Cornering
Lesson 5: Presentations and Parking

Australian Curriculum: Mathematics (Year 8-10A)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (Year 8)

ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles. (Year 9)

ACMMG224: Apply trigonometry to solve right-angled triangle problems. (Year 9)

- Selecting and accurately using the correct trigonometric ratio to find unknown sides (adjacent, opposite and hypotenuse) and angles in right-angled triangles.

ACMMG272: Prove and apply angle and chord properties of circles. (Year 10A)

Lesson abstract

After time to complete their work, students make their road safety presentations, and give each other feedback. An optional activity uses the model in a new way to design a car park. The class reflects on the modelling process throughout the unit.

Mathematical purpose (for students)

To effectively communicate what we have learned about road safety and road design.

Mathematical purpose (for teachers)

The two assignments (a short presentation to inform the public about road safety and a report to inform the council about an aspect of road design), require students to communicate what they have learned to different audiences, giving an appropriate amount of explanation and evidence.
The final reflection is intended to help students realise the power of the iterative approach of mathematical modelling. The models developed from a very simple beginning (paper cut-outs) to using sophisticated geometry. Students did not make all the models themselves, but they experienced understanding, using, critiquing, and recommending refinements. They can review these processes. The final discussion about applying the model in other situations highlights how a model can be used beyond its current context, providing care is taken.

Lesson Length 75 minutes approximately

Vocabulary Encountered

Lesson Materials

- Slide show *ST7_Cornering_5a_Presentations.pptx*
- GeoGebra file *ST7_Cornering_5b_Parking.ggb* (for optional activity)
- Student Sheet - Parking a Car (for optional activity)
- Student sheet "Using Real Data" from Lesson 3 (or spreadsheet)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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Lesson structure

Finalising and giving Presentations and Reports (Group work – 60 minutes)
Reflections on The Modelling Cycle (Class discussion - 15 minutes)
Optional Activity: Design a Car Park (Group or individual work – 30 minutes)

Presentations and Reports

- Allow students time to finish their road safety presentations and perhaps their written council reports.
- For students who have time available, use the optional activity Design a Car Park given below. Alternatively, this could be set as an assignment.
- The road safety presentations should only take a few minutes each. Organise sufficient to fit the time available, selecting for variety and use of the models.
    - Some presentations may need to be given in parallel to parts of the class.
- As students watch and listen to the presentations, they should make notes to constructively critique each other's presentations You might use the questions on Slide 2 for structure.
    - What did you learn from the presentation?
    - Is it suitable for a member of the target audience (e.g. a person from another class)?
    - Is it important to change or add anything?
- Organise for students to submit their written council reports, as desired.

Reflecting on the Modelling Cycle and Models

Lead a discussion on the modelling processes (Slide 3) across the unit. Hopefully students will feel a sense of accomplishment as they look back and see how much they have learned about the situation! Working lesson by lesson (see below), ask students what they learned from each model, and to recall some assumptions, relationships strengths and limitations of each model. (There may be some overlap between these questions, e.g. being able to learn something from a model is a strength of the model!).

Lesson 1: Sharing the Road (Slide 4)

- We started with the "Do not overtake turning vehicle" sign. Why is it used?
- How did we get started trying to understand the situation? What did we learn? What were the strengths and limitations of this model?
    - Cut-out paper scale model.
    - We learned that vehicle length can cause a problem when going around corners.
    - The model was easy to make, but it was hard to work out exactly where the vehicle would go.
- How did we develop a better model?
    - We realised we needed wheels to really see what was happening.

Lesson 2: A Better Model (Bicycle Experiments) (Slide 5)

- What did we learn from our experiments?
    - We could clearly see that the rear wheel was cutting in.
    - We could see that both wheels followed circles when the wheel angle stays constant.)
    - We saw that the circles had the same centre, and the wheels were tangents to their circles.
- What were the strengths and limitations of this experiment and using a real bicycle as a model?
    - It was using an actual vehicle, so it really helped us understand.
    - It took a lot of time to do the experiment and get accurate measurements.
    - It helped us draw a diagram of the situation and see the geometry.

Lesson 3: Software Models (Bicycle Models) (Slide 6)

- What did we learn from the software models?
    - We could clearly see how far the rear wheel cut in each time.

    - We could change the wheelbase and the roundabout size and see how it affected the cut in distance.
    - We saw that having a small wheelbase compared to the roundabout size meant that the rear wheels didn't cut in very much. In real life this could be a car going around a big roundabout.
- What were some of the assumptions of these models?
    - They assume that the vehicles are driving in a perfect circle, meaning that the steering wheel is turned to a particular angle and held there.
    - The vehicle has no width. It is a single line the length of the wheelbase. Real vehicles have width and overhangs at front and back.
- What were the strengths and limitations of this model?
    - We could clearly see how far the rear wheel cut in each time.
    - We could easily change the wheelbase and the roundabout size and see how it affected the cut-in distance.
    - We could enter real vehicle data and find the cut-in distance, without having to do an experiment.
    - It is difficult to imagine exactly where a wide vehicle would go, because the model only shows one set of wheels (because it was based on a bicycle/scooter/motorbike).
    - We could only give road safety advice about the inner set of wheels. It would be better to be able to see the whole vehicle in the model.
    - We needed the width of the vehicle to help us get a better idea of lane usage on roundabouts.

Lesson 4: Four Wheels (Slide 7)

- What did we learn from these more complicated models?
    - We could see that the left front corner usually sticks out the most, and that the rear corners don't get in the way very much at all.
    - We learned that the model is quite complicated: we needed to be systematic in our investigations, so we could be sure that we had covered all the possibilities. For example, it looked at first as if the rear overhang didn't affect lane usage, but when we tried an extreme example it did.
    - We learned about how to relate the model to the real world – e.g. thinking about where a pedestrian would be standing, or how close another car might be.
- What were some strengths and limitations?
    - We could try a lot of different real vehicles using the width and overhang sliders.
    - The 'lane usage' was calculated for us.
    - When we wanted to investigate a truck turning a corner to the right instead of going around a roundabout, we had to imagine what was happening.

Lesson 5: Presentations (Slide 8)

- What challenges did you face when you were planning and giving your safety presentations and writing your council reports?
    - It was sometimes hard to know what to put in and what to leave out.
    - Sometimes it is hard to communicate your ideas. We knew what we wanted to say but it was hard to explain.
    - We thought we knew what was happening when the bus was turning the corner, but when we tried to explain it we realised that we had left something out.

Lesson 5: Design a Car Park (optional activity)

Students who have worked on the car parking activity can present it here.

Further Applications of the Models

Conclude the unit by asking students to think of other real-life contexts in which a model like this could be used.

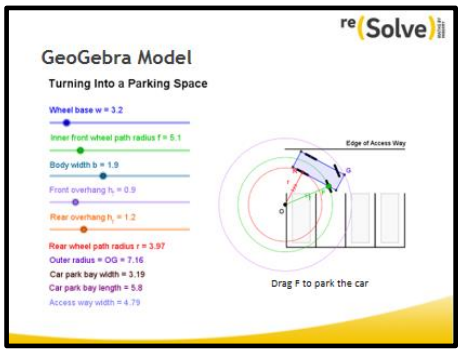
- It is most appropriate to situations where a vehicle or object moves in an arc on wheels. Examples include parking aircraft at an airport, moving goods in and around warehouses, and fitting and manoeuvring some furniture on wheels.
    - Not everything with wheels turns like these vehicles. A sports wheelchair, for example, can spin on its axis.
    - Designing shipping lanes is a 'no wheels' possibility because the turning circles still apply.

- If students are interested in trains, they may wonder how train carriages turn corners when their front and rear wheels must run on the same track (i.e. have the same path radius). They may like to research this. (A train carriage is not the same as a road vehicle. Each set of wheels is attached to a platform (called a bogie) which is attached to the carriage by a pivot. Put simply, this means that unlike a bus, the 'rear' wheels can turn as well as the front wheels. The situation is more complex than this, because each bogie has 4 wheels and hence its own wheelbase. The very slight 'cut in' here is dealt with by having sloping wheels that can shift horizontally slightly but still be on the track. Wikipedia has some nice diagrams of bogies and train wheels: https://en.wikipedia.org/wiki/Bogie and https://en.wikipedia.org/wiki/Train_wheel)

Optional Activity: Design a Car Park

This activity is for students who have additional time available. It shows that a model can be reused in a new situation, but also to show that care needs to be taken when doing so. The limitations of the model in this new situation are investigated. Some useful images are supplied on Slides 10 – 12.

Students need the GeoGebra file *ST7_Cornering_5b_Parking.ggb*, Student Sheet – Design a Car Park, and the vehicle data handout from Lesson 3. They should be able to work through the activity without assistance, but these hints may help.



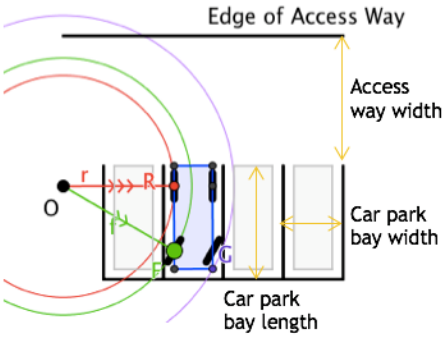
- The car is parked by dragging the front inner wheel (point F). The car follows a circular arc into the bay, after lining up 'horizontally'.
- Access way width.
    - The Access way width reported on the screen is the minimum road width *required by the given vehicle*, turning into the parking bay with the selected degree of turning (i.e. using selected inner front wheel path radius).
    - The Access way width required cannot be directly altered. Instead, it can be set to a desired width for any vehicle by manipulating the front wheel path radius. (It need not be exactly 4.8m etc.)
- Observations
    - A wider real access road allows a bigger front wheel path radius, hence smaller cut-in distance and smaller lane usage. This results in a smaller parking bay width.
    - Students may notice that the front left corner grazes the next vehicle as it passes, and that the model has been set up so that this always happens. We wouldn't want this to happen in real life.
    - Students may further realise that the parking bay width is the same as the 'lane usage', but the parking bay has been drawn slightly to the left so that the car finishes neatly in the car space.
    - The underlying assumption of the model, that a driver turns in a single circular arc to park, may not be sensible to students. It is this assumption that results in the car 'invading' the adjacent bay. You may want to describe your parking technique to help students understand!
- "Which parking bay dimension do you think is most important: the width or the length?"
    - Students should realise that a vehicle being a bit longer than the car space will probably not cause any problems. In the model, if the grey vehicles were a bit longer they would not be in the way.
    - If the parking bay is not quite wide enough, vehicles will find it difficult to get in to the space without touching other cars.
- "Based on the model, would you recommend any modifications to the council's rules?"
    - Students may suggest some minor modifications, but the model's results are fairly well aligned to the council's rules.
    - Students may feel that they cannot trust the model because of its assumptions. Encourage them to think about what refinements they could make, or how they would remake the model.

Another GeoGebra model to investigate turning vehicles is available at https://www.geogebra.org/m/RTP9f8Em. It is quite sophisticated and fun, with a turning steering wheel, and forward and reverse motion. Enjoy!

Design a Car Park

Name: _____

The file *ST7_Cornering_5b_Parking.ggb* shows a free car parking space, with cars parked in the bays on either side.



- Drag point F to see the car drive into the parking bay.
- Enter vehicle data for the vehicles below. Record the required parking dimensions as predicted by the model.
- Which vehicle needs the widest parking bay? Explain why.
- Adjust the 'inner front wheel path radius' (i.e. how sharply vehicles are turning) to be larger and smaller. How does this affect the parking bay and access way widths required?

Inner front wheel path radius 6.5 m							
Vehicle	Wheelbase (m)	Width (m)	Front Overhang (m)	Rear Overhang (m)	Total car length (m)	Predicted parking bay width (m)	Access way width required (m)
Ute A	3.0	1.7	0.6	1.2			
Ute B	3.2	1.9	0.9	1.0			
Car A	2.7	1.5	0.8	1.0			
Car B	2.5	1.7	0.7	0.5			

The local council wants to make the following car park bay dimensions the rule for all carparks. They would like you to decide whether the dimensions will allow vehicles to park safely.

- Use the model to test if each vehicle will be able to park in each of these parking bays.
- Are there any other vehicles that might want to park in these bays? Test some more vehicles.
- Which parking bay dimension is most important: the width or the length?
  (Think about what would happen if the parking bay were not quite wide enough, or not quite long enough.)

Access way width (m)	Parking bay width (m)	Parking bay length (m)
6.4	2.6	5.0
5.8	2.8	5.0
5.2	3.0	5.0
4.8	3.2	5.0

- Based on results from the model, would you recommend any modifications to the council's rules?

What are some of the assumptions in this model?

What are the strengths and limitations of this model?

How would you improve the model for giving advice to the council on car park design?

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