# THE TASTE OF WATER

## Australian Curriculum: Mathematics (Year 10/10A)

**ACMSP253:** Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data.

**ACMSP277:** Investigate reports of studies in digital media and elsewhere for information on their planning and implementation.

**ACMSP278:** Calculate and interpret the mean and standard deviation of data and use these to compare datasets.

## Lesson abstract

Students test whether they can taste the difference between tap and bottled water by collecting and interrogating experimental data. This lesson can serve as an introduction to the concept of the p-value as students compare their results to the likelihood of finding the same result by sheer chance.

## Mathematical purpose (for students)

Does bottled water taste different to tap water? How could we prove/disprove that? What proof would be convincing?

## Mathematical purpose (for teachers)

To devise an experiment testing whether bottled water tastes differently to tap water, and to determine which conditions are necessary to prove their hypotheses.

Suggested presentation — Two lessons of one hour each.

Vocabulary encountered

- hypothesis
- p-value

Lesson materials

- *1a Random Trials* Spreadsheet
- a large number of cups or glasses
- access to tap water
- bottled water

We value your feedback after this lesson via our website.

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# Introduction

Discuss reasons why consumers might choose to buy and drink bottled water rather than use tap water. Students may consider factors such as taste, convenience, and health/safety, as well as 'green' issues such as global health, ecology, ethical consumption, sustainability and human rights issues.

A Queensland Urban Utilities survey found that 35% of respondents preferred bottled water over tap water. Of those 35%:

- 47% said it was for the taste
- 38% believed bottled water is better quality
- 19% said that bottled water is better for their health
- 18% said they know what's in bottled water.

# Experimentation

Focus on the claim *bottled water tastes different to tap water*. In small groups, students will need to conduct an experiment to test this claim. Their experiment should generally resemble:

1. Choose one student in the group who claims they can taste the difference between tap and bottled water. This student will be the "taster".
2. Without the taster watching, arrange 20 cups in a row and number each cup from one to twenty.
3. Pour a small amount of either tap water or bottled water into each cup. To decide whether a cup should contain tap or bottled water, flip a coin—this ensures that the allocation of water is random. Secretly record which type of water is in each cup, ensuring that the taster does not know.
4. The taster tastes one mouthful from each cup in turn and states whether it is tap water or bottled water as they go. Record the answers but do not tell the taster if they are correct until they have tried every cup.

# Analysing data

There are two ways for students to analyse their results: an informal approach where students use 'common sense' reasoning to draw a conclusion, or a formal approach in which students consider the probability that their result could have been obtained by chance.

## Informal approach

Students discuss what findings would convince them that someone can taste the difference between tap and bottled water:

- Would identifying 50% of the cups correctly be enough?
- Does the taster need to get 100% accuracy?
- Or is it somewhere in between?

On the basis of this discussion students can infer whether or not the taster can taste the difference.

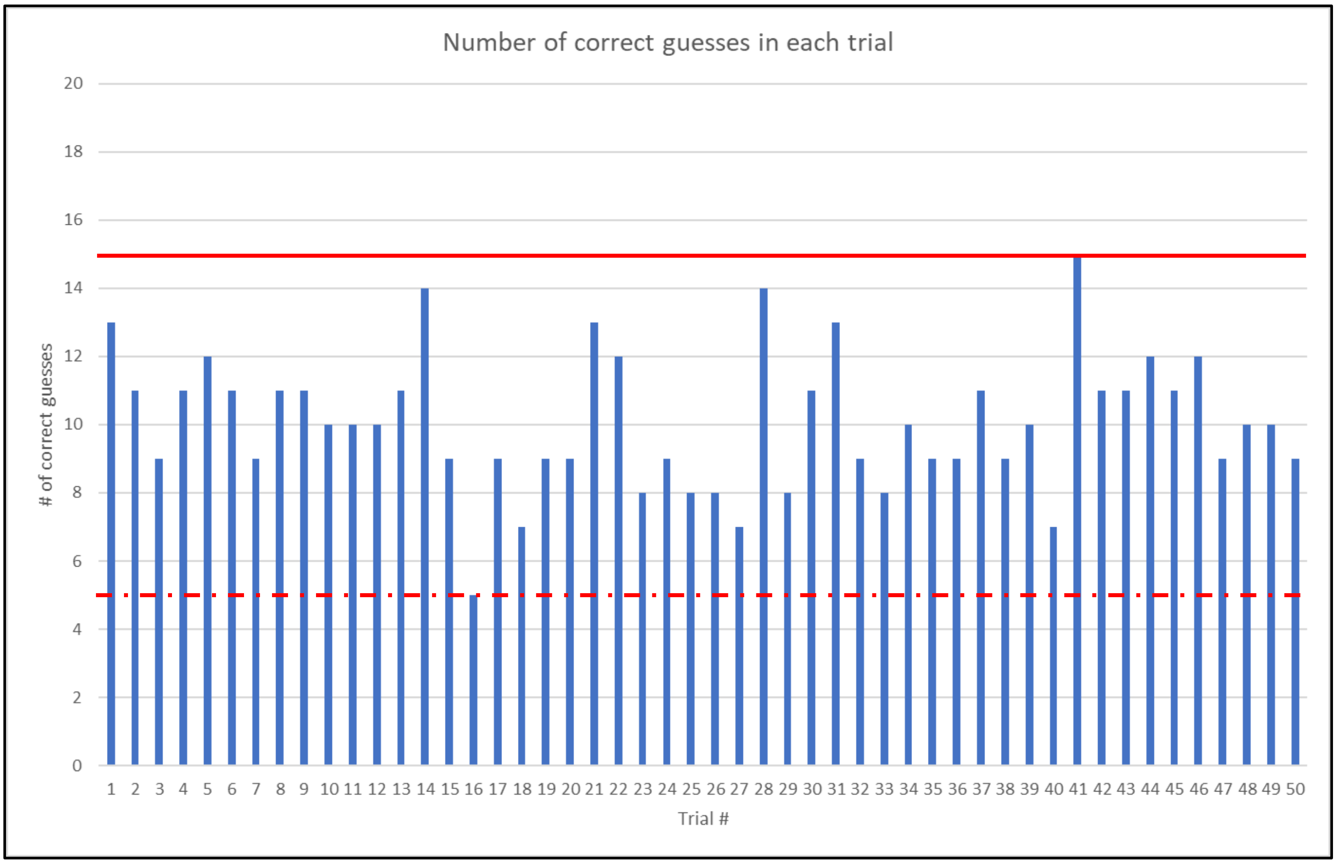
## Formal approach

In this approach, students will compare their findings with the likelihood of their findings being obtained by chance. This is an introduction to simple inferential statistics, in which the idea of "somewhere in-between" is quantified. Students will need to work in Excel or other spreadsheet software.

Ask students: *what sort of result would you expect if someone was randomly guessing the contents of each cup? Would you expect that they would guess 50% correct?*

Explain to students that they need to see how often a random set of guesses for 20 cups can be expected to be correct:

1. Use a spreadsheet command such as “=randbetween(0,1)” to create at least 50 randomly generated sets of 20 0s or 1s (where 0 represents an incorrect guess, and 1 represents a correct guess).
2. Count the number of correct guesses in each set by adding the 20 values, then plot the results of all 50 experiments. The chart below shows an example using spreadsheet *1a Random Trials*.




Draw a horizontal line across the chart to show the number of correct guesses in the actual experiment – the solid red line above shows 15 correctly identified samples.

Ask students: *if the taster identified every single cup* ***incorrectly****, what might this tell us? Can the taster tell the difference between tap and bottled water?*

- If a taster identified every cup incorrectly, this suggests that they can tell the difference between tap and bottled water, but they cannot *identify* tap and bottled water. For this reason 15 incorrectly identified samples has the same statistical significance as 15 correct samples, and is shown as a dashed horizontal line on the chart.

Students can now compare the observed result with what happens by chance. Using the above chart they would see that a result of 15 or more correct guesses occurred by chance only once out of 50 trials. This may be sufficiently convincing evidence that the taster can tell the difference between tap and bottled water. On the other hand, a result of 12 correct identifications would not be very convincing, as 12 or more correct guesses occurred in 14 trials.

## Teacher Notes

- A key idea in statistics and probability is *variation*. Students are unlikely to correctly identify tap and bottled water 100% of the time, so the question is whether random variation can account for the observed result.
- If we were setting this up as a formal experiment, we would first write a null hypothesis ($H_0$). In this case the null hypothesis would be that the student cannot taste the difference between tap and bottled water. The alternative hypothesis ($H_1$) would be that the student can taste the difference. In analysing the data, the researcher decides whether or not there is sufficient evidence to reject the null hypothesis.

- The probability of obtaining at least 15 correct out of 20, or no more than 5 correct out of 20, can also be calculated using a graphics calculator, Pascal's triangle extended to 20 rows, or online calculators such as http://vassarstats.net/textbook/ch5apx.html. It is 0.0414. If we were using a P-value of 0.05 (that is the probability of the observed result occurring by chance is <0.05), we would, in the case of a student correctly identifying 15 samples, reject the null hypothesis and conclude that they can taste the difference.
- If the student could only correctly identify 14 samples, the P-value would be 0.115, which means that there is a 11.5% chance of obtaining 14 or more (or 6 or less) by chance if the student cannot taste the difference. In this case we would not reject the null hypothesis. This does not mean that the student cannot detect a difference in taste, but it does mean that we cannot be sufficiently confident from this experiment that they can detect the difference. We may wish to collect more data.
- This is an introduction to a sign test, which assumes a binomial distribution with probability ½ of correctly identifying each sample. A sign test is a simple introductory statistical test that can be used whenever there is a yes/no question.

# Reflection

Students discuss:

- *How many trials do we need in the experiment? Would 10 be enough? If we used 10 trials, how many would need to be correct to make a reasonable conclusion?*
- *If you were to present this as an argument for or against selling bottled water in the school canteen, how would you present your conclusions? What disclaimers would you include? What are the limitations of the research? What other factors (e.g. environmental impact) would be important to consider?*

# Further activities

## Activity 1

Students might pose their own questions and design their own experiments that can be analysed in a similar way. For example, to determine whether people can hop more quickly on their dominant leg than on their non-dominant leg, each student in the class could time themselves over a given distance hopping first on one leg and then on the other. In this case, the only result we are interested in is whether they are quicker on their dominant leg or slower, not how much quicker or slower. All other variables such as time of day, surface, shoes etc. must be kept identical.