Lesson

Introduction

A Magic V is a number puzzle. The aim is to arrange five consecutive numbers so that the sum of each 'arm' of the V is the same. Below are two Vs. The left V is a Magic V ($4+2+3=9$ and $5+1+3=9$); the V on the right is not. Each number can be used only once.



Students should try to make some Magic Vs and explore some of their features before they are given the reasoning task. Use [Student Sheet 1 - Magic V Template](#) together with the [Number Cards](#) or students can draw their own.

Ask students to write down their understanding of the definition using [Student Sheet 2 - What is a Magic V?](#)

Discuss a good answer - it will point to the facts that all numbers are used just once, and the totals along the arms are the same. Students may also notice other features, such as the equality of the arm sums without the vertex.

The teachers' page [All Magic Vs Using Numbers 1-5](#) shows all possible Magic Vs with the numbers 1 to 5.

Reasoning Task

Lead into the main task by introducing a discussion of what students noticed about the position of numbers in Magic V's, particularly the number in the vertex.

Hand out [Student Sheet 4 - Sam's Conjecture](#) and [Student Sheet 3 - Magic V Record Sheet](#) (this is for students to quickly record the Magic Vs they have tried).

Allow students time to work, individually or in groups, on this task.

Sam said, "It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5."

Is Sam right? Explain why or why not.

Reasoning Prompts

For more prompts in the context of this task, see this [table](#)

- What is the same and different about [these two Magic Vs]? ([Analysing](#))
- Alter an aspect of something [your Magic V] to see (such and such an effect). If we change this what will happen? ([Analysing](#))
- What is the pattern here? ([Generalising](#))
- Is that ... (pattern) always going to work? ([Generalising](#))
- Convince me. ([Justifying](#))
- Explain why this result works? ([Justifying](#))

What is the same and different about ...?	Alter an aspect of something to see an effect. If we change this what will happen?	What is the pattern here?	Is that... (pattern) always going to work?	Convince me...	Explain - why does this work?
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Enabling Prompts

- Convince me that this is (or is not) a Magic V.
- Suggest students use the number cards to try more options.
- Make as many Magic Vs as you can with 5 at the vertex. Write the totals of the numbers on the arms.
- Make as many Magic Vs as you can with 2 at the vertex. What totals are needed on the arms?

Extending Prompts

- What would happen if we used the numbers 2 to 6 instead of 1 to 5? (This has 3 even and 2 odd numbers.)
- Generalise Sam's conjecture to all sets of five consecutive numbers. (ANS: A Magic V can only be made if the number at the vertex is

Summary Phase

Invite students to share their arguments in support of Sam's conjecture, in order of complexity. The Formative Assessment [Table](#) shows the likely variation in responses. Use these inputs to develop a whole class mathematical discussion that reveals the reason why Sam's conjecture is true in increasing depth generality. You might:

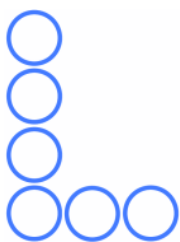
- Encourage students to explain each other's thinking.
- Ask:
 - What is one thing you know now about making a logical argument that you did not know before?"
 - What have you learned about explaining your reasoning to others?

Further Activities

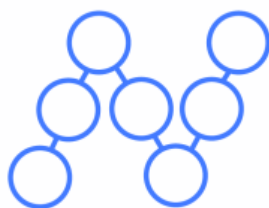
Having listened to a range of students' strategies in the initial task, students can apply those strategies to follow-up tasks as shown below.

1. Magic Vs can be made from any set of 5 consecutive numbers such as [2,3,4,5,6], [6,7,8,9,10] or [22,23,24,25,26]. Students can explore what stays the same and what changes. Student can refine their conjectures and generalisations about properties of odd and even numbers.
2. Investigate Magic Vs made with the numbers 2,4,6, 8, 10.
3. Magic Vs can be made from other sets of numbers. Students can choose a set of number and look for the Magic Vs (if any) that can be made with them.
4. Investigate Magic Vs that have longer arms (e.g. arm length 4 using the numbers 1-7?
5. Investigate other magic letters, e.g. L, N and W.

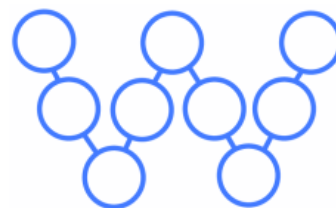
Magic L?



Magic N?



Magic W?



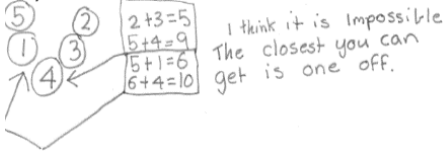
More information on the Magic V can be located at <https://nrich.maths.org/6274> , <https://nrich.maths.org/7821> and <https://nrich.maths.org/1835>.

A somewhat different task using the Magic W is found at <https://nrich.maths.org/1835>

Formative Assessment

The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students' reasoning during the lesson. Suitable prompts are suggested to support or extend such students' reasoning.

Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers' assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student's reasoning.

ANALYSING		
Possible Student Response	Level	Suggested Prompts
The student tries to make a Magic V with an even number on the bottom. (See Annotated Work Sample 1)	Beginning	Do you think we can have an even number at the bottom if we use different numbers (not 1 to 5)? Why? Can you convince us?
The student compares possible Magic Vs to show how each time you use an even number at the bottom there is a difference of one or more in the total of the arms. 	Developing	If you have 4 at the bottom, what would the total of equal arms each have to be? (Ans $4 + (1 + 2 + 3 + 5) \div 2 = 10\frac{1}{2}$)
The student is examining the total of the arms on magic and not magic Vs, and sorting according to even and odd numbers at the vertex. (See Annotated Work Sample 2)	Developing	You have calculated the totals of the 4 numbers not at the vertex. What do you notice? How can you use this information?
The student analyses the odd and even characteristics of the numbers and totals, and notices the significance of 'balance'. (See Annotated Work Sample 3)	Consolidating	Analyse how the situation is different if you use a different set of numbers like 2 to 6?
The student analyses the properties of odd and even numbers and communicates the significance of 'balance'. (See Annotated Work Sample 4)	Extending	Analyse how the situation is different if you use a different set of numbers like 2 to 6?
GENERALISING		
Possible Student Response	Level	Suggested Prompts
Does not communicate a common property or rule (conjecture) for a pattern. (See Annotated Work Sample 1)	Not Evident	Do you notice any pattern in the Magic Vs you have found?
The student attempts to communicate a common property using diagrams, counting, oral language and/or concrete materials to draw attention to properties of a Magic V. (See Annotated Work Sample 2)	Beginning	You have written down some useful calculations. Explain the pattern you were observing.
The student records other Vs, and compares and contrasts them in order to communicate patterns that they notice.	Developing	What is the pattern here?
The student is beginning to recognise how the properties of odd, even and balance, contribute to making a 'V' a Magic V. (See Annotated Work Sample 3)	Consolidating	What if we have more odd than even numbers?
The student describes the rule to demonstrate the notion of balance by comparing an odd and even case. (See Annotated Work Sample 4)	Extending	Will this hold true for any set of consecutive numbers?

JUSTIFYING		
Possible Student Response	Level	Suggested Prompts
"No, it is possible. There's about a hundred ways to make it."	Not Evident	Can you make a Magic V with 2 at the vertex? What do you notice?
"Sam is right, it is impossible to make a Magic V with an even number at the bottom." (See Annotated Work Sample 2)	Not Evident	If you put 2 at the vertex, what must be the total of each arm to make the V magic?
"I think you can't put an even number at the bottom with numbers from 1-5 because it is odd number of cards."	Beginning	Explain more fully. Is there another reason?
"It is impossible to make a Magic V because with the 2 and a 4 at the bottom it would not get the same total." (See Annotated Work Sample 2)	Developing	You say that if we have an even number at the bottom then the total of the two arms are not equal. Is it true for all different Vs with 2 and 4 at the bottom? Why?
"We think that Sam is right because we tried the two even numbers and none of them worked and we mixed the numbers around and only the odd numbers worked."	Developing	You have explained what you found. Now look for a reason why it might be true.
The student extends the generalisation using logical argument about the properties of odd and even numbers. (See Annotated Work Sample 3)	Consolidating	Apply your argument to see if the same going to be true for different sets of numbers?
The student verifies the statement is true or the generalisation holds for <i>all</i> cases using logical argument. e.g., that the total of both arms must be divisible by two. (See Annotated Work Sample 4)	Extending	What if we use different numbers in the V and have more even than odd numbers? Use the same argument to see if Sam's conjecture is still true.

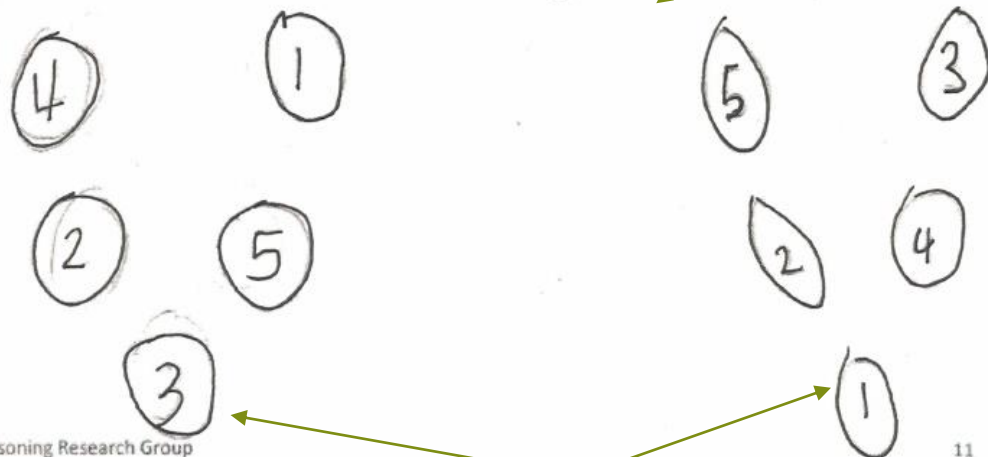
Annotated Work Sample 1

JUSTIFYING: Recognises what is correct or incorrect using materials, objects or words.

However, the second part of the statement does not make sense. The teacher would need to speak to the student to clarify what they mean. (Maybe just a slip?)

JUSTIFYING: The argument may not be coherent or include all steps in the reasoning process.

Sam is right it is impossible to make a magic V with an even number at the bottom.
even One can make a magic V with an number in the vertex.



ANALYSING: Beginning

GENERALISING: Not evident

JUSTIFYING: Beginning

Teacher Prompt:

What is the same and different about these two Magic Vs?

What other examples did you look at?

Can you explain why the even number cannot be at the bottom?

ANALYSING: Notices similarities across examples.

The student notices the importance of the vertex in creating a Magic V but cannot explain 'why' it is important.

Student Name: Work Sample 1 Reasoning Task: MAGIC V Date: _____

Observation of student's reasoning:

* Beginning to notice + describe role of vertex in making a Magic V - gave an example but explanation didn't make sense.

	Analysing	Generalising	Justifying
Not Evident	<ul style="list-style-type: none"> Does not notice common property or pattern. 	<ul style="list-style-type: none"> Does not communicate a common property or rule (conjecture). 	<ul style="list-style-type: none"> Does not justify.
Beginning	<ul style="list-style-type: none"> Recalls random known facts or attempts to sort examples or repeats patterns. 	<ul style="list-style-type: none"> Attempts to communicate a common property or rule for the pattern. 	<ul style="list-style-type: none"> Describes what they did and recognises what is correct or incorrect. Argument is not coherent or does not include all steps.
Developing	<ul style="list-style-type: none"> * Notices a common property, or sorts and orders cases, or repeats and extends patterns. Describes the property or pattern. 	<ul style="list-style-type: none"> Generalises: communicates a rule (conjecture) using mathematical terms and records other cases or examples. 	<ul style="list-style-type: none"> Attempts to verify by testing cases and detects and corrects errors or inconsistencies. Starting statements in a logical argument are correct. *
Consolidating	<ul style="list-style-type: none"> Systematically searches for examples, extends pattern or analyses structure to form a conjecture. Makes predictions about other cases. 	<ul style="list-style-type: none"> Generalises: communicates a rule using mathematical symbols and explains what the rule means or explains how the rule works using examples. 	<ul style="list-style-type: none"> Verifies truth of statements by confirming all cases or refutes a claim by using a counter example. Uses a correct logical argument.
Extending	<ul style="list-style-type: none"> Notices and explores relationships between properties. 	<ul style="list-style-type: none"> Generalises cases, patterns or properties using mathematical symbols (including algebraic symbols) and applies the rule. Compares different expressions for the same pattern or property to show equivalence. 	<ul style="list-style-type: none"> Uses a watertight logical argument. Verifies that the generalisation holds for all cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Focus on explaining 'why?'

* Need to go back & ask student to clarify statement

Annotated Work Sample 2

JUSTIFYING: Starting statements in a **logical argument** are correct and accepted by the class.

ANALYSING: Sorts and classifies according to a common property.

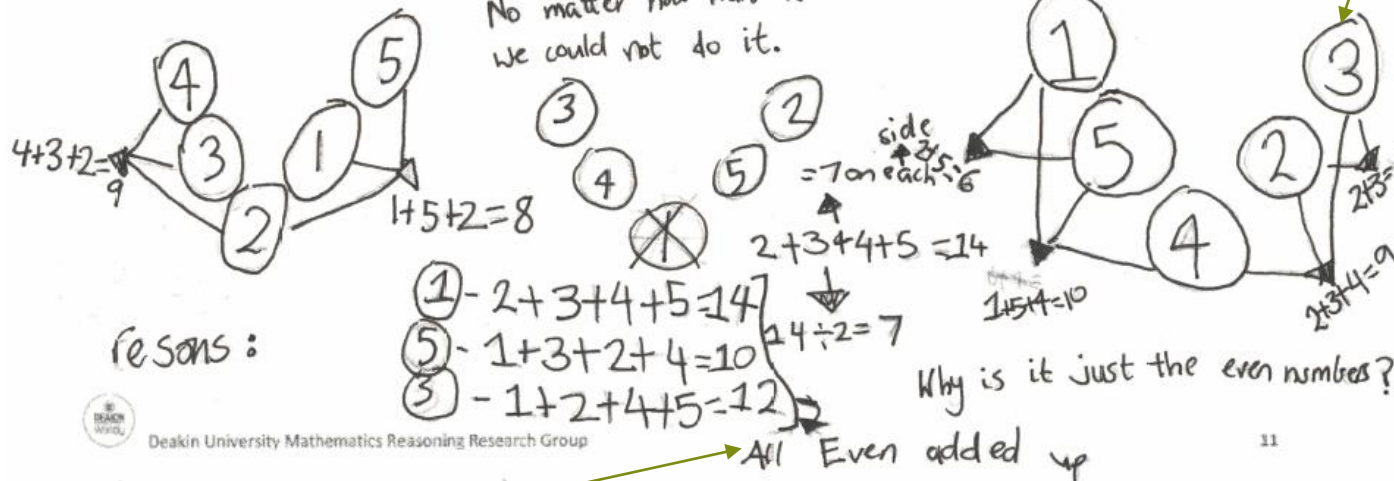
The student is analysing the total of the arms and sorting according to even and odd numbers at the vertex.

Sam said "It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5".

Is Sam right? Explain why or why not? [You can use sentences, number sentences and drawings in your explanation.] We both think Sam is right/correct.

When we tried to make the V with the 2 or the 4 at the vertex, we couldn't find a way to make a magic V with the 2 or the 4 at the bottom/vertex.

No matter how hard we tried we could not do it.



ANALYSING: Developing

GENERALISING: Beginning

JUSTIFYING: Developing

Teacher Prompt:

You have made a very interesting remark that if we have an even number at the bottom then the total of the two arms are not equal. Is it true for all different Magic Vs with 2 and 4 at the bottom? Why?

What do we need to have at the bottom to make the two arms equal?

JUSTIFYING: Attempts to **verify** by testing cases or explaining the meaning of a conjecture using one example.

GENERALISING: Detecting and correcting errors and inconsistencies using materials, diagrams and informal written methods.

The next step is for the student to think about the properties of odd and even numbers.

Student Name: Work Sample 2 Reasoning Task: MAGIC V Date: _____

Observation of student's reasoning:

- * Adding total of arms & comparing 'Vs' with an odd/even number at vertex.
- * developing a logical argument but no 'why'

	Analysing	Generalising	Justifying
Not Evident	<ul style="list-style-type: none"> Does not notice common property or pattern. 	<ul style="list-style-type: none"> Does not communicate a common property or rule (conjecture). 	<ul style="list-style-type: none"> Does not justify.
Beginning	<ul style="list-style-type: none"> Recalls random known facts or attempts to sort examples or repeats patterns. 	<ul style="list-style-type: none"> Attempts to communicate a common property or rule for the pattern. 	<ul style="list-style-type: none"> Describes what they did and recognises what is correct or incorrect. Argument is not coherent or does not include all steps.
Developing	<ul style="list-style-type: none"> Notices a common property, or sorts and orders cases, or repeats and extends patterns. Describes the property or pattern. 	<ul style="list-style-type: none"> Generalises: communicates a rule (conjecture) using mathematical terms and records other cases or examples. 	<ul style="list-style-type: none"> Attempts to verify by testing cases and detects and corrects errors or inconsistencies. Starting statements in a logical argument are correct.
Consolidating	<ul style="list-style-type: none"> Systematically searches for examples, extends pattern or analyses structure to form a conjecture. Makes predictions about other cases. * 'what will...' 	<ul style="list-style-type: none"> Generalises: communicates a rule using mathematical symbols and explains what the rule means or explains how the rule works using examples. 	<ul style="list-style-type: none"> Verifies truth of statements by confirming all cases or refutes a claim by using a counter example. Uses a correct logical argument.
Extending	<ul style="list-style-type: none"> Notices and explores relationships between properties. 	<ul style="list-style-type: none"> Generalises cases, patterns or properties using mathematical symbols (including algebraic symbols) and applies the rule. Compares different expressions for the same pattern or property to show equivalence. 	<ul style="list-style-type: none"> Uses a watertight logical argument. Verifies that the generalisation holds for all cases using logical argument.

Comments (feedback, reasoning prompts for further development):

- * Look closer at properties of odd/even numbers
- * Develop justification → look at reasoning prompts.

Annotated Work Sample 3

ANALYSING: Notices more than one common property by systematically generating further cases and/or listing and considering a range of known facts.

The student is analysing the properties of odd and even numbers and noticing the significance of 'balance'.

because if an even goes at the bottom it unbalances

JUSTIFYING: Extends the generalisation using logical argument

The next step is to develop a clearly worded watertight argument.

because there always has to be one even and one odd on each side (arm)

if we put a even number at the bottom

even odd even odd

it will unbalance. so there would be two odd on ~~one~~ 11 numbers side with one even left

ANALYSING: Consolidating

GENERALISING: Consolidating

JUSTIFYING: Consolidating

Teacher Prompt:

Do you think it is going to be true for different sets of numbers such as numbers from 2-6?

What if we have more even than odd numbers? Can you convince us that an even number at the bottom will then make a Magic V? Why?

GENERALISING: Extends the pattern using an example to **explain how** the rule works.

The student is beginning to recognise how the properties of odd, even and balance, contribute to making a 'V' a Magic V.

Student Name: WORK SAMPLE 3 Reasoning Task: MAGIC V Date: _____

Observation of student's reasoning:

- Identified/analysed odd/even & concept of 'balance' → needs more work.
- Used logical argument

	Analysing	Generalising	Justifying
Not Evident	<ul style="list-style-type: none"> Does not notice common property or pattern. 	<ul style="list-style-type: none"> Does not communicate a common property or rule (conjecture). 	<ul style="list-style-type: none"> Does not justify.
Beginning	<ul style="list-style-type: none"> Recalls random known facts or attempts to sort examples or repeats patterns. 	<ul style="list-style-type: none"> Attempts to communicate a common property or rule for the pattern. 	<ul style="list-style-type: none"> Describes what they did and recognises what is correct or incorrect. Argument is not coherent or does not include all steps.
Developing	<ul style="list-style-type: none"> Notifies a common property, or sorts and orders cases, or repeats and extends patterns. Describes the property or pattern. 	<ul style="list-style-type: none"> Generalises: communicates a rule (conjecture) using mathematical terms and records other cases or examples. 	<ul style="list-style-type: none"> Attempts to verify by testing cases and detects and corrects errors or inconsistencies. Starting statements in a logical argument are correct.
Consolidating	<ul style="list-style-type: none"> Systematically searches for examples, extends pattern or analyses structure to form a conjecture. Makes predictions about other cases. 	<ul style="list-style-type: none"> Generalises: communicates a rule using mathematical symbols and explains what the rule means or explains how the rule works using examples. 	<ul style="list-style-type: none"> Verifies truth of statements by confirming all cases or refutes a claim by using a counter example. Uses a correct logical argument.
Extending	<ul style="list-style-type: none"> Notifies and explores relationships between properties. 	<ul style="list-style-type: none"> Generalises cases, patterns or properties using mathematical symbols (including algebraic symbols) and applies the rule. Compares different expressions for the same pattern or property to show equivalence. 	<ul style="list-style-type: none"> Uses a watertight logical argument. Verifies that the generalisation holds for all cases using logical argument.

Comments (feedback, reasoning prompts for further development):

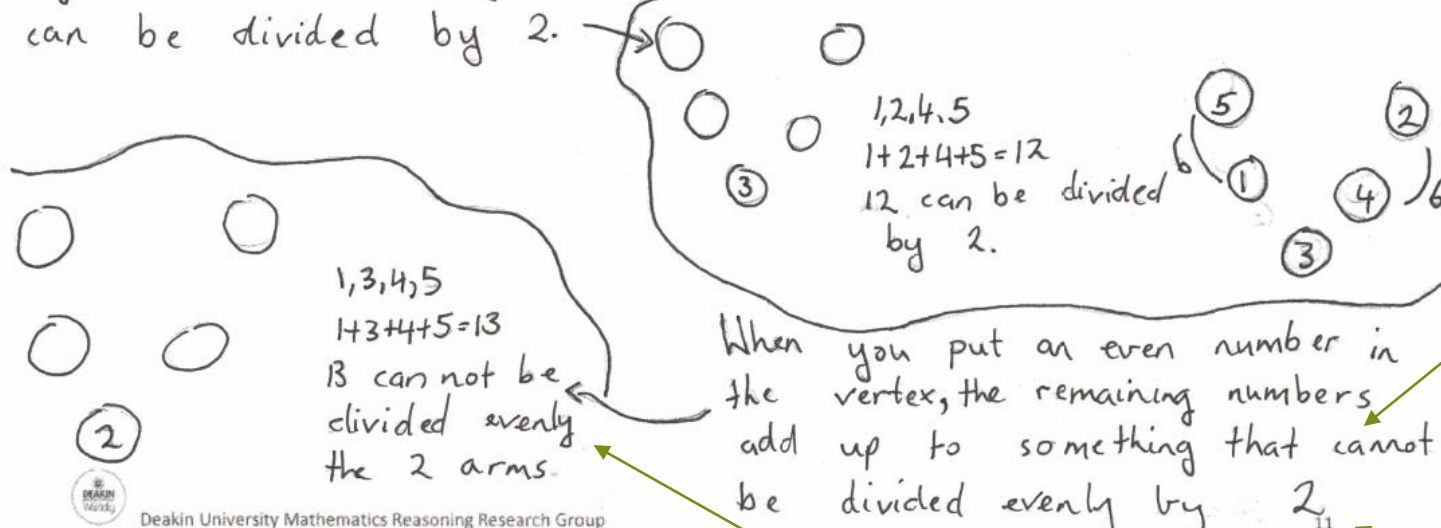
- * Develop watertight argument → prompts
- * Explore 'balance' further & totals of arms.

Annotated Work Sample 4

JUSTIFYING: Verifies the statement is true or the generalisation holds for *all* cases using logical argument.

The next step is to develop a watertight argument.

I think Sam is right because after you put a number in the vertex of the V, the remaining numbers should add up to something that can be divided by 2.



GENERALISING:

Generalises properties by forming a statement about the relationship between common properties.

ANALYSING: Extending

GENERALISING: Extending

JUSTIFYING: Extending

Teacher Prompt:

What if we use numbers from 2-6?

What if we have more even than odd numbers? Can you convince us that an even number at the bottom will make a Magic V? Why?

ANALYSING: Notices and explores

relationships between numerical structures of patterns.

The student is analysing the properties of odd and even numbers and communicates the significance of the two arms being equal in total.

JUSTIFYING:

This is an important part of the logical argument.

Student Name: Work Sample 4 Reasoning Task: MAGIC V Date: _____

Observation of student's reasoning:

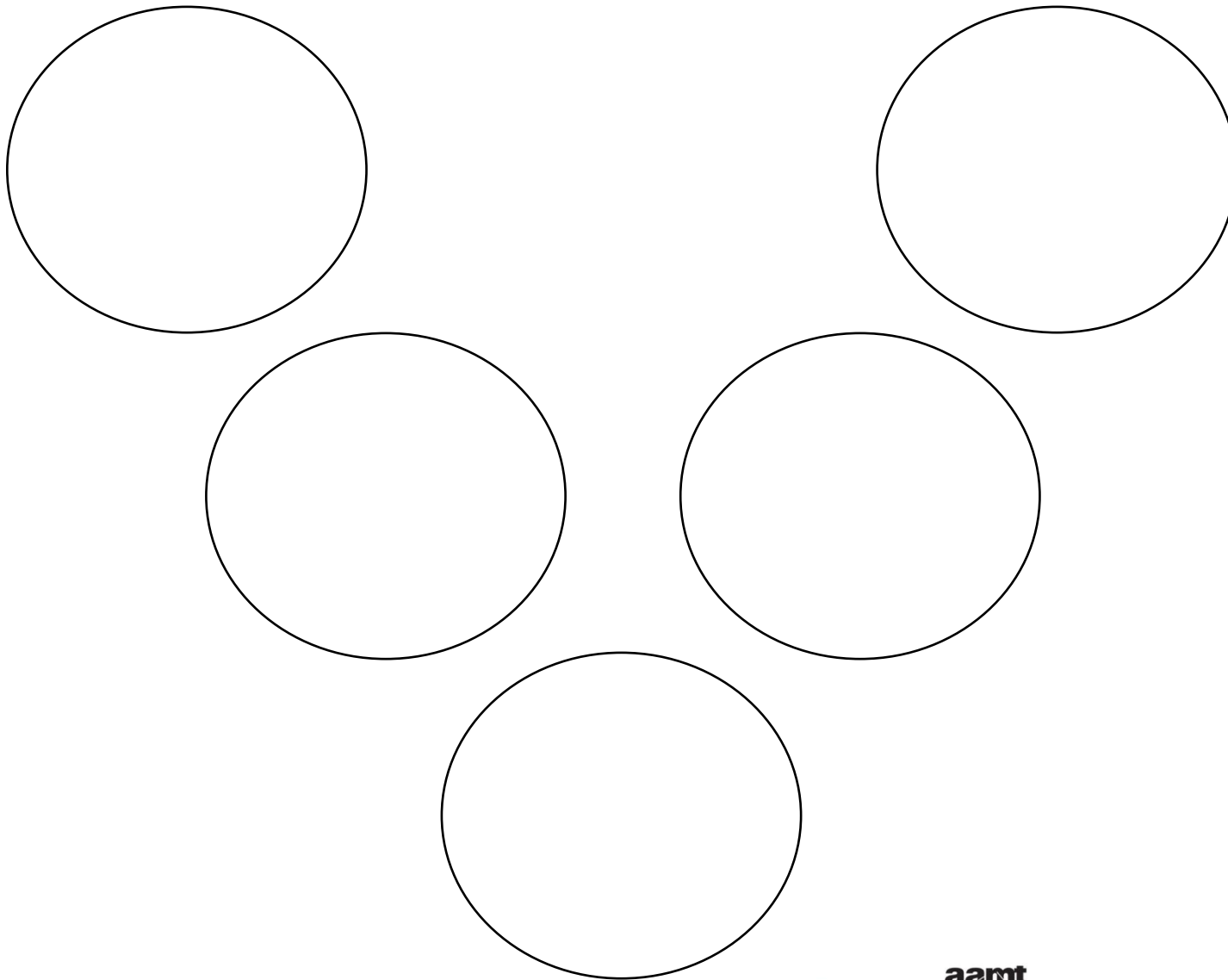
* Watertight argument about odd/even & totals of arms - gave examples to support justification

	Analysing	Generalising	Justifying
Not Evident	<ul style="list-style-type: none"> Does not notice common property or pattern. 	<ul style="list-style-type: none"> Does not communicate a common property or rule (conjecture). 	<ul style="list-style-type: none"> Does not justify.
Beginning	<ul style="list-style-type: none"> Recalls random known facts or attempts to sort examples or repeats patterns. 	<ul style="list-style-type: none"> Attempts to communicate a common property or rule for the pattern. 	<ul style="list-style-type: none"> Describes what they did and recognises what is correct or incorrect. Argument is not coherent or does not include all steps.
Developing	<ul style="list-style-type: none"> Notices a common property, or sorts and orders cases, or repeats and extends patterns. Describes the property or pattern. 	<ul style="list-style-type: none"> Generalises: communicates a rule (conjecture) using mathematical terms and records other cases or examples. 	<ul style="list-style-type: none"> Attempts to verify by testing cases and detects and corrects errors or inconsistencies. Starting statements in a logical argument are correct.
Consolidating	<ul style="list-style-type: none"> Systematically searches for examples, extends pattern or analyses structure to form a conjecture. Makes predictions about other cases. 	<ul style="list-style-type: none"> Generalises: communicates a rule using mathematical symbols and explains what the rule means or explains how the rule works using examples. 	<ul style="list-style-type: none"> Verifies truth of statements by confirming all cases or refutes a claim by using a counter example. Uses a correct logical argument.
Extending	<ul style="list-style-type: none"> Notices and explores relationships between properties. <p><i>balance division equal</i></p>	<ul style="list-style-type: none"> Generalises cases, patterns or properties using mathematical symbols (including algebraic symbols) and applies the rule. Compares different expressions for the same pattern or property to show equivalence. 	<ul style="list-style-type: none"> Uses a watertight logical argument. Verifies that the generalisation holds for all cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Enhance justification by exploring other possible Magic Vs e.g using numbers 2-6.

Magic V Template



1 2 3 4 5

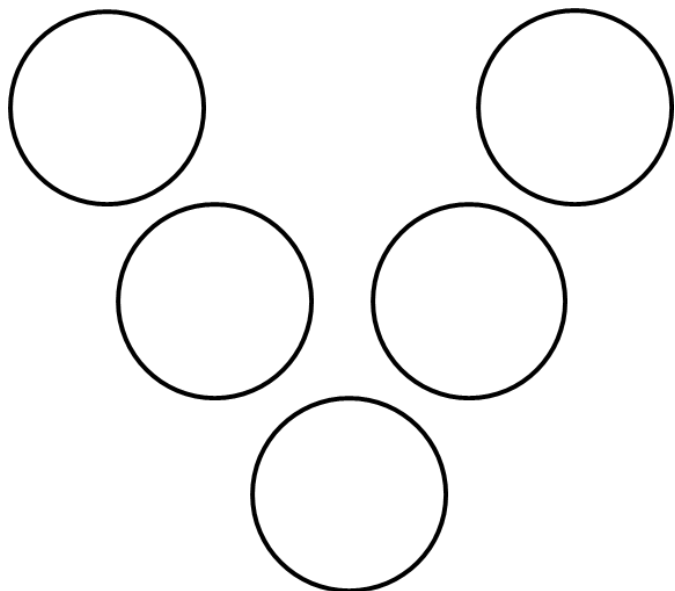
1 2 3 4 5

1 2 3 4 5

1 2 3 4 5

What is a Magic V?

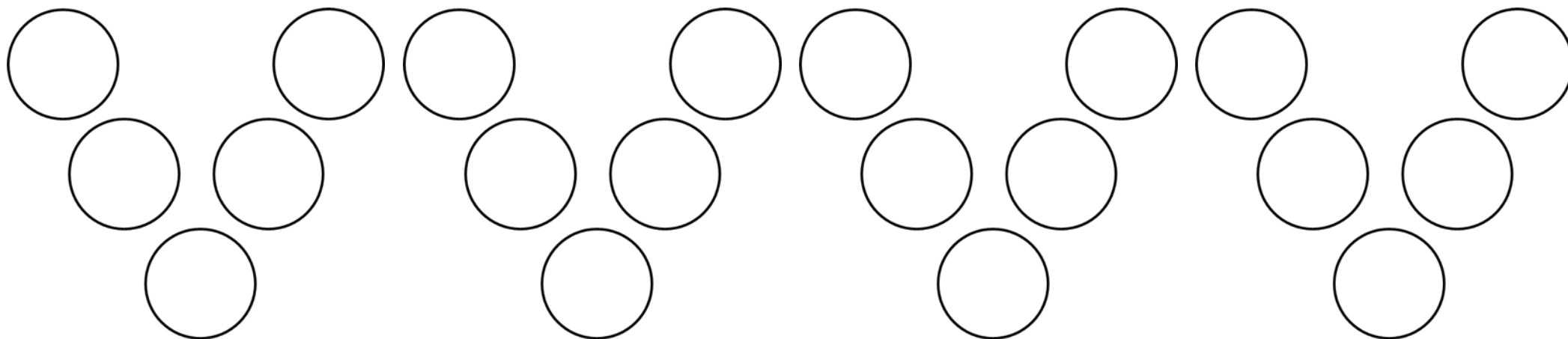
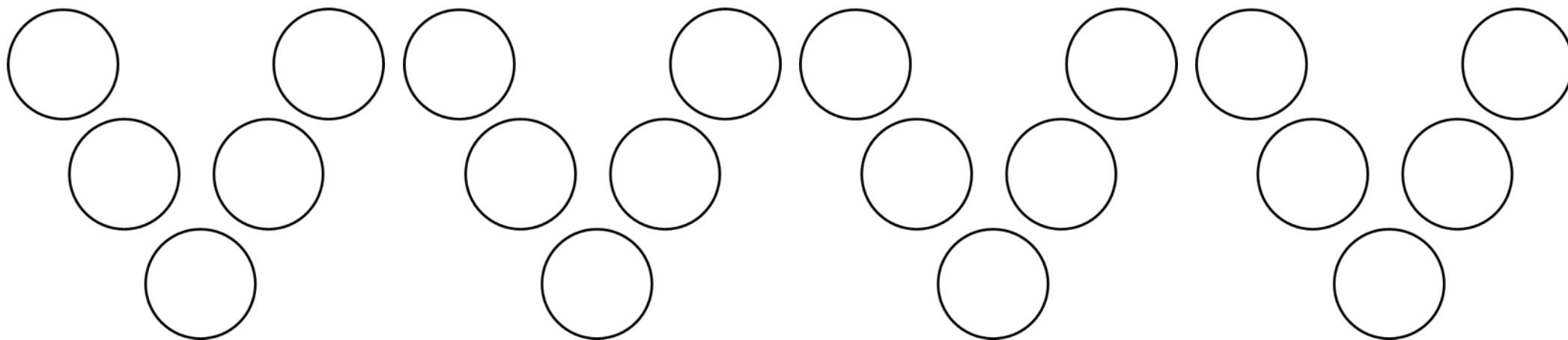
Name: _____



How do you know this is a Magic V?

Magic V Record Sheet

Name: _____



Sam said, “It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5.”

Is Sam right? Explain why or why not.

All Magic Vs Using Numbers 1-5

5 4
2 3
1

2 4
5 3
1

5 3
2 4
1

2 3
5 4
1

4 5
3 2
1

3 5
4 2
1

4 2
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3 4
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Student Name:

Reasoning Task:

Date:

<u>Observation of student's reasoning:</u>			
	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	<ul style="list-style-type: none"> Does not notice common property or pattern. 	<ul style="list-style-type: none"> Does not communicate a common property or rule (conjecture) for a pattern. 	<ul style="list-style-type: none"> Does not justify.
BEGINNING	<ul style="list-style-type: none"> Recalls random known facts or attempts to sort examples or repeats patterns. 	<ul style="list-style-type: none"> Attempts to communicate a common property or rule (conjecture) for a pattern. 	<ul style="list-style-type: none"> Describes what they did and recognises what is correct or incorrect. Argument is not coherent or does not include all steps.
DEVELOPING	<ul style="list-style-type: none"> Notices a common property, or sorts and orders cases, or repeats and extends patterns. Describes the property or pattern. 	<ul style="list-style-type: none"> Generalises: communicates a rule (conjecture) using mathematical terms, and records other cases or examples. 	<ul style="list-style-type: none"> Attempts to verify by testing cases, and detects and corrects errors or inconsistencies. Starting statements in a logical argument are correct.
CONSOLIDATING	<ul style="list-style-type: none"> Systematically searches for examples, extends patterns, or analyses structures, to form a conjecture. Makes predictions about other cases. 	<ul style="list-style-type: none"> Generalises: communicates a rule (conjecture) using mathematical symbols and explains what the rule means or explains how the rule works using examples. 	<ul style="list-style-type: none"> Verifies truth of statements by confirming all cases or refutes a claim by using a counter example. Uses a correct logical argument.
EXTENDING	<ul style="list-style-type: none"> Notices and explores relationships between properties. 	<ul style="list-style-type: none"> Generalises cases, patterns or properties using mathematical symbols and applies the rule. Compares different expressions for the same pattern or property to show equivalence. 	<ul style="list-style-type: none"> Uses a watertight logical argument. Verifies that the generalisation holds for <i>all</i> cases using logical argument.
<u>Comments (feedback, reasoning prompts for further development):</u>			