Packaging Designer
Lesson 5: Back to Reality

Australian Curriculum: Mathematics (Years 8 and 9)

ACMNA188: Solve a range of problems involving rates and ratios, with and without digital technologies. (Year 8).
ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

Lesson abstract

Students undertake three short activities that highlight the interpreting and evaluating phases of modelling. Writing a commentary for a race using only a distance time graph, focusses on interpretation. Predicting running world records focusses on evaluating a model. Predicting future numbers of teachers focusses on selecting a model that uses the best available data, and appreciating which data influences the outcome most.

Mathematical purpose (for students)

The outcomes of modelling must be interpreted carefully, and models must be evaluated thoroughly.

Mathematical purpose (for teachers)

The intention of this lesson is to focus on the interpretation and evaluation phases of mathematical modelling. The interpretation phase takes mathematical results, expressed in mathematical language and representations and translates them into real world language, with real world considerations. The evaluation phase asks whether the model is likely to provide sufficiently good answers for real world use.

By considering three real world contexts, students gain experience of interpreting the outcomes of mathematical modelling in terms of the real situation that the model represents; understanding how outcomes of mathematical modelling are dependent on assumptions and decisions made when setting up the model; evaluating the effectiveness of a mathematical model; and considering how to refine mathematical models.

Lesson Length 50 minutes approximately

Vocabulary Encountered
- interpretation
- evaluation
- validity

Lesson Materials
- Slide show: ST7_Packaging_5a.pptx
- Student Sheet 1 - Hurdles race (1 per student)
- Student Sheet 2 - One mile world records. (1 per student, optional)
- Spreadsheet: ST7_Packaging_5b_milerecords.xslx (optional)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

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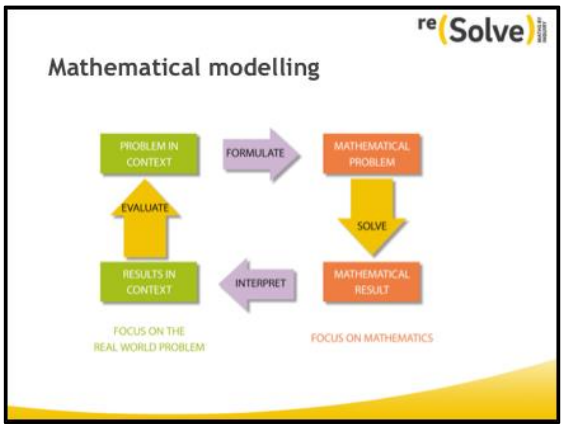
Lesson structure

- Whole-class introduction/collaborative small-group work: Hurdles Race (about 20 minutes)
- Small-group work: World records (about 20 minutes)
- Small-group work: Future Teachers (about 10 minutes)
- Whole class discussion: Interpreting and evaluating in the Modelling Cycle (about 5 minutes)

Hurdles Race



Start by showing the slide Mathematical modelling.

In this lesson, students will focus on issues of interpreting and evaluating mathematical models. This first activity is about interpretation step, where results of mathematical work are described in real world terms.

Organise students to work in pairs.

Show the slide Hurdles race.

Explain that the graph on the slide shows how three athletes ran a race. The graph shows the distance from each athlete from the start during the couple of minutes that the race took.

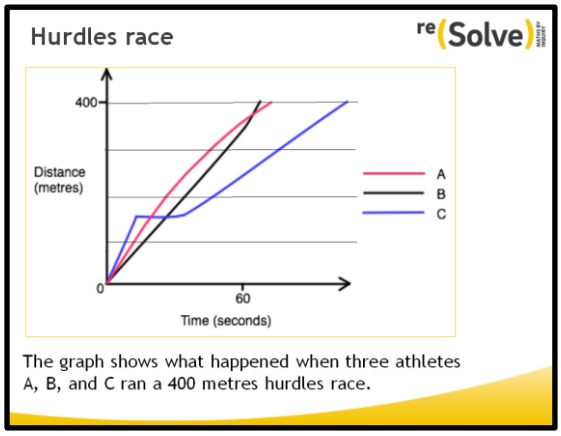
The graph has been drawn from data on the positions of the athletes during the race. This is the result of mathematical work, or possibly even carried out automatically from data loggers.

Handout Student Sheet 1- Hurdle Race. Students should imagine that they are the race commentators for the radio and describe what is happening as carefully as possible. They do not need to measure distances accurately.

Discuss student responses, focussing on interpreting the mathematical information in real world terms. Students can use their imagination for this.

Expected student response

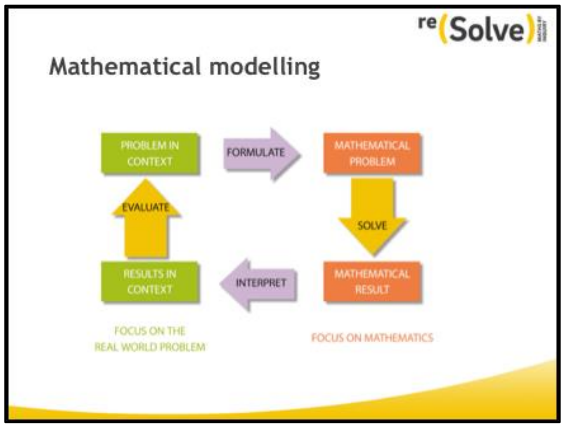
- C gets a good start, followed by A and then B
- At about the 160m mark, C falls and A and B pass her.
- A accelerates, increasing her lead on B
- By the 300 m mark, B is closing the gap with A, running steadily whilst A slows.
- The race comes to an exciting finish as B overtakes A in the last few metres.
- Meanwhile C was on the ground for about 20 seconds after the fall, and then progressed slowly, obviously injured, to cross the line about 30 seconds after the others.



Again, show the slide Mathematical modelling and ask students to relate the situation to the modelling diagram.

Expected student responses

- The commentating task illustrates the work that goes into interpreting mathematical results in a context.
- This involves mathematical knowledge (e.g. how to read a line graph) and knowledge of the situation (e.g. why a hurdler might stop during a race.)
- The interpretation needs to be in real world terms, such as seeing that B passes A. The graph just shows that the distances travelled from the start of the race were equal just before the end of the race.
- Interpreting a mathematical result is not always easy!



Draw out as many features as possible from students. Emphasise that in this case we were working with a mathematical model that had been developed by someone else: the graph. Importantly the activity involved interpreting the mathematical representation in terms of the problem context. This involves moving from the mathematical world (the graph) to the real world (what happens in the race). For example, the real world interpretation is "passing"; the mathematical evidence is equal distances at a given time.

Move on to consider the graph as a model. Discuss which is better and why, the graph or the word description?

- Highlight how the graph reports the data accurately, whereas the word description emphasises how the three athletes are positioned relative to each other at any given time.
- The word description emphasises what race fans are interested in.
- A different description in words could give other information (e.g. the running speeds) that could be derived from the same data.

World Running Records

This activity focusses on evaluating a mathematical model i.e. considering whether it is likely to give good predictions.

- All models need to be evaluated against real world criteria.
- If not satisfactory, the model will have to be improved.

This activity uses the men's world records for the mile run (1.609344 km).

- The data provided is for records approved by the IAAF (International Association of Athletics Federations).
- The men's records are used because they go back to 1913, about 50 years longer than women's records.
- The spreadsheet file ST7_Packaging_5b_MileRecords.xls gives the data in table form along with other information (such as the record by Australian John Landy in 1954).
- Students can use this to draw a graph and use it to make their predictions.
- An alternative is to use the (metric) Olympic 1500 m records for men and women, using data from IAAF website.

Show the slide World record times for one mile races or use the spreadsheet.

The challenge is to predict the world record in 2020. First test the method by making predictions from the data to 1975. Depending on time available, students can make careful predictions using spreadsheet data or just predictions from the computer display.

Show the slide **World Records from 1913 - 1975**. **(see below)** Each dot shows a new world record.

Discuss how this data could be used to make predictions e.g. for 1980, 2000 and 2020.

- The data appears to be tracking downwards in a straight line.
- Students can use the spreadsheet a to find a trendline, or draw a straight line through the data by eye, on paper or work directly from the class computer display.

Show the slide **Predictions from straight line model,** note the predictions and discuss reasonableness. The straight line model assumes that the rate of improvement in the records is constant.

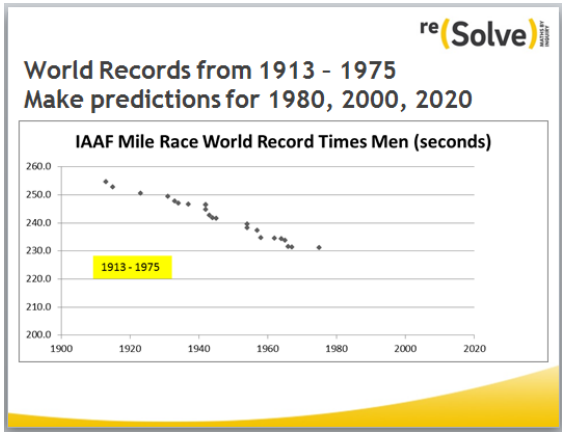
Students predictions will be slightly different to these, if they use a different straight line. Be careful with rounding if using formulas.

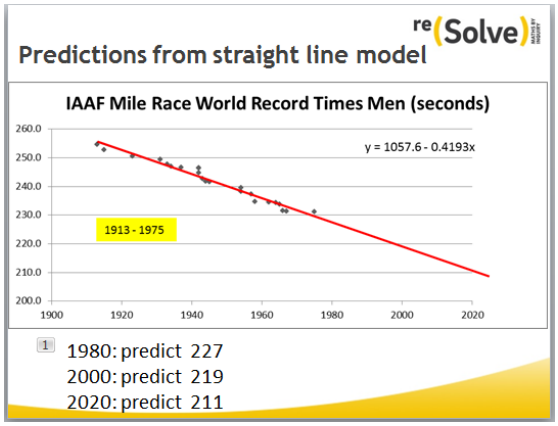
Check the predictions from the 1975 data using the data to 1999 as on the slide World records from 1913 - 1999 and ask students to use the additional data to make a new prediction for 2020.

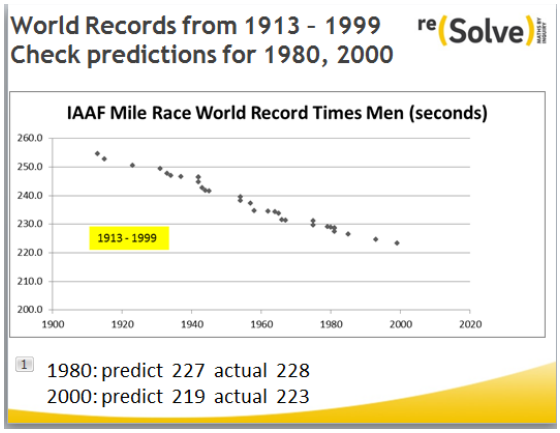
Now Student Sheet 2 One Mile World Records can be handed out and used to make predictions.

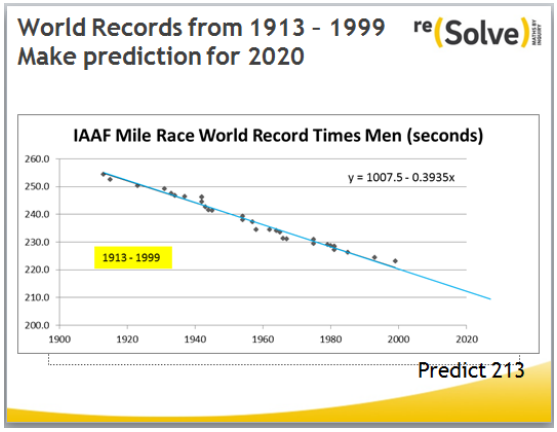
Organize students to work in small groups to discuss whether the prediction of 205 seconds for 2020 is likely to be reasonable and report back to the class.

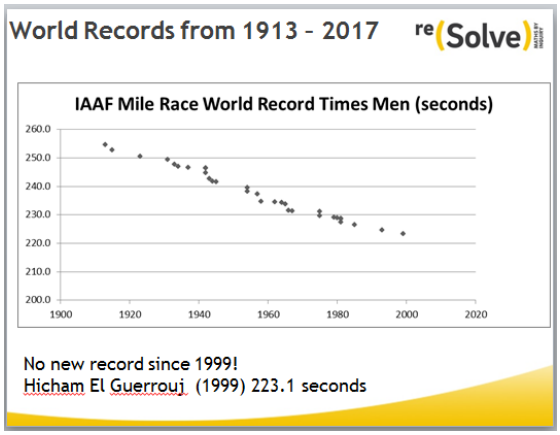
Finally show the slide World records from 1913 – 2017, which shows that there have been no new world records in the twenty years since 1999. The reasons for this are unclear, but it may be due to exceptional athletic performance, changes in popularity of the mile race, changed drug testing procedures or other factors. Emphasise that the straight line model assumes that the rate of improvement is constant over time, and there is no good reason why this should reflect real life. The straight line assumption made to formulate the model is not very good.

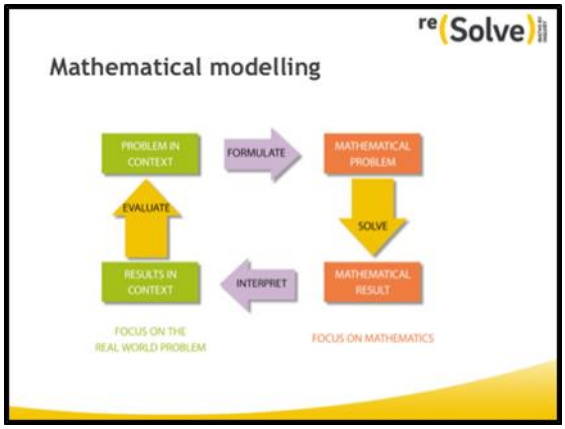












Apart from checking for errors in the mathematics or the logic, useful ways to evaluate the validity of a model and its predictions are to:

- ask yourself or experts if you really believe the answer obtained;
- make predictions using the same method of things that are already known (as in this case) and test them.

If the solution is not sufficiently valid for its real world purpose, the modelling cycle begins again at the formulation phase.

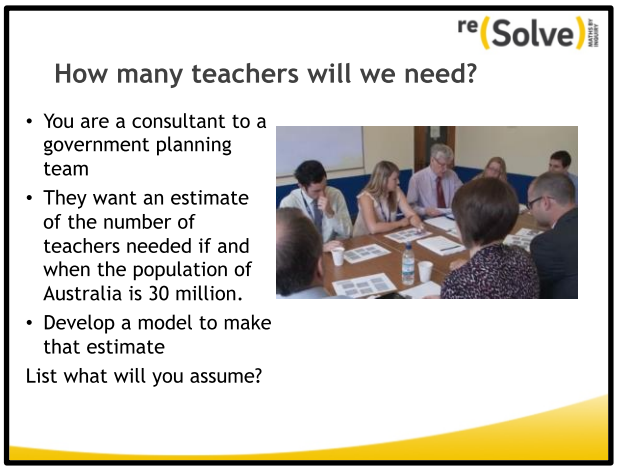
Future Teachers

This modelling task is a variation of a famous 'Fermi problem' which asked how many piano tuners are in Chicago. Such problems are named after Enrico Fermi a physicist who made reasonable estimates with just a little commonly known data.

The purpose is to focus students' thinking on how the assumptions that we make when setting up a model might be refined, and alternative approaches explored when we know more information.

The task is to predict the number of teachers needed in Australia when the population is 30 million. Some students might know that the population of Australia reached 25 million in August 2018. In 2018, demographers estimated that the population will reach 30 million in about 2030 (https://mccrindle.com.au/insights/blog/australias-population-to-reach-25-million/).

Introduce the task with the slide How many teachers will we need?



Making a first estimate

First, make a rough estimate of this number from 'common knowledge', without collecting accurate data:

- identify the key variables and create a simple model, and
- be prepared to explain you reasoning.

The focus is on the reasoning (i.e. the model used) not the answer. This is a useful way to think through a problem before you go looking for lots of data - at the least it helps you to identify variables and data you will need.

Give the groups 5-10 minutes to work on this, then collect some responses.

Sample response 1	Sample response 2
Students identify and select the variables (usually in words) • the fraction of a lifetime spent as a student (f) • the number of students in a class (c). • the population (P) They then estimate the number of teachers needed (T) as T = P * f / c If f = 13/80 ~ 1/6, c = 25, then T ~ 30 million/150 = 200,000.	Our school has about 1500 students and about 100 teachers. We guess that a third of the population of Australia would be at school, so 10 million students. So, to find number of teachers needed (T) solve $\frac{T}{10\,000\,000} = \frac{100}{1500}$ Answer: T = 600 000 approximately. Variables: T = number of teachers needed t = number of teachers in typical school s = number of students in the typical school f = fraction of population who are students

Discuss the responses noting:

- The similarities and differences in different estimates.
- The similarities and differences in the assumed data (e.g. class numbers), and also in methods used.
- The similarities and differences in the variables that have been identified (e.g. the two fractions in the sample responses - both called f - are different variables),
- Which numbers have a very big effect on the answer obtained (e.g. in sample response 1, the estimate is that one sixth of the population is of school age, but in sample response 2 the estimate was one third),

Just for fun

Get students to record their estimates and find the class average of these very simple initial estimates. Later, compare this average to the very best estimate that you get. When individuals use their own information and judgement, it is often found that the average of 'the crowd' can give a very good estimate (see https://en.wikipedia.org/wiki/The_Wisdom_of_Crowds).

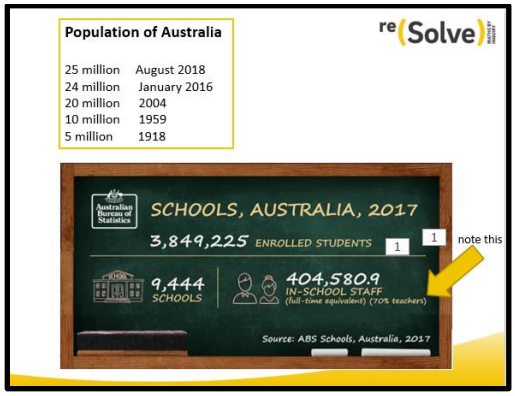
Improving your model with new information

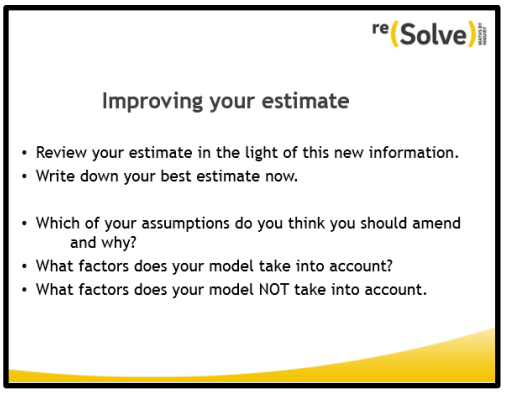
Now give groups 5 – 10 minutes to review their assumptions critically, considering

- how accurately do you know the numbers (e.g. P, c, f) you used? How would your estimate change if they moved to the possible limits? (f is probably good to 10% for up to age 18, c is probably smaller by ~20-30%, so T estimate could increase by ~30%)
- difficulty in knowing what is typical around Australia
- What other factors might be important? (teachers don't teacher every lesson and class sizes are sometimes much smaller (reducing c), the expanding population is probably younger (increasing f,,.....)

Discuss a few responses, then present some 2017 data which students can use to make a new estimate.

Show the slides Schools, Australia, 2017 and then Improving your estimate.





Expected student response

Assume ratio of teachers to whole population stays the same so T ~ t * P / p, so about 340,000 teachers.

This approach assumes that a constant proportion of the population will be teachers. It does not allow for any effect of the population having more young people (perhaps), or changes in government staffing policies, or changes in number of students staying at school.

Finally, discuss how evaluation of the model needs to take careful account of the structure of the model, the quality of the data used, and the assumptions that are made in setting it up. The actual model that is best to use may be the one with the best data available for the variables.

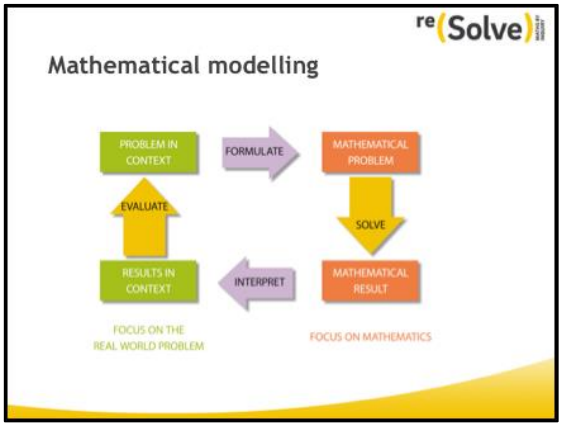
Conclusion

Show the slide Mathematical modelling with the modelling cycle.

The Hurdle Race focussed on interpretation of mathematical results in real world terms. Mathematical data can be 'read' in many different ways, depending on the concerns of the reader

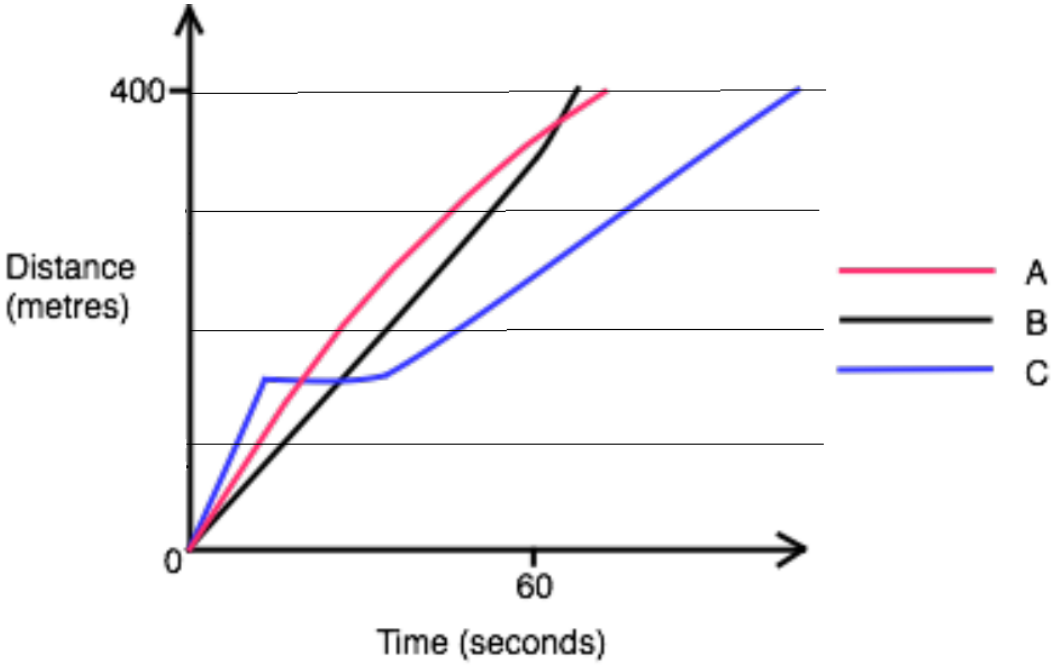
World Records showed that the evaluation of a model is very important. We can easily make unwarranted assumptions, such as that the apparently linear trend in running times will continue.

Future Teachers highlighted the many factors that need to be considered in making a good model, but also the possibility of obtaining rough estimates fairly easily. The test of the estimates is whether they are good enough for the real world purpose.



Hurdles Race

Name: _____



The graph shows what happened when three athletes A, B, and C ran a 400 metres hurdles race.
The distance of each athlete from the start is plotted against time after the start of the race.
Imagine you are the race commentator for the radio.
Describe what is happening as carefully as possible.
You do not need to measure the distances or times accurately.

WRITE YOUR COMMENTARY HERE

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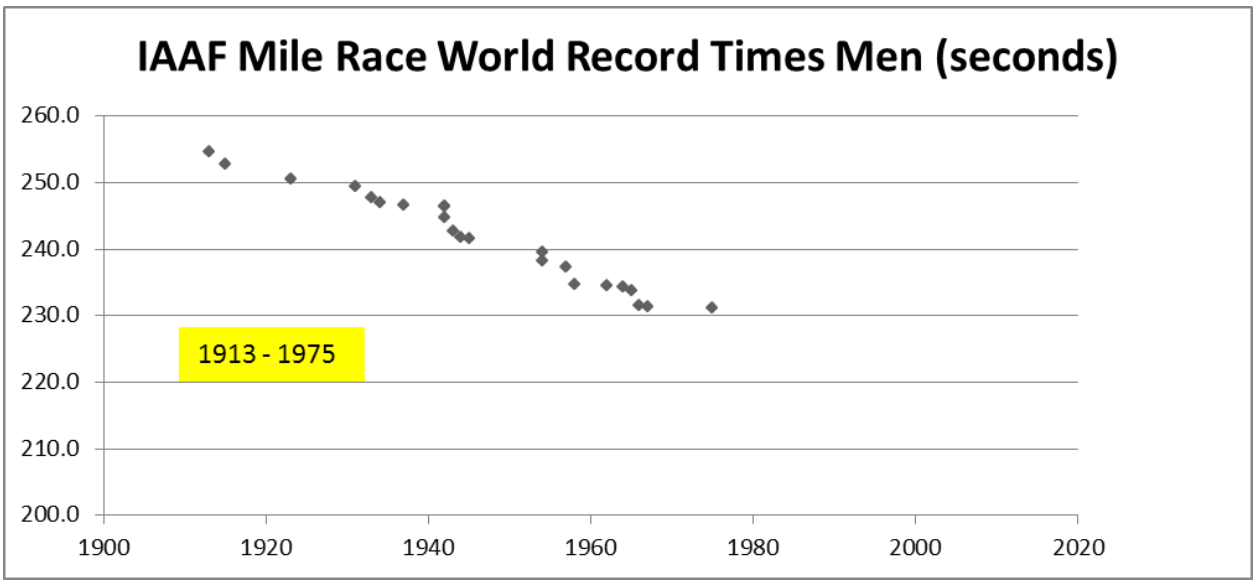
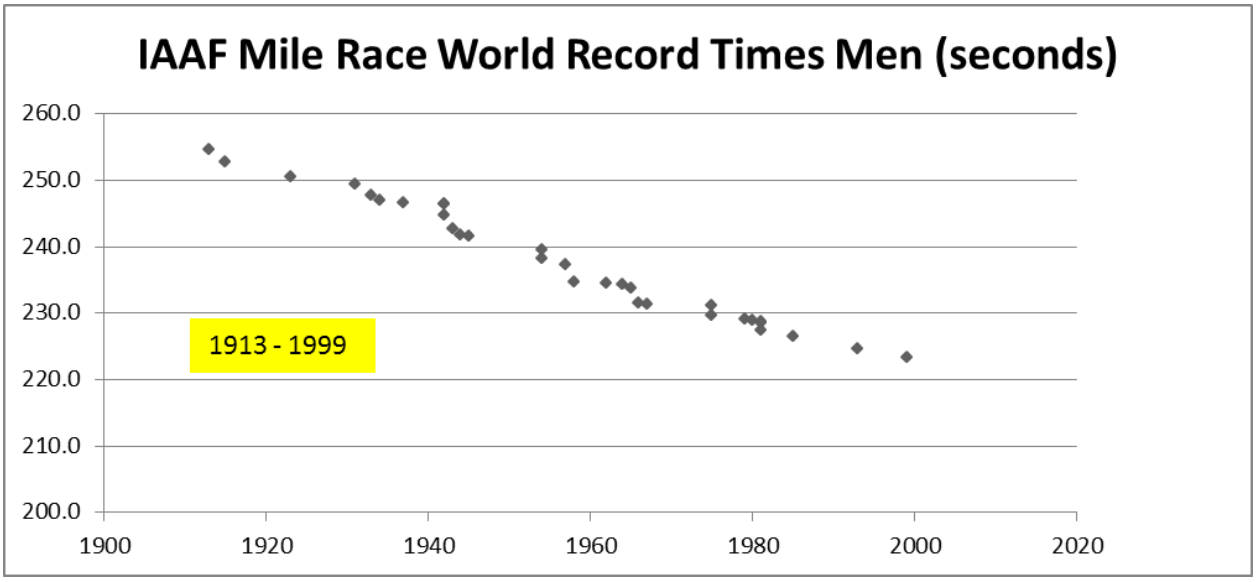
One Mile World Records

Name: _____





These graphs show official IAAF world record times for one-mile races for men from 1913 until 1975 and in the second graph until 1999.

It was predicted that by 2020, the world record will be 3.25 minutes (20 seconds).

Is this a good prediction?

First use the data to 1975 to make predictions for 1980, 2000 and 2020, and see how accurate the predictions are using the data to 1999.

Then use the real data to 1999 to improve your prediction for 2020.

Are these good predictions? What has your model assumed? Explain the reasons for your findings.

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