Packaging Designer
Lesson 3: Critiquing Reports

Australian Curriculum: Mathematics (Years 8 -10)

ACMMG195: Choose appropriate units of measurement for area and volume and convert from one unit to another. (Year 8)
ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (Year 8)
ACMMG217: Calculate the surface area and volume of cylinders and solve related problems. (Year 9)
ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles. (Year 9)
ACMMG224: Apply trigonometry to solve right-angled triangle problems. (Year 9)
ACMNA234: Substitute values into formulas to determine an unknown. (Year 10)

Lesson abstract

Students are given three pre-prepared student reports on the packaging problem to critique. They work with a partner to understand, complete and correct a report, and identify the mathematical modelling involved. Then they explain their report to another pair. This task provides an opportunity to consider alternative approaches students might take to develop their own work further, and it enables them to see characteristics of a good report.

Mathematical purpose (for students)

Design involves compromises. Mathematical modelling provides information to help make them.

Mathematical purpose (for teachers)

The three pre-prepared student reports involve different levels of mathematics. Each report provides some ideas how students might deal with varying the height and diameter of the cylinders, and this may assist students to improve their own approach. A key point is to consider the modelling behind the reports and discuss how design involves compromises, just as mathematical modelling requires simplification of real situations.

Lesson Length 50 minutes approximately

Vocabulary Encountered

Lesson Materials

- Slide show: ST7_Packaging_3a.pptx
- Student Sheets for Reports A, B and C (one of the three reports per student)
- Student Sheet 4 - Critiquing Reports (one per student)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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Lesson structure

- Collaborative pairs work: Looking at a report (about 10 minutes)
- Collaborative small-group work: Sharing and critiquing reports (about 25 minutes)
- Class reflection: Design involves compromises and mathematical modelling (10 minutes)

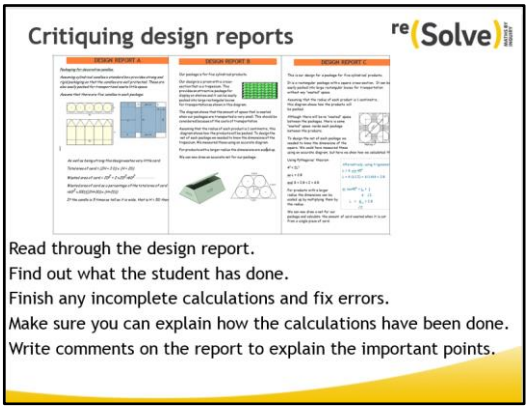
Understanding a Report

Show the slide Critiquing design reports. Explain to the class that they are going to work in pairs to look carefully at the work of other students who started to produce a report for the product manufacturer.

Give each pair of students one of the three reports - Student Sheet 1 - Design Report A or Student Sheet 2 - Design Report B or Student Sheet 3 - Design Report C. Also hand out Student Sheet 4 - Critiquing Reports.

Take into account the different mathematical levels of the reports when allocating them to students.

- Report A has a simple package design, but uses algebra.
- Report B is a complex design but the dimensions are only found by measuring a scale drawing.
- Report C is a simple design and includes Pythagoras' Theorem and optional trigonometry.



Give students time to critique their first report. Students will need to follow the working shown in the report they are allocated, making sure they can explain what has been done to another pair of students later. They will also need to complete some of the calculations.

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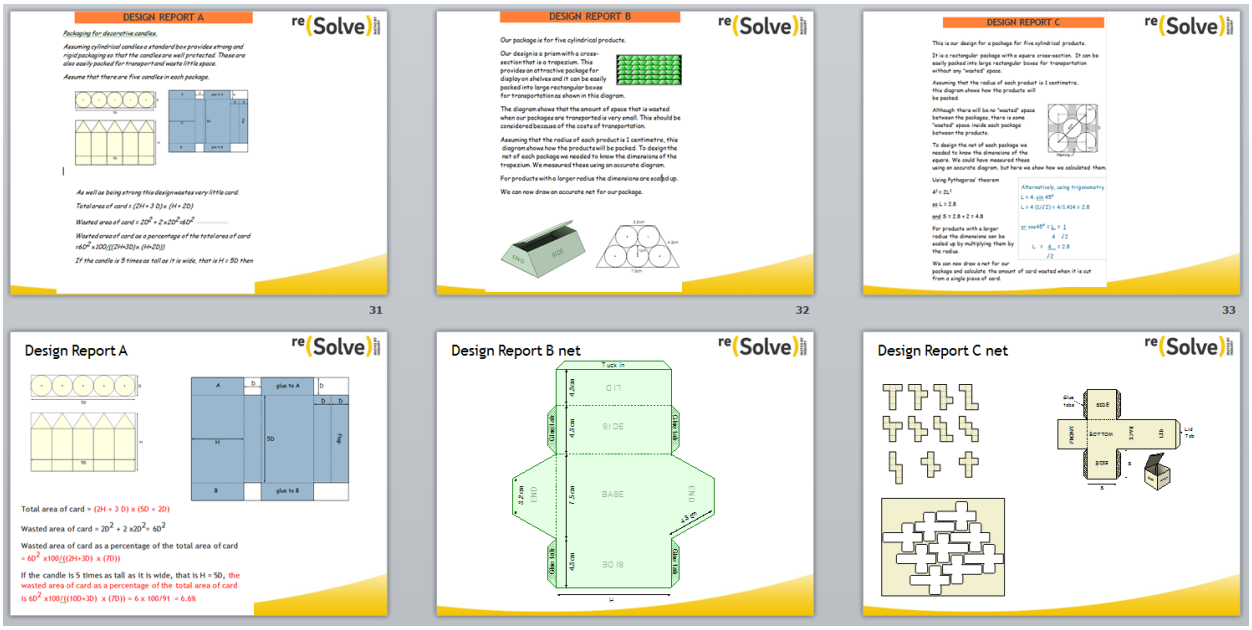
The three teacher sheets supplied for Report A, Report B and Report C highlight errors and show missing information.

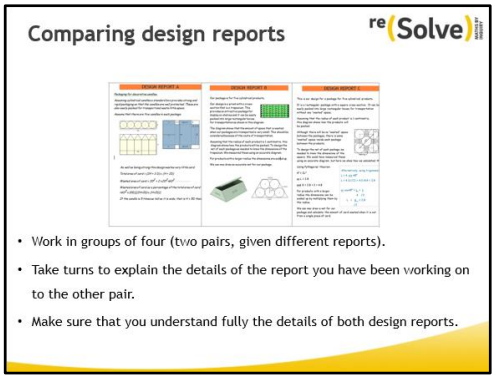
Students should annotate the report they are working on and correct errors so that they will be able to explain the details.

The following slides of Reports A, B and C and solutions are included for use as required during the discussion.



Sharing and Critiquing Reports

Show the slide Comparing design reports and later Improving Reports

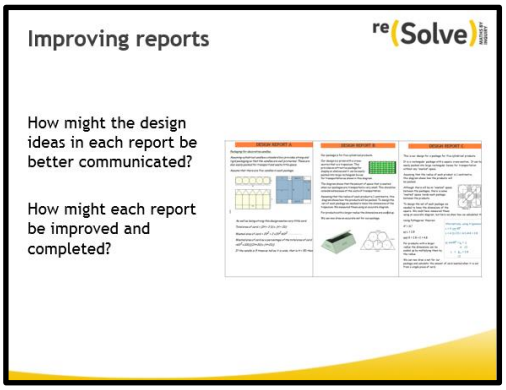


Ask pairs of students to join another pair, who have looked at a different report. Both pairs should then explain to each other what they found in the report they looked at. They should explain to the other pair the details of the work they have been looking at.

Students continue working with the points on Student Sheet 4 - Critiquing Reports.

Encourage students to:

- Take turns to explain the details of the design report they have been working on, guided by the
- Point out to each other the modelling that has been done in each report
- Ask questions if they don't understand the explanations of the other pair.
- Make sure that they understand fully the details of both design reports.
- Complete the worksheet questions, including how to improve the reports, and what is learned from them.



Allow students some time to review and revise their designs and reports, in preparation for the presentations in the next lesson.

Class Reflection

This should be a whole-class reflection on the issues that have emerged from this activity.

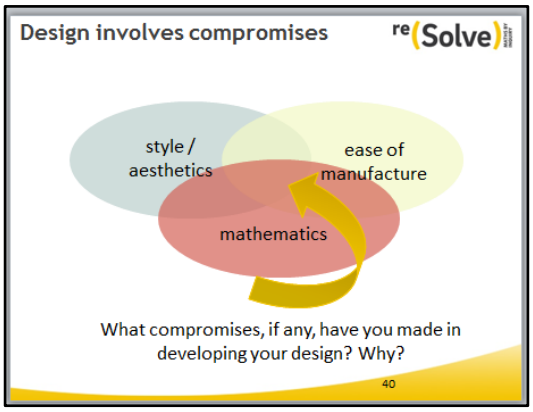
Briefly review features of the three reports.

It is likely that students will identify how Report A proposes a very much simpler design, which may be less attractive. However, the use of cut-out windows that allow customers to view the attractive products inside may prove a useful compromise – and the design is certainly simple to manufacture and stack. How would the trapezium of Report B work in practice? What about the square-based design of Report C – would the candles need internal support (e.g. more cardboard pieces inside) to stay upright?

Remind students that designers have to make compromises in terms of style and aesthetics, as well as ease of manufacture (including efficiency) and convenience. Mathematics can guide the options.

Thinking about design

Show slide Design compromises



Ask students to identify any compromises that appear to have been made in designs A and B and C.

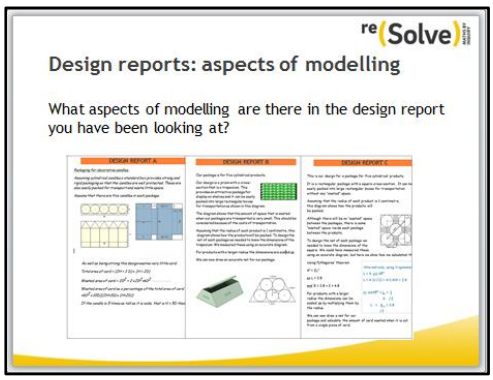
Then, ask students to identify any compromises that they decided to make in their own designs.

Emphasise that:

- all design involves making compromises.
- you can't please all the people all the time!
- the most aesthetically pleasing or interesting package may be too expensive to manufacture or transport or display, or inconvenient for the customer.

Discuss these points with the whole class, indicating that this is something that they need to consider as they work on a redrafted design.

Thinking about modelling



Now move the conversation on so that students start to think about the design reports in terms of the modelling cycle. Ask each pair to consider the modelling and design processes in the report they are looking at. Show the slide Design reports: aspects of modelling which prompts students to think about what aspects of modelling the design reports focus on.

- What modelling has the student done?
- What design decisions did they make?
- What assumptions have they made?

Expected responses

- This student assumed that the products were exactly cylinders.
- This student assumed that the largest radius of the product was at the bottom.
- This student assumed that the cylinders could pack right up alongside each other.
- This student assumed that the gluing flaps were very small.

Preparation for reporting

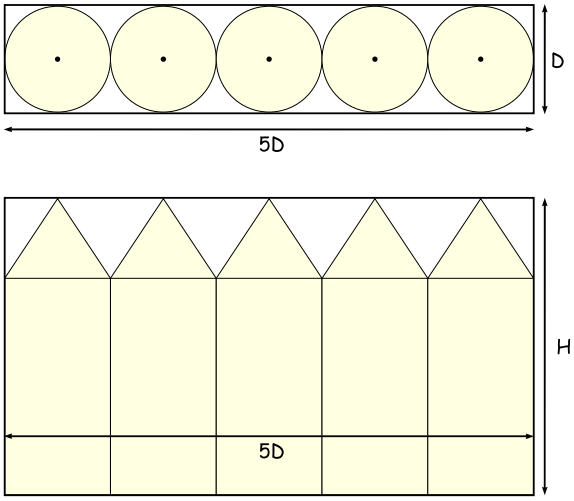
Designs and reports need to be completed before the next lesson, and students need to be prepared for the presentation (mode to be decided by teacher).

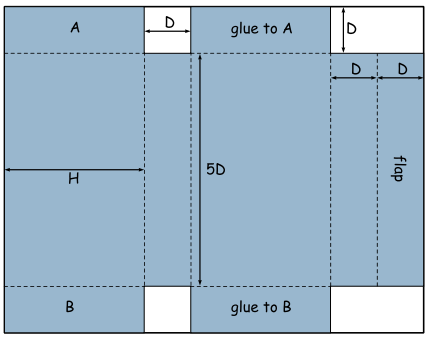
Teacher Sheet 1 - Design Report A

This is a good design. Some errors have been made in calculating the areas.

Assuming cylindrical candles a standard box provides strong and rigid packaging so that the candles are well protected. These are also easily packed for transport and waste little space.

Assume that there are five candles in each package.





As well as being strong this design wastes very little card.

Total area of card = (2H + 3 D) x (5D + 2D)

Wasted area of card = $2D^2 + 2 \times 2D^2 = 6D^2$

Wasted area of card as a percentage of the total area of card
$= 6D^2 \times 100/((2H+3D) \times (7D))$

If the candle is 5 times as tall as it is wide, that is H = 5D, the wasted area of card as a percentage of the total area of card
is $6D^2 \times 100/((10D+3D) \times (7D)) = 6 \times 100/91 = 6.6\%$

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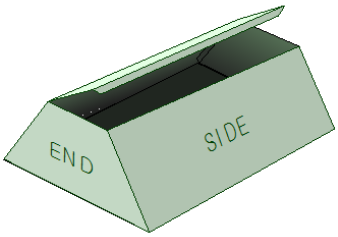
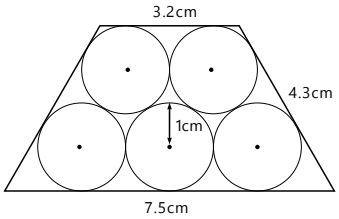
Teacher Sheet 2 - Design Report B



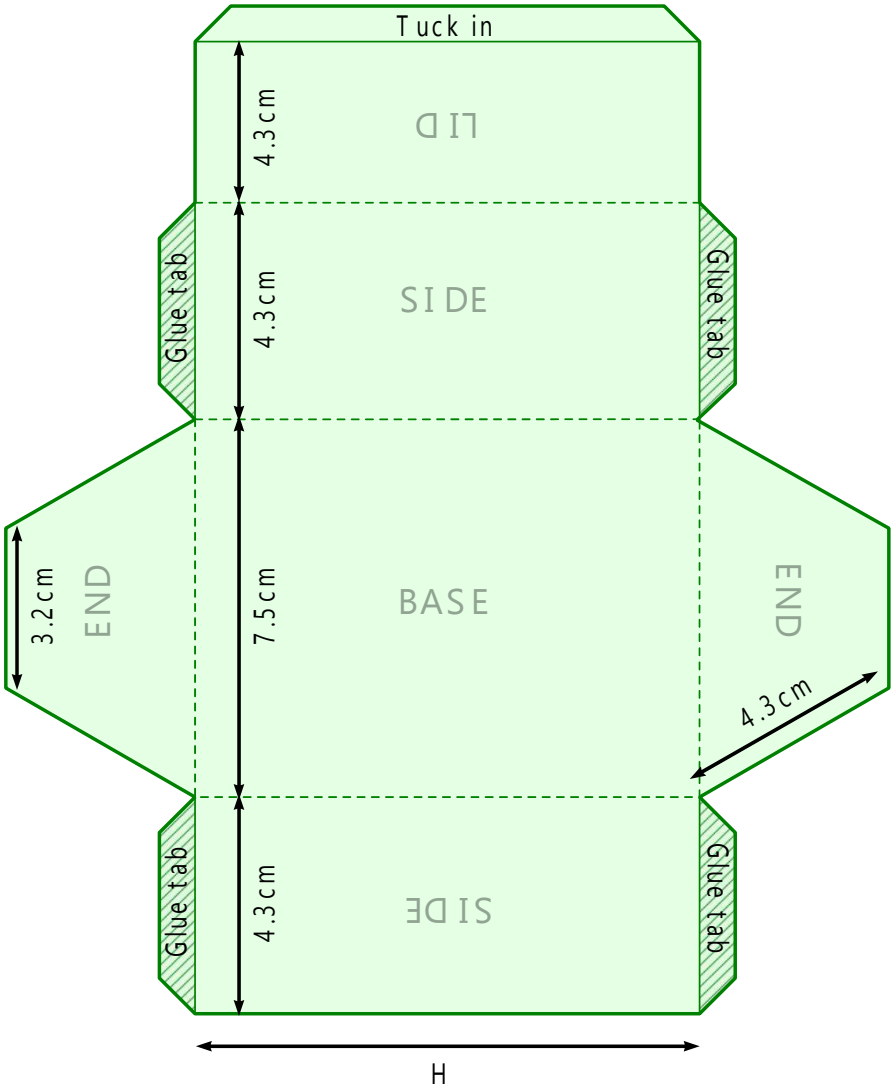


This is a good design. The net needs to be completed.
Some students could use trigonometry instead of measuring.

The trapezium diagram in Design Report B shows the end cross-section of the package, which is a prism.
The diagram shows what the package looks like. In this diagram, the bottles lie down flat.

The height of the end trapezium measures 3.7cm.

Here is an accurate drawing of the net.



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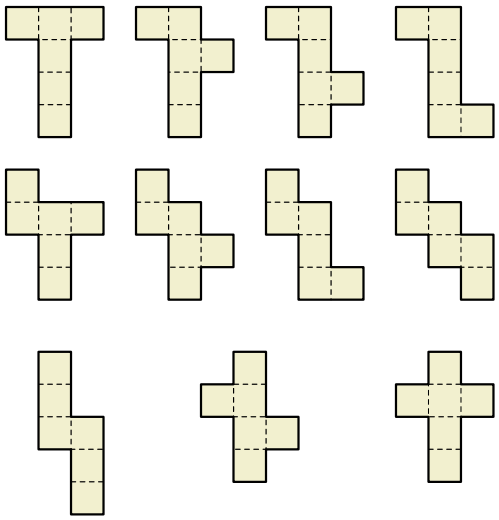
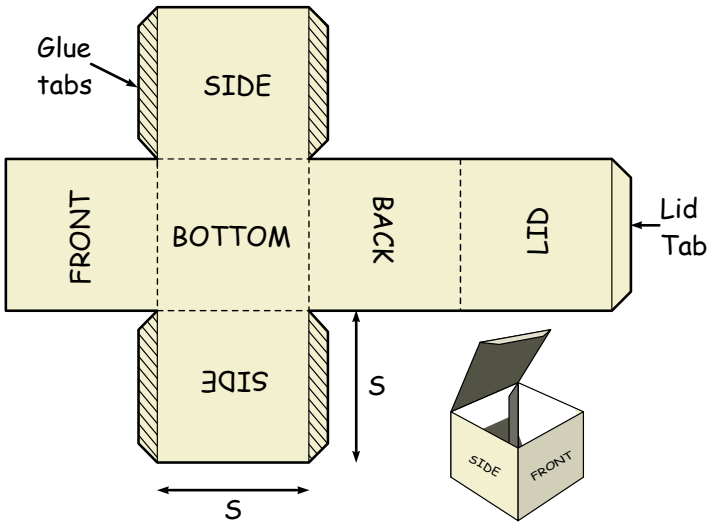
Teacher Sheet 3 - Design Report C

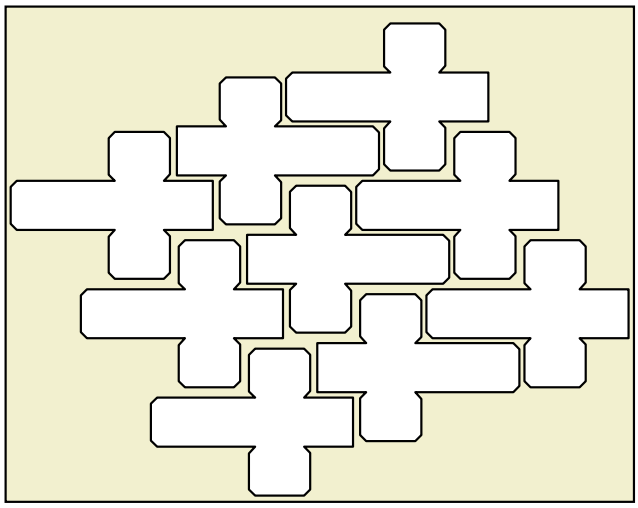
This is a good design. The net is to be completed.

There are eleven possible nets for a cube, as shown here.

If the most common net is used, it tessellates without wasting much material when cut from a large sheet of card.







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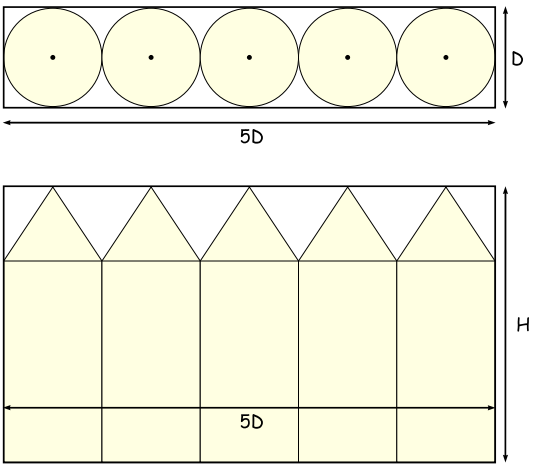
Design Report A

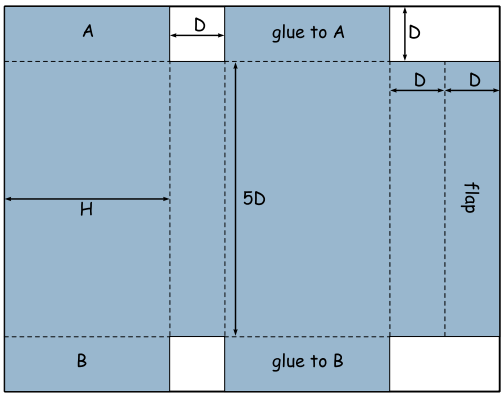
Name: _____

Packaging for decorative candles.

Assuming cylindrical candles a standard box provides strong and rigid packaging so that the candles are well protected. These are also easily packed for transport and waste little space.

Assume that there are five candles in each package.





As well as being strong this design wastes very little card.

Total area of card = (2H + 3 D) x (H + 2D)

Wasted area of card = $2D^2 + 2 \times 2D^2 = 6D^2$

Wasted area of card as a percentage of the total area of card

$= 6D^2 \times 100/((2H+3D) \times (H+2D))$

If the candle is 5 times as tall as it is wide, that is H = 5D then

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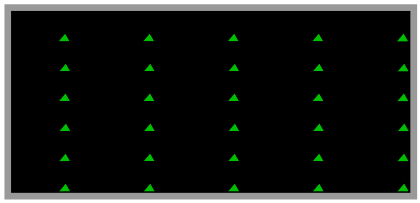
Design Report B

Name: _____

Our package is for five cylindrical products.

Our design is a prism with a cross-section that is a trapezium. This provides an attractive package for display on shelves and it can be easily packed into large rectangular boxes for transportation as shown in this diagram.

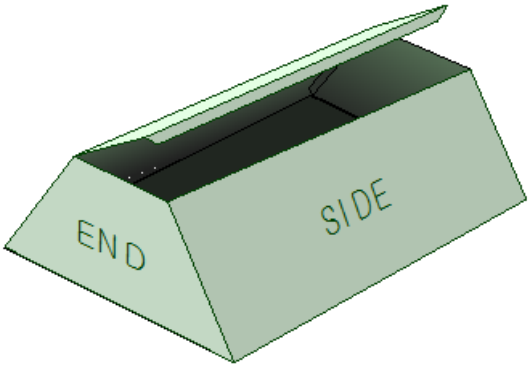


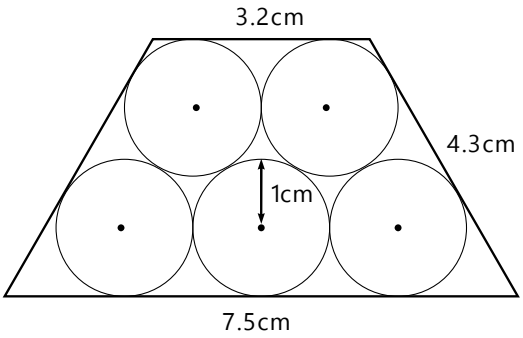
The diagram shows that the amount of space that is wasted when our packages are transported is very small. This should be considered because of the costs of transportation.

Assuming that the radius of each product is 1 centimetre, this diagram shows how the products will be packed. To design the net of each package we needed to know the dimensions of the trapezium. We measured these using an accurate diagram.

For products with a larger radius the dimensions are scaled up.

We can now draw an accurate net for our package.





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Design Report C

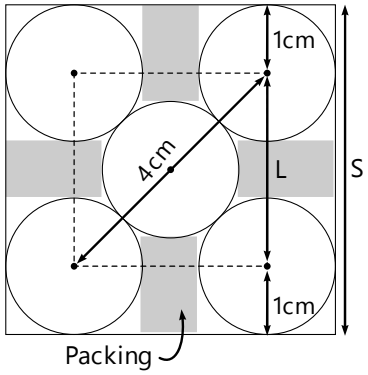
Name: _____

This is our design for a package for five cylindrical products.

It is a rectangular package with a square cross-section. It can be easily packed into large rectangular boxes for transportation without any "wasted" space.

Assuming that the radius of each product is 1 centimetre, this diagram shows how the products will be packed.

Although there will be no "wasted" space between the packages, there is some "wasted" space inside each package between the products.



To design the net of each package we needed to know the dimensions of the square. We could have measured these using an accurate diagram, but here we show how we calculated them.

Using Pythagoras' theorem

$4^2 = 2L^2$

so $L = 2.8$

and $S = 2.8 + 2 = 4.8$

Alternatively, using trigonometry

$L = 4.\ \sin 45°$

$L = 4\ (1/\sqrt{2}) = 4/1.414 = 2.8$

or $\cos 45° = \frac{L}{4} = \frac{1}{\sqrt{2}}$

$L = \frac{4}{\sqrt{2}} = 2.8$

For products with a larger radius the dimensions can be scaled up by multiplying them by the radius.

We can now draw a net for our package and calculate the amount of card wasted when it is cut from a single piece of card.

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Critiquing Reports

Name: _____

Review the design report.

- Work out what the student has done.
- Correct any errors.
- Finish off any incomplete calculations, draw missing nets.
- Make sure you can explain how the calculations have been done.
- Annotate the work with comments that explain the important points.
- Begin to think about:
    - What you can learn from the report.
    - How this approach could help you improve your work.

Consider:

- Does the design work? How do you know?
- When the candles/bottles are standing up on the shelf, what shape is the bottom of the package?
- Is there much wastage of card? How much?
- Is there much waste of storage space for transport? How much?
- What assumptions have the students made?
- Does the manufacturer have enough information to make the package?
- Do you think the manufacturer would be pleased with the design? Why or why not?

- How could the report be improved?
    - Would any additional diagrams help?
    - Could current diagrams be clearer?
    - Could the layout be improved?
    - Would annotations help explain what has been done in any places?
    - Are additional explanations needed?
    - Can the report be made more comprehensive?

Make some notes about:

- What you have learned from these reports that might help you improve their own work.

- How the design ideas in each report might be better communicated.

- How each report might be improved and completed.

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