Introduction to Modelling
Lesson 3: Understanding the Modelling Process

Australian Curriculum: Mathematics (Years 8 and 9)

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

- identifying direct proportion in real-life contexts

ACMNA 192: Simplify algebraic expressions involving the four operations. (Year 8)

Lesson abstract

Students work through the modelling process as summarised in the modelling diagram. After reviewing their work from Lesson 2, they link their work on theme park queues to each stage to the modelling process, and reflect on how this helps them to analyse real problems and get realistic solutions.

Mathematical purpose (for students)

The modelling diagram provides insight into mathematical modelling.

Mathematical purpose (for teachers)

The aim is to develop a deeper understanding of mathematical modelling and the key processes involved, summarised with the diagram of the modelling cycle. The mathematics already introduced is looked at in more depth - for example in the transformation from model statements to solution. The focus is mainly on mathematical proficiencies of problem solving and reasoning, with an emphasis on proportional reasoning, and optionally algebraic formulas.

Lesson Length 45 minutes approximately

Vocabulary Encountered

- Formulating
- Solving
- Interpreting
- Evaluating
-)

Lesson Materials

- Slide show: ST7_Intro_To_Modelling_3a.pptx
- Student Sheet – Mathematical Modelling Cycle (1 per student)
- Access to spreadsheet software (optional)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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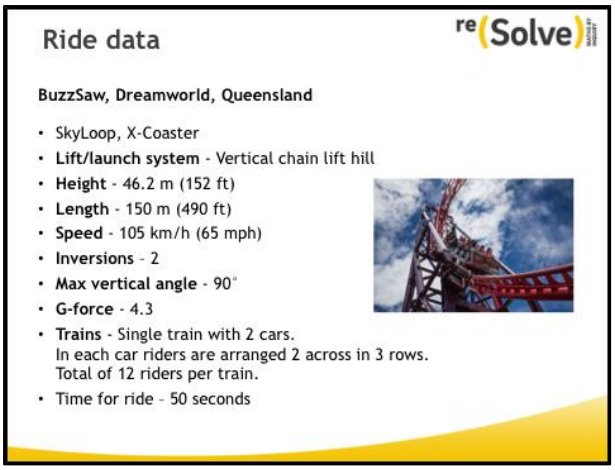
Lesson structure

- Reflections on theme park models (Whole class discussion -5-10 minutes)
- The modelling process (Teacher introduction -10 minutes)
- Linking your model to the modelling diagram (Collaborative small-group work -15 minutes)
- Thinking about modelling (Whole-class discussion -10 minutes)

Optional: Generalizing the model (10 minutes

Reflections on Theme Park Models

This introductory section sets up the reflective tone of the lesson, asking students to reconsider their assumptions and the results they have obtained



Show the slide Ride data - repeated here

Gather estimates for the number of riders per hour from some groups' reports and your observations. Ask the class to consider if these are realistic. (The official data for this ride suggests 600 riders per hour).

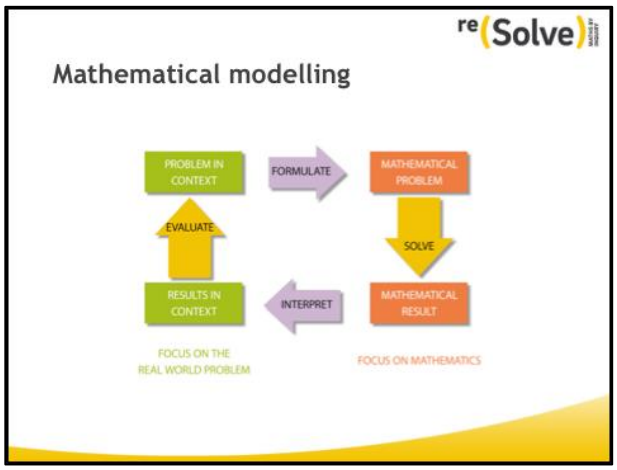
Also share some of the different solutions of where to place the signs for 30 and 60 minute wait times.

Encourage students to evaluate how reasonable their recommended positions are. For example, they could measure the distance outside to get a better idea of the length proposed or explore on Google Earth how far around the school the line would reach for their calculated distances for 30 minute and 60 minute wait times.

Discuss how changing the assumptions they made may affect the predicted waiting times and queue length (e.g. having a faster or slower change-over of riders). Also discuss how the information about the ride influences the results – for example, students might consider the effect of having a different design for the car system with say four riders in each row of a car. This would double the number of riders per ride, but it may slow down the change-over time.

The Modelling Process

This section moves on to look in more depth at the modelling process itself, and prompt groups to reflect on how the four phases worked out in their group's solution to the ride queueing problem.



Show the slide Mathematical modelling

Use these prompts to lead a discussion that highlights key aspects of modelling.

- The diagram summarizes the various phases of modelling.
  We'll be coming back to it many times to link it to the models we build throughout these lessons.
- First we have to understand the problem situation, identifying the factors that may be important in that context. We can assign known or estimated values to each of these factors or take them to be variables.
- We choose which factors to include as variables in our model, and identify the relationships between them. This is model formulation and the product is our model - a mathematical problem statement of our assumptions and how we see the structure of the problem.

- We then employ mathematics to find mathematical results from the model – answers to the questions we are interested in. We check the working, of course – unrecognised errors waste our time and effort.
- We can now interpret our mathematical results to understand what they mean in the problem context.
- We then you evaluate them. Are our answers sensible, believable and do they adequately answer the questions? If not we need to again check our working - if that's seems right, we may try to improve our model (better data, more factors, more accurate relationships) going round the process again.
- When satisfied, we can report what we found out and the reasoning we used.

Linking Your Model to the Modelling Diagram

Make sure that each student has a copy Student Sheet – Mathematical Modelling Cycle.

In this phase of the lesson students, working in their pairs/small groups, pro-actively relate the various steps in the modelling they developed for the theme park queuing to the modelling phases in the diagram. The intention is that they develop a clear understanding of the nature of the mathematical activity of modelling.

Ask students to relate their work lesson to the four 'state boxes' in the diagram and the four 'process arrows' that connect them.

It may useful to ask small groups to develop a joint poster of their thoughts. These can then be used to share ideas in whole-class discussion.

Groups will need time for this challenging activity. They may not find it easy this first time. Suggest they look carefully at what they wrote down when working on the problem to see if they can link what they were doing to the modelling diagram.

Suggest students may find it helps to start by listing the factors they chose to include and how they represented them mathematically (as either estimated or known values or as variables). Deciding not to include a factor is also important in modelling.

Enabling prompts

As you circulate and observe the discussions, you may want to inject guidance in the form of questions that point to the elements in the modelling diagram.

- What did you know about the problem context?
- What did you assume?
- Which values did you have to find out?
- Which values did you estimate?
- What did you do when you were formulating your model?
- What exactly was your model?
- What was it you solved?
- What was your mathematical result?
- What did this mean in the context of the problem?
- What did you interpret?
- How did you find out how good your estimate was (evaluating)?

Thinking about Modelling

Review the modelling

The aim of this section is to clarify students' thinking about the modelling processes by sharing their ideas in whole class discussion.

In whole-class, work through the modelling processes (arrows in the diagrams, starting with 'Formulate'), calling on groups in turn to explain what they did at each stage of their own modelling. Help clarify their thinking and understanding through rephrasing. (It is good to ask a student, after a muddled explanation: *Can you say a bit more about that?* or just *Say it again, Sam*).

You might use the enabling prompts above to help draw out how students' work matches the modelling process.

- What did you know about the problem context?
  (The ride comes into the station, passengers get on and get off before it leaves again, people wait in the line)

- What did you assume?
  (Times for getting on and off the ride)
- Which values did you have to find out?
  (The number of people who are on each ride, the time each ride lasts)
- Which values did you estimate?
  (The number of people for a given length of line)
- What did you do when you were formulating your model?
  (calculated how many people were able to go on the ride each hour)
- What exactly was your model?
  (the calculation/formula that was used)
- What was it you solved?
  (the calculation/formula that had been set up)
- What was your mathematical result?
  (Answers for the number of people getting on the ride each hour, how far the queue would stretch with this number of people in it, the lengths of queue for people waiting 30 minutes, 60 minutes, 90 minutes and so on).
- What did this mean in the context of the problem?
  (How far away from the ride the different signs would be placed)
- What did you interpret?
  (The numerical values to give answers in context)
- How did you find out how good your estimate was (evaluating)?
  (Considering if it was likely)

Consider the variables

Illustrate how the proposed variables might be assigned particular names or letters for shorthand if we wish to work algebraically.

Some discussion of the precise meaning of the variables will be important - for example, time of ride for queueing may not represent how long the ride lasts but the time between beginning to load successive cars. 'Time" would not be a good variable name.

Emphasise that there are many other factors that we could have included but we have to **simplify** if we are going to be able to handle the problem - but also not to leave out anything essential. That's the challenge of formulating a useful model. Deciding to leave factors out is an important step.

Ask students how their variables are connected i.e. what are the mathematical **relationships** between them? Ask groups if they can share one connection, one relationship that they used? Try to ensure that you can illustrate each of them in both words and letters – drawing on students' answers as much as possible. For example,

Wait time = Length of queue*Queue density/Flow rate T = L*Q/F

Expected student responses

- Queue density (number of people per metre) = number of people in 10 metres ÷ 10 Q= n/10
- Number in queue = Length of queue * Queue density N = L*Q
- Flow rate (number of people per minute on ride) = number on each ride/time of ride F = r/t
- Wait time = time of ride*number in queue when joined/number on each ride W=t*N/r
- I hour sign position (metres) = 60 * flow rate/queue density

Solving and interpreting

Emphasise that this gives a mathematical **model**. We **solve this** using our mathematical skills to transform the model into results. These results need **interpreting** in terms of the real situation (we move back to the real world side of the diagram) - we **interpret** the result in the context of the queueing problem. If the result is in algebra, you have to translate it back into words. If the result is in words, that is already done - but we still have to think what it means.

We should think about the structure of our model, for example, if the management want you to advise them on how to reduce the waiting time on a popular ride, how could they do it? e.g. by shortening time of ride, by adding more cars (maintaining safety though). …

Evaluating and reporting on a model

We then move on to the fourth arrow - we **evaluate** how good the model is in the context of the problem. Have we got the essential features? How could we improve our model? Any suggestions?
Of course, in the end we need to check our model experimentally - by observing rides and seeing how well the model fits.

Then finally we prepare our report – allow students a little more time to work on their report reviewing what they have written, relating the various elements to the modelling process.

Before Lesson 4 students should research some data for different types of rides. They should choose some rides and in particular they should find the how long the ride takes and it's capacity (to get an idea of flow rate). Use the slide Different types of ride to stimulate this research.

Generalising the Model (Optional)

Algebra is a natural modelling language for this kind of problem - but students may not have used it in this way.

Show the slide Extending the challenge

Explain that so far the mathematics we have used has been mostly arithmetic - using specific values that we have estimated or found from real data. But the thinking we have done is more general than the particular numbers.

The language that expresses that generality is algebra, using symbols for the variables.

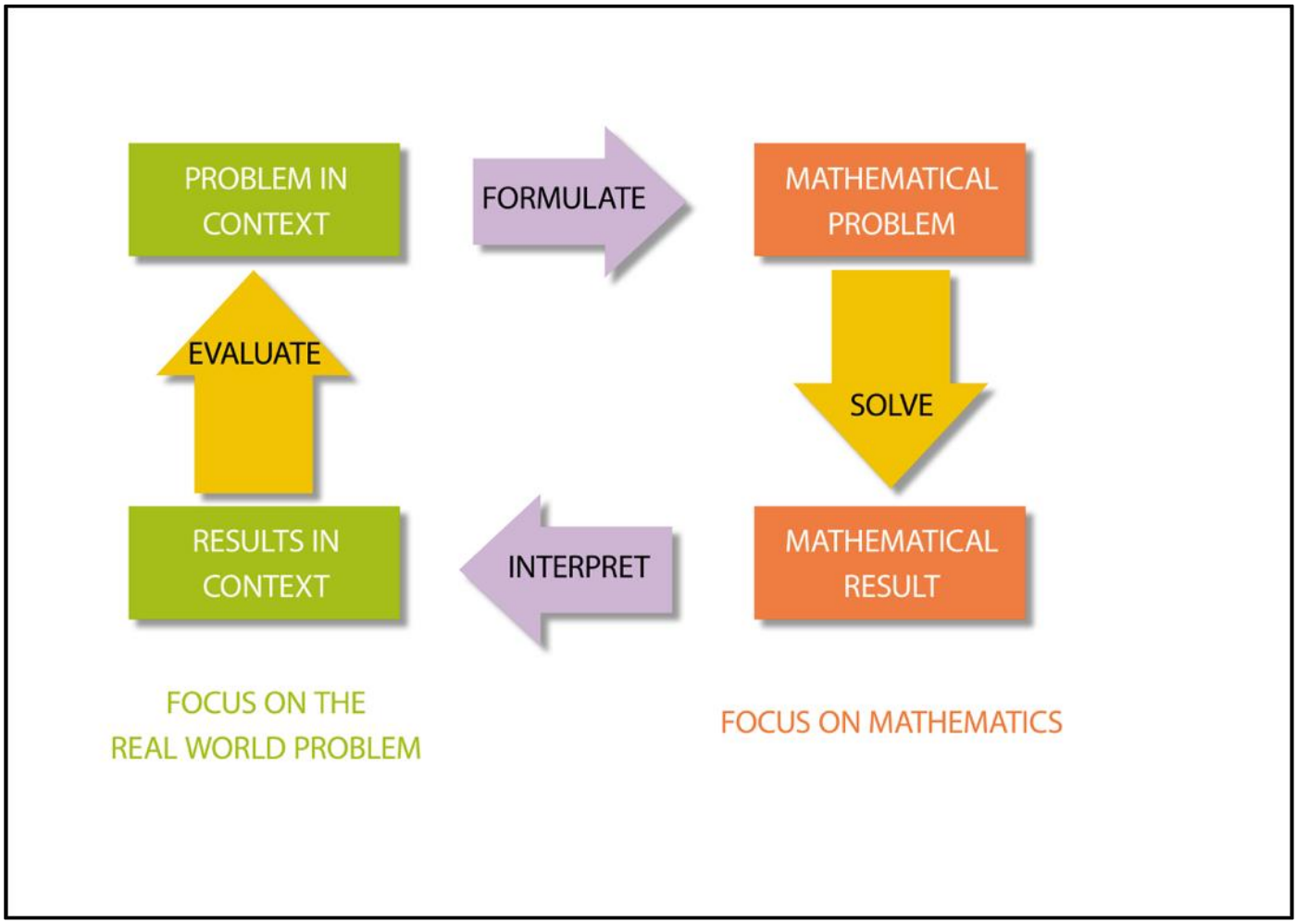
Suggest that students extend their model by considering how the wait time, (*W*) changes if they increase or decrease the number of people going on the ride in 1 hour (60**F*).

Ask students to write a rule or formula for (*W*) in terms of (*F*) and sketch a graph of (*W* against *F*). This gives a representation that provides insight into how the flow rate (*F* people per hour) influences the result.

Spreadsheets can be a particularly useful way to express algebraic modelling – some students could put the formulas into a spreadsheet to give tables and graphs of results for various choices of the data.

Mathematical Modelling Cycle

Name: _____



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