Cornering
Lesson 3: Software Models

Australian Curriculum: Mathematics (Year 8-10A)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (Year 8)

ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles. (Year 9)

ACMMG224: Apply trigonometry to solve right-angled triangle problems. (Year 9)

- Selecting and accurately using the correct trigonometric ratio to find unknown sides (adjacent, opposite and hypotenuse) and angles in right-angled triangles.

ACMMG272: Prove and apply angle and chord properties of circles. (Year 10A)

Lesson abstract

Students use dynamic geometry models that match their bicycle experiments. They observe what happens to the rear wheel path as they vary the wheelbase or the tightness of the corner. Students then use the models to predict how much the rear wheels will cut in for a variety of real vehicles. The final products and human factors in vehicle safety are discussed.

Mathematical purpose (for students)

To explore, use and critique some dynamic geometry models of the bicycle experiment.

Mathematical purpose (for teachers)

Students firstly make links between their bicycle experiments and the resulting diagrams and the provided dynamic geometry models (which are noticeably more abstract and more general). The models are then linked to the scenario of a very long bicycle moving around a corner or a roundabout. Students see a relationship between the size of the corner, the length of the wheelbase and the absolute and relative size of the rear wheel cut-in distance. They also see the how the model goes beyond reality, predicting physically impossible behaviour, thus showing the need for constraints in modelling.

Using real vehicle data in the model helps students make connections to the real world context, prompting sensible questions about how to decide what is safe, and what effects the width and full length of the vehicle may have.

Lesson Length 50 minutes approximately

Vocabulary Encountered

- Critique
- Dynamic geometry

Lesson Materials

- Slide show: *ST7_Cornering_3a_Software_Models.pptx*
- GeoGebra files *ST7_Cornering_3b_Radius_Bicycle.ggb* and *ST7_Cornering_3c_Angle_Scooter.ggb* (students to save a copy to their devices if possible)
- Student Sheet – Vehicle Data (1 each, or supplied spreadsheet)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

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Lesson structure

Software Models Based on the Bicycle (Teacher introduction followed by small-group work – 15 minutes)
Our Final Products (Class discussion - 5 minutes)
Using Real Vehicle Data (Small-group work – 15 minutes)
The Modelling Cycle (Class discussion - 5 minutes)
Refining the Models (Small-group work – 10 minutes)

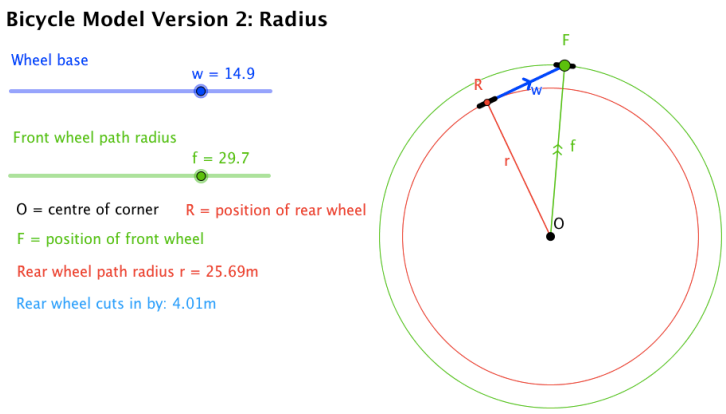
Software Models Based on the Bicycle

In this section, students gain further insight into the mathematical structure of the problem using interactive geometrical models of a bicycle or scooter. It is a good idea for each group to use both models, to get a broader understanding of the situation. The Radius model *ST7_Cornering_3b_Radius_Bicycle.ggb* is the basis for the four-wheeled model in Lesson 4, so must be used by all groups.

Slide 2 of *ST7_Cornering_3a_Software_Models.pptx* can be used to recap the previous lessons.

Show the file *ST7_Cornering_3b_Radius_Bicycle.ggb*, and then *ST7_Cornering_3c_Angle_Scooter.ggb*. (Images are also given on slides 3 and 4). Students should recognise them as similar to their diagrams from Lesson 2. Demonstrate the 'moving parts'.

- Enter some data from the experiments and confirm that the model predicts fairly well the rear wheel cut-in distance.
- Ask students how the model is similar/different to the sketches they made last lesson. They may need time to 'play' with the models before they answer.
- Emphasise to students that these working diagrams are mathematical models. No 'wheels' are being used for the predictions. The models are working on formulas that reflect reality as well as we understand it. You might like to show the "algebra window" in the GeoGebra files to highlight the variables and relationships that the dynamic diagrams are built on, without going into details.



Features:

- Sliders to input wheelbase and front wheel path radius. The situation depicted is that the front wheel is driven along the green circle.
- Front wheel can be dragged to visualise the movement.
- Paths of both wheels are circles. Values for rear wheel path radius and cut-in distance are automatically calculated and displayed.
- Optional: the model has been constructed based on the known geometric features from last lesson (i.e. the right-angled triangle).

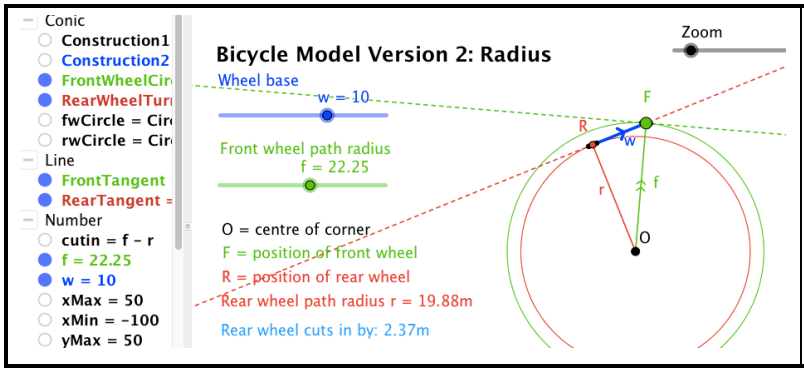
Comparison to Lesson 2 Sketched Model:

Similarities:

- It has 2 circles with the same centre.
- The front wheel is shown turned in to the corner, while the back wheel stays parallel to the bike frame.
- The bike is shown as a line.

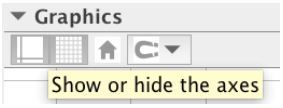
Differences:

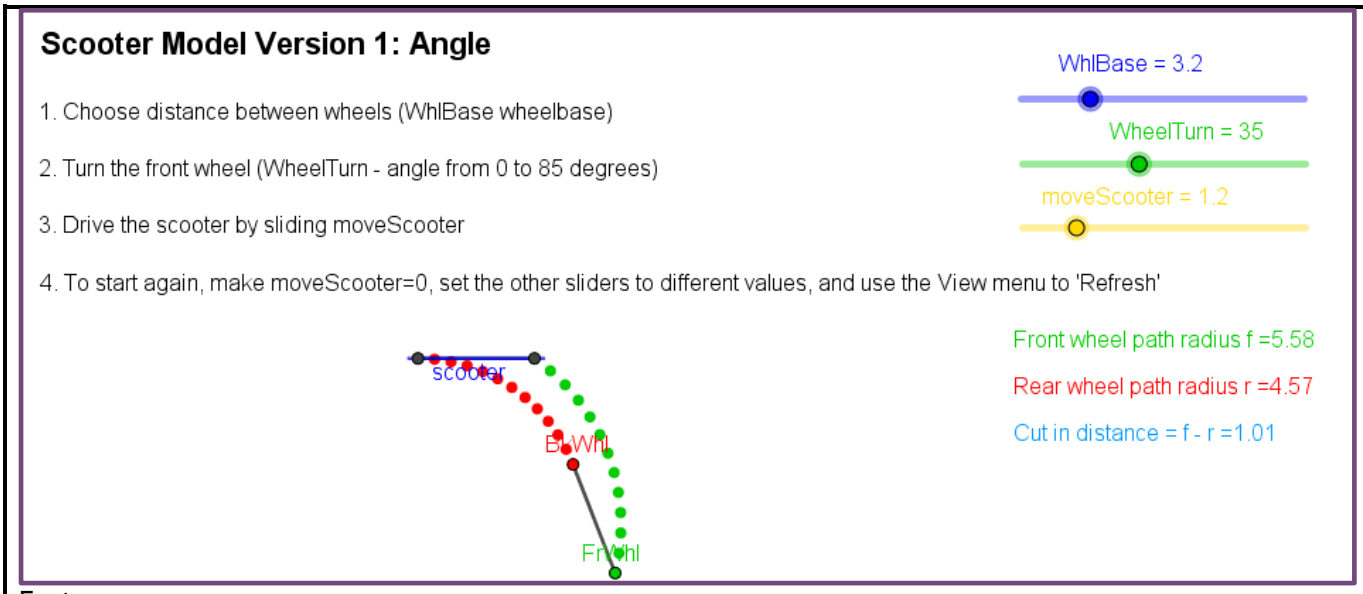
- You can move the model around in a circle (representing moving the front wheel along the green circle).
- You can change the size of the corner, and the length of the bike. It is easy to see how these things affect how much the back wheel cuts in.
- It calculates the cut-in amount (difference in radii) for you.



Construction lines are hidden – for the curious they can be found by clicking on their entries in the Algebra window (e.g. from the View menu select 'Algebra' to see the Algebra window, then click the dot for FrontTangent and RearTangent to make them visible).

The axes and grid can also be shown if desired:





Features:

- Sliders to input wheelbase and front wheel angle.
- Paths of wheels are shown to be circles, by the trace as the scooter moves (using the slider moveScooter). The image below shows the partially completed path.
- Values for front and rear wheel path radii and cut-in distance are automatically calculated and displayed.

Note: the model has been constructed by using only the angle of turn and wheelbase to find the centre of rotation, then path radii.

Comparison to Lesson 2 Sketched Model:

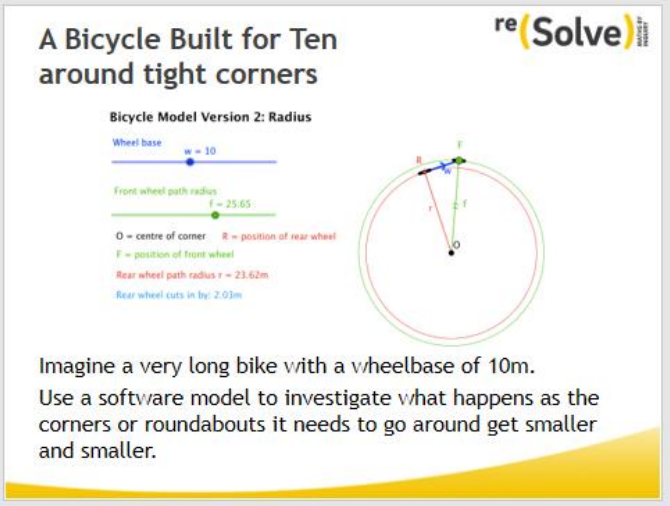
Similarities:

- It has 2 circles with the same centre.
- The scooter is shown as a line.

Differences:

- You can move the model to demonstrate that the scooter goes around in a circle.
- You can't see the wheels or where they are turned – only the path that they follow.
- You can change angle of the front wheel, and the length of the scooter. You can then see how these things affect how much the back wheel cuts in.
- It calculates the cut-in amount (difference in radii) for you.

The bicycle built for ten around tight corners



Students now examine how wheelbase length influences turning, by imaging a very long bicycle. They investigate with the models what happens as a long bicycle with a wheelbase of 10m tries to turn around smaller and smaller corners or roundabouts.

- The activity is shown on Slides 5 and 6 A Bicycle Built for Ten around tight corners.
- Screenshots of the models are given in the table below. Set the wheelbase to 10 (metres) for both models.
- Encourage students to try always to keep a picture in their minds of how the real bicycle would be moving.
- For some students, a roundabout is a simpler context than a corner, because the model is a complete circle. Other students may be happy imagining the corner as a quarter of the circle in the model. Either interpretation will work here, and in the subsequent lessons.

Expected Student Responses

- Both models show that when we make the roundabout smaller (travel around in a tighter circle), the rear wheel cuts in more and more.
- The cut-in distance also becomes a larger fraction of the roundabout radius.
- Both models eventually reach a stage where the front wheel travels in a circle, but the back wheel is just pivoting on the spot. Then the cut-in distance is as long as the wheel base. This requires the angle of the front wheel to be perpendicular to the body of the vehicle.
- Ask students to discuss and report whether the results would apply to long bicycles, buses and trucks.
    - It is not possible for the front wheel to turn to 90° on any conventional vehicles.
    - The maximum angle through which wheels can turn is generally around 40° to 50°. The table below shows an approximate limit. The turning circle of a vehicle relates to this maximum angle and the vehicle dimensions.
    - An interesting invention is this wheel which can drive in more than one direction! https://www.youtube.com/watch?v=_tmiu1wpp_E

What are the implications for vehicles with long wheelbases?

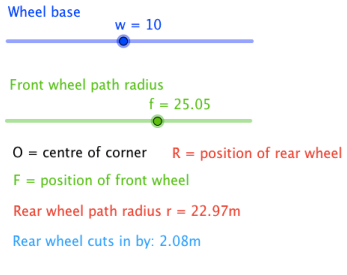
- Vehicles with long wheelbases will use extra sideways room when turning around small roundabouts or corners. This may mean going over the kerb, or into the next lane.

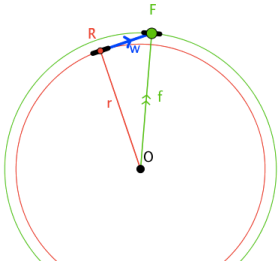
What happens if you make the wheelbase very short compared to the roundabout radius?

- This simulates the situation of something like a real bicycle going around a large roundabout. This combination does not cut in very much at all. The rear wheels travel around almost the same circle as the front wheels.

Modelling a long bicycle with 10m wheelbase and decreasing roundabout size

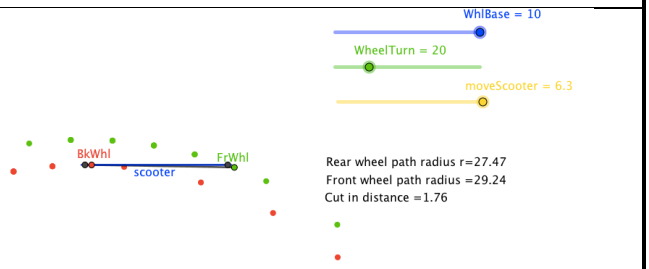
Version 2 Radius Model



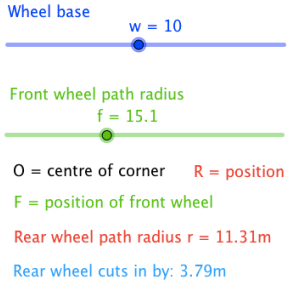


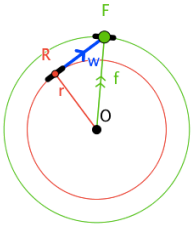
This 10m bicycle could travel around a roundabout with radius 22.97 m. The lane would need to be at least 2.08m wide.

Version 2 Angle Model

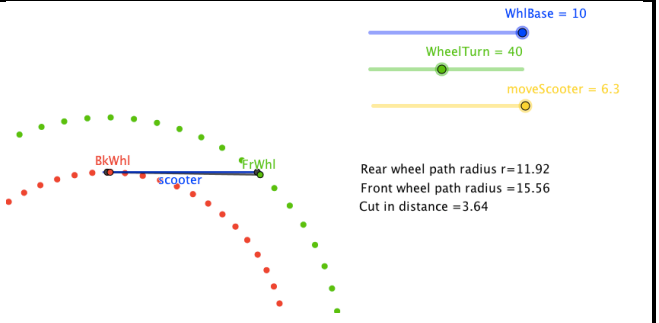


Setting wheels at angle of 20 degrees, this 10m bicycle could travel around a roundabout with radius 27.47 m. The lane would need to be at least 1.76m wide.

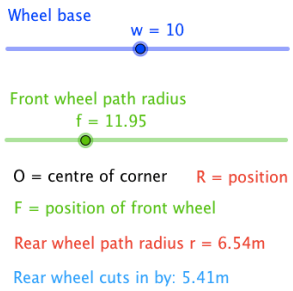


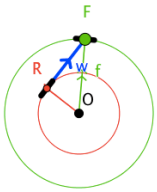


A car with this wheel base is often not able to turn with a small front wheel path radius than this.

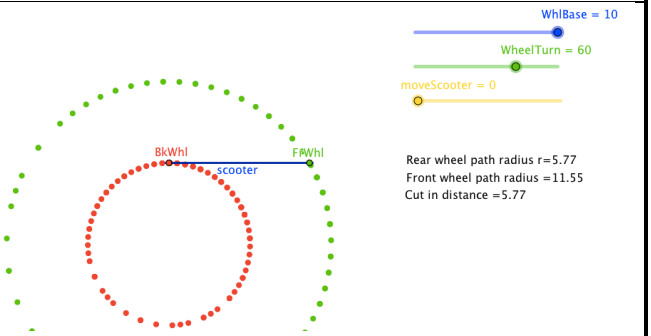


This wheel turn angle is about the maximimum for many cars.





Most cars can't turn as sharply as this.



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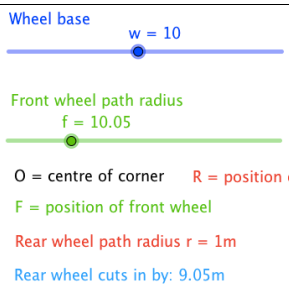
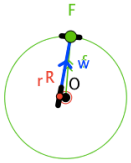
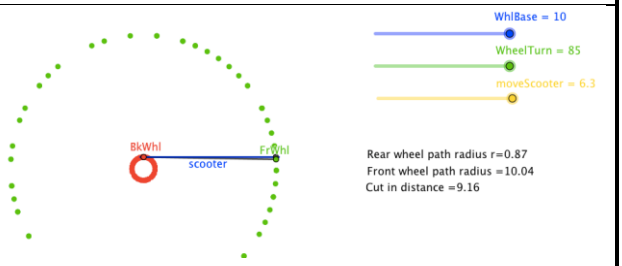




The diagram shows that the front wheel is nearly at right angles to the bicycle. In reality, wheels are not made to turn so much.



The front wheel is nearly at right angles to the bicycle. It almost pivots on the spot, around the back wheel.

Towards the Final Products

Slide 7 offers suggestions for two final products with different audiences to assess this unit. Modify to suit your requirements and discuss expectations and procedures with students. These suggestions give a lot of choice; you may wish to reduce this.

Road Safety Presentation

- To inform road users about being safe around long vehicles.
- Poster or a very short presentation (e.g. 2 mins)
- For a general audience (e.g. TV audience, local primary school)

Report to local council on road design

- To make recommendations to technical department about an aspect of road design (giving mathematical evidence)
- Examples:
    - Size of roundabouts so that long vehicles can get around safely,
    - Width of city streets for long vehicles to go around corners
    - Size of car parking bays (some students could use the optional activity in Lesson 5 for this)

Discuss your expectations for the two products, and review how what has been found so far might assist.

A discussion of some of the 'human' factors involved in road safety may be useful here. For example, when a pedestrian decides how far to stand from the path of an approaching bus, what reasoning are they using? Are they looking at the path of the front wheels and standing 1m back? Or are they just standing on the edge of the curb? And how risky is each of these decisions?

Using Real Data

Some real data is presented on Slide 8 and on Student Sheet 1 – Vehicle Data. A spreadsheet *ST7_Cornering_4d_Vehicle_Dimensions.xlsx* is also supplied.

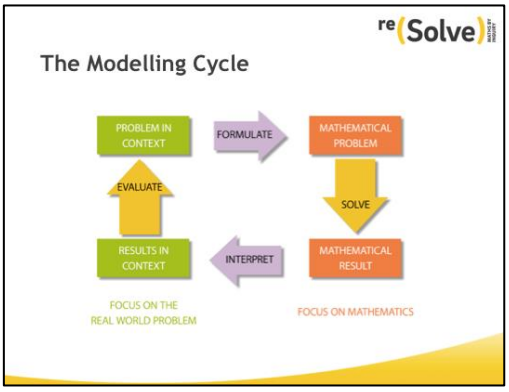
Students should try a variety of combinations of vehicle and roundabout size, and record the rear cut-in for each. Note: by entering the roundabout size as the front wheel path radius we are assuming that the driver matches the path of the front wheel to the edge of the roundabout. A knowledgeable driver would not do this!

Ask students to record which combinations seemed 'OK' and which might be problems for other road users.

- This will involve discussion of what an acceptable cut-in distance would be. It is not intended that students come up with a definite answer, but that students be evaluating the data they are gathering in terms of their final products - asking themselves what they need to find out. They may for example think about:
    - How far back will pedestrians tend to stand?
    - How much 'cut-in' will make the vehicle run over the edge of the roundabout? This will also depend on the lane size and the width of the vehicle. For now, students may decide to approximate lane usage by adding the width and the cut-in distance. This is a rough approximation of lane usage, because the vehicle width and the path radius are not in line, but at an angle. This is discussed further in Refining the Models below.
- The vehicle data includes width and overhangs, which are not included in the current models. They will be included in the next models (Lesson 4).
- Students may wish to look online (or in the real world) for other vehicles to enter into the model. They will need to get data for wheelbase, width, and front and back overhangs for use in Lesson 4. Note: if they find rigid trucks with two rear axles it is best to define the wheelbase as from the midpoint of the two rear axles to the front axle.
- Students may wish to know the standard lane width – it is around 3.3-3.5 metres.
- Note: the sliders on the models will sometimes skip a value. If student have trouble getting the slider to show a value they want to test, the value can be directly entered: right click on the slider, select Object Properties->Basic, and change the Definition to be the desired value. Close the Object Properties window. The slider will still work as before but will now be displaying the desired value.

The Modelling Cycle

Show slide 9: The Modelling Cycle



- *What modelling have we been doing in this lesson?*
  - We have learned to use models made by someone else. (It is quite common in workplaces to have to work with models that have been developed by others).
  - We have found mathematical results using the models, and then interpreted the results in terms of the real world problem.
- *What are the limitations of our models? What has been simplified?* (This is an opportunity for students to be critical about the bicycle models.) Slide 10 shows images for the discussion.
  - The models only use the wheelbase of the vehicle – they do not consider the width or front and back overhangs.
  - The model gives us the cut-in distance, but there is more thinking needed to work out what this means for road safety and design.
- *How do you think we could improve our models? What extra information could the model show to help us think about safety, roundabout size and lane width?*

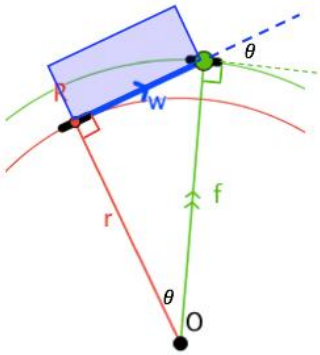
Refining the Models

In the next lesson, students will be given a dynamic geometry model that incorporates width, and front and back overhangs. Before seeing this, it is useful for students to attempt to modify their existing model by adding at least one of these factors.

The reason incorporating width is obvious, but the need to think about overhangs on the front and back might not be so obvious. You might like to have a class poll of answers to the following questions, which will be answered next lesson:

- On a real bus, which point has the greatest path radius (so requires the largest turning space)?
- On a real bus, which point has the smallest path radius (so the greatest cut-in)?

Example of modification of existing model



To include vehicle width a student might draw a sketch of their bicycle model, then change the line representing the bicycle to a rectangle. This image can be demonstrated with slide 11. It will become clear that the front left corner of the vehicle is the furthest point from the centre of the roundabout. Finding this distance is difficult (the cosine rule can be used), highlighting the usefulness of a dynamic geometry model which automatically calculates the distance.

Students who are proficient with dynamic geometry software may wish instead to modify one of the software models. They may need to be aware of angle and chord properties of circles e.g. *the angle in a semicircle is 90°*.

It is not vital that students refine the model fully – attempting to add features will help them think more deeply about the model, and be prepared for next lesson's models.

Looking ahead

In the next lesson, students will be exploring new models, deciding on the scope of their final products, and using the models to investigate and gather information for their road safety presentation and roundabout report.

Vehicle Data

Name: _____

Below are the dimensions of some real vehicles. Try some of them in the model with different roundabout sizes. Roundabouts have radii from 5m to 25m or more.

Keep a record of which combinations might be a problem for other road users.

Vehicle	Wheelbase (m)	Width (m)	Front Overhang (m)	Rear Overhang (m)	**Comments**
Truck A	3.8	2.4	1.4	1.8	
Truck B	6.0	2.4	1.4	3.5	
Truck C	5.0	2.5	2.2	3.0	
Bus A	5.8	2.5	2.7	3.5	
Bus B	6.3	2.5	2.6	3.1	
Bus C	6.0	2.6	2.6	3.3	
Bus D	7.0	2.4	1.0	3.9	
Bus E	7.3	2.3	1.2	2.3	
Bus F	8.2	2.5	2.6	3.7	
Minibus A	3.5	1.9	0.8	0.8	
Van A	3.1	1.8	0.8	0.7	
Van B	4.0	2.5	0.9	1.0	
Ute A	3.0	1.7	0.6	1.2	
Ute B	3.2	1.9	0.9	1.0	
Car A	2.7	1.5	0.8	1.0	
Car B	2.5	1.7	0.7	0.5	
Car C	4.4	1.9	0.9	1.1	
Car D	8.4	2.0	0.7	0.7	

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