Cornering

Lesson 4: Modelling Four Wheels

Australian Curriculum: Mathematics (Year 8-10A)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (Year 8)

ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles. (Year 9)

ACMMG224: Apply trigonometry to solve right-angled triangle problems. (Year 9)

- Selecting and accurately using the correct trigonometric ratio to find unknown sides (adjacent, opposite and hypotenuse) and angles in right-angled triangles.

ACMMG272: Prove and apply angle and chord properties of circles. (Year 10A)

Lesson abstract

Students use new software models, first to investigate the effect of front and rear overhangs on space required for turning, and then including variable width. They investigate the effect of changing these variables on the space required for a vehicle to turn, including the lane usage. Students use the models to gather data for their final products, and sharpen their focus and scope. Clear communication of mathematical ideas is discussed.

Mathematical purpose (for students)

To investigate variables affecting turning space with a final model and to work towards final products.

Mathematical purpose (for teachers)

This lesson increases students' understanding of the underlying geometry of turning. It requires them to interpret the software models in real world terms, visualising how vehicles turn. They must frequently link components of the mathematical model with physical movements and spaces, using concepts such as lane usage. The lesson also allows the opportunity to practise investigative skills. Focussing on the final products (presentation and report) informs students' decisions about what to investigate and what data to collect from the model to be able to confidently make statements and recommendations. The process of drafting presentations and reports helps students clarify their objectives and evaluate if their investigations are giving them the information needed.

Lesson Length 65 minutes approximately

Vocabulary Encountered

- Lane usage
- Central island radius
- Circle terms such as tangent, concentric

Lesson Materials

- Slideshow *ST7_Cornering_4a_Four_Wheels.pptx*
- GeoGebra files *ST7_Cornering_4b_Four_Wheels.ggb* and *ST7_Cornering_4c_Angle_Overhang.ggb* (to display in class, and students to access a copy to their devices)
- Student Sheet "Vehicle Data" from Lesson 3 (students to bring)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

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Lesson structure

Models with Width and Overhangs (Teacher introduction followed by small-group work – 15 minutes)

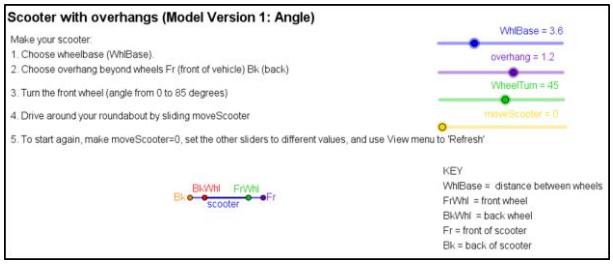
Investigating with the Model (Class discussion - 5 minutes followed by small-group work – 25 minutes)

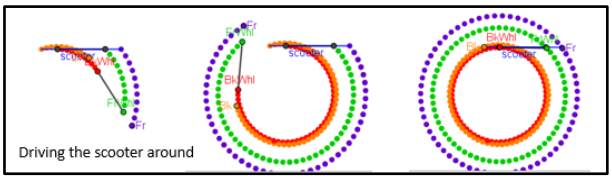
Drafting the Presentation and Report (Teacher introduction followed by small-group work – 15 minutes)

The Modelling Cycle (Class discussion – 5 minutes)

Models with Overhangs and Width

Scooter with overhangs (angle model)





This section improves the model of the last lesson by including overhangs beyond the front and back wheels. It is good to examine the effect of overhangs separately before introducing the added complexity of width (with the next model).

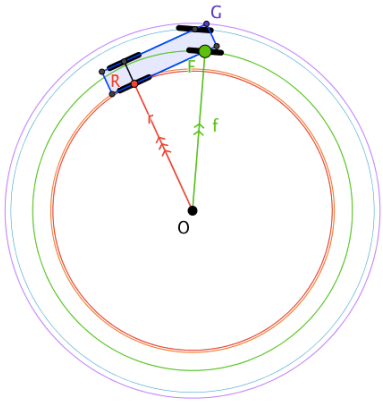
Introduce the model *ST7_Cornering_4c_Angle_Overhang.ggb* and show how the slider can be used to add an overhang at front and back to a scooter. Moving the scooter around (using the slider moveScooter) will show the traces of each corner point. Slide 2 shows the starting screen.

NOTE: We have called this no-width 'vehicle' a scooter only because it is easy to tape the front wheel of a scooter at a given angle. A bicycle behaves in the same way.

The software and Slide 3 of *ST7_Cornering_4a_Four_Wheels.pptx* can be used to draw out student observations and the following discussion points:

- The front overhang makes a lot more difference to the space required for turning than the back overhang. Do you see this taken into account in vehicle design?
- An interesting thing to notice is that the rear of the scooter does not 'cut in' as far as the rear wheel does, i.e. its path has a larger radius (traced in orange). This is not intuitive for students, but it can be very useful knowledge for drivers parking cars!
- Unless the rear overhang is extremely long, the back does not go outside the front wheel path.
- This model has a few limitations addressed in the next model:
    - It does not include the vehicle width.
    - The front and rear overhangs are always equal.
    - The model doesn't show the resulting radii (although these could be easily added to the model).

Radius model for vehicle with overhangs and width



Introduce the model *ST7_Cornering_4b_Four_Wheels.ggb*. Students will need a copy on their own device. This model is used for the rest of the unit. Be sure students understand the basic principles and terminology involved. Slide 4 shows a sample screen.

- You may wish to 'hide' the final text that says 'Lane usage =' to allow students to decide themselves what quantity indicates the lane usage. (To do this, go the View menu and select 'Algebra'. In the Algebra window, scroll down to 'Text 13' and click on the blue dot to hide it.)
- Be sure that students can see the whole screen as shown on Slide 4.
- Explain that the model is based on the Bicycle Radius model of Lesson 3, replacing the line segment by a rectangle. Although both front wheels steer a real vehicle, the model vehicle is moved by dragging just the inner front wheel F.

Allow students some time to familiarise themselves with the model, by working on the two questions below. The intention is that they begin to see which corners of a vehicle are most likely to 'leave the lane' during a turn and impinge on other road users. Highlight this during the discussions.

Which parts of the vehicle 'stick out' most? (Slide 5)

- Unless the rear overhang is very long, the rear inside corner does not go outside the red and green circles (i.e. paths of the inside wheels). (Noted in scooter model above.)
- The front corners seem to stick out more, especially on the outside of the curve (left hand side).
- Draw out from students' observations that the space used by the vehicle as it turns (lane usage) will be the outer radius OG minus the rear wheel path radius. This is important later.
- Note: road planners often use the 'swept path' of a vehicle, which is a diagram with a shaded area showing everywhere the vehicle covers as it turns. An internet image search for 'bus swept path' gives these diagrams for a variety of different situations (e.g. a bus driving straight toward a corner and then gradually turning). Students may find these visualisations helpful.
    - The shape of these swept paths can be linked to our model: the swept area clearly bulges out as the vehicle turns, that is, the vehicle takes up more width when turning than when driving straight ahead. In our model, the swept path is just a ring, because the steering wheel is set in one position.

Should we ever be concerned about the rear of a vehicle 'swinging out' to the left as it goes around a roundabout? (Slide 5)

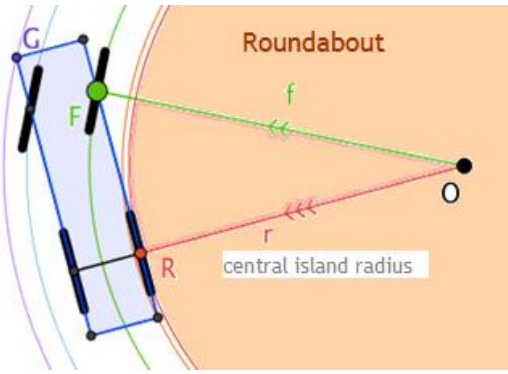
- When using their paper scale model in Lesson 1, students may have noticed the rear of the truck sticking out into the next lane, because this is one way of moving a paper truck with no wheels around a corner!
- If we model an extreme situation: a vehicle with a long rear overhang (e.g. 5m, with a front overhang of 0m, and a short wheelbase of 3m), the left rear corner sticks out further than the left front. This suggests that the left rear corner could sometimes be a problem for other vehicles, BUT ensure that students visualise what this extreme vehicle would look like:
    - A ute with a *very* long pipe hanging over the back might get close to these dimensions, but otherwise vehicles like this are not generally seen on the roads. The vehicle would need to be very heavy at the front and very light at the back to be stable even when standing still.
- It is unlikely that a vehicle like this would be on the road, so the left rear corner is not our main safety concern. However, we can learn from this to be careful around turning utes with long loads sticking out.
- Students may have seen the rear of a vehicle swinging out in a dramatic fashion, e.g. in rally driving. This is due to a loss of traction which allows the wheels to skid sideways. This is not a normal occurrence on our roads – high speed or a slippery surface is required – and not part of our 'rolling wheel' model.

Lane usage around a roundabout

Now students work independently or in groups with the model to apply what they have learned to driving around a roundabout. Assume that the vehicle must not drive over the central island of the roundabout. This means that the real wheel radius must be at least equal to the central island radius – the front wheel radius will need to be larger than that.

Vehicle	Wheel base	Width	Front Overhang	Rear Overhang
Van B	4.0 m	2.5 m	0.9 m	1.0 m

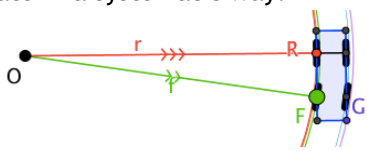
Roundabout central island shaded in red; vehicle 'uses' width of (OG-r) of the road

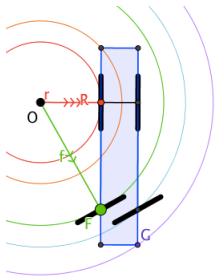


- *How much lane width will Van B need to go around a large roundabout with central island radius of 15m? (Slide 6)*
    - Set the vehicle dimensions first, and then play with the inner front wheel radius until the rear wheel radius is at least 15m, the roundabout radius.
    - The model gives a lane usage of 3.17m for an inner front wheel radius 15.6 m. This is for travel in the lane nearest the roundabout.
    - Lane usage is less if the vehicle travels in an outer lane (e.g. radius 18m has lane usage of 3.07m)
- *How big would a roundabout need to be for Van B to use no more than 3.5m of lane as it goes around? (Slide 7)*
    - A central island radius of at least 8.95 m is big enough.
- *Which vehicle has the largest 'lane usage' on a 12.5m radius roundabout? (Slide 8)*
    - Ask each group to test a few vehicles and provide their largest result to the class review.
    - The Student Sheet 'Vehicle Data' from Lesson 3 contains required data, as does Slide 8.
    - The spreadsheet *ST7_Cornering_4d_Vehicle_Dimensions.xlsx* with the same data is also provided.

Which new variable has the biggest effect on the lane usage? (Slide 9)

- Encourage students to work systematically changing one variable at a time. A good way of doing this is to increase one variable in steps of 0.1 and see how much the lane usage increases each step.
- Students will find that changing the rear overhang does not seem to increase the lane usage.
- There will be disagreement about the other variables (front overhang and body width), because the relationship between them and the distance OG is a complicated one. It is not necessary for students to understand in depth what is occurring here, but encourage them to investigate in a systematic way.
    - When the roundabout is quite large compared to the wheelbase, the angle of the front wheels is small, meaning that the vehicle is 'straighter', i.e. more in line with the lane. In this case the width has a greater effect on OG than the length, which is possible to see in the model by moving the width slider and watching how fast the purple circle increases, then repeating with the front overhang slider.





    - When the roundabout radius is similar to the wheelbase, the vehicle is oriented more 'across the lane' and you may find the front overhang has a greater effect on OG.
    - The effect of each variable is not exactly linear – e.g. as you increase the width by steps of 0.1 with the slider, the increase in OG varies slightly along the slider.
    - The effect of each variable also depends on which values of other variables.
    - **The most important thing is for students to be 'hands on' with the model and learning to use a structured approach as they investigate and test ideas. A conclusion that the situation is complicated and that both variables affect the lane usage significantly will be sufficient.**

Investigating with the Model

Show Slide 10 to review your requirements for the two assignments – road safety presentation and report to council. Talk through a few examples of what they may investigate and how the model might be helpful, such as:

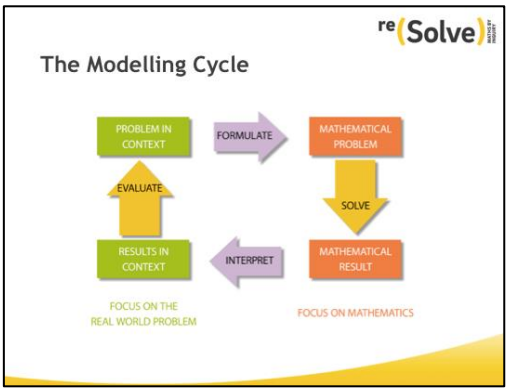
- students already know that the rear wheels of a vehicle cut in significantly if the vehicle is long and the turn is tight, and some data from the model could produce a helpful road safety presentation about not standing too near the edge of the curb when a bus is turning;
- students might provide advice to council on the space a new size of stretch limousine would require in several scenarios (e.g. turning left, right, entering and leaving a roundabout);
- some students might work through the optional activity 'Design a Car Park' in Lesson 4 to report to council;
- some students will be able to modify the GeoGebra models to answer their own questions;
- remind students that the present models do not work for articulated vehicles.

Students should now use any of the models supplied to work on the two assignments. It is likely they will be planning the scope of their investigation and using the model to investigate simultaneously. As they investigate, they may decide to alter the scope of their investigation.

Slides 11 and 12 may be helpful for a discussion with students about clear communication, tailored to the audience.

The Modelling Cycle

Conclude the lesson with slide13: The Modelling Cycle



- *What modelling have we been doing in this lesson?*
  - We have found mathematical results using the models, and then interpreted the results in terms of the real world problem, e.g. we have seen the paths of the corners of a vehicle as it turns, and have thought about which corners will have an impact on other road users, and we have found 'lane usage' and evaluated how it applies to the real world.
  - We have thought about how we can apply this model to slightly different situations.
  - We have encountered some assumptions used to build the models and to use them.
  - We have thought about how to communicate our advice based on results clearly.
- *Why is it important to be systematic in our investigations?*
  - So we don't 'jump to conclusions' without looking at all the possibilities.
  - So we can convince our clients that we have been careful, and our results are reliable.

Looking ahead

In the next lesson, groups will be finishing their presentations and reports, and then giving the presentations to the class.

An optional activity in Lesson 5 applies a modification four-wheel model to car parking, recommending parking bay widths and access road dimensions. Students also discuss the model's strengths and weaknesses for investigating this situation. Some students might use this activity for their report to council.