re(Solve) MATHS BY INQUIRY The Tiler

Y4

Summary of learning goals

- Students build their multiplicative reasoning skills through the context of tiles arranged in a courtyard. The tiling array is made up of tiles of different sizes, so students' understanding of the array is extended to work with partially abstracted arrays.

Australian Curriculum: Mathematics (Year 4)

ACMNA075: Recall multiplication facts up to 10 × 10 and related division facts.

ACMNA076: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder.

ACMNA082: Solve word problems using number sentences involving multiplication or division where there is no remainder.

Summary of lessons

Who is this sequence for?

- Students should be familiar with arrays and using them to solve multiplication problems. They should understand that arrays are made up of coordinated rows and columns.

Lesson 1: The Tiler

This task explores arrays through the context of tiling a courtyard. Students are given the total cost of tiling a courtyard and use this to calculate the price for individual tiles. They then explore the cost of different tiling designs to determine if one is cheaper than another.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Australian Academy of Science

Reflection on this sequence

Rationale

Arrays are commonly used to represent multiplication as coordinated rows and columns. There are many real-world examples of arrays, making arrays an accessible and powerful tool for students. This sequence extends and abstracts the array as a multiplicative structure. In this task, differently sized tiles abstract the array. Students are forced to think about the rows and columns simultaneously. This requires them to recognise the coordinated structure of the array, allowing them to multiply the number of rows by the number in each row.

reSolve mathematics is purposeful

- Students develop their understanding of arrays as a multiplicative structure.
- A connection between multiplication and area is emphasised.

reSolve tasks are inclusive and challenging

- Students are challenged to think differently and more abstractly about arrays.

reSolve classrooms have a knowledge-building culture

- Students can approach answering the task in different ways. Sharing these different ways of seeing and working with the array builds a collective understanding of ways of reasoning with the structure.

LESSON 1

reSolve MATHS BY INQUIRY — The Tiler

The Tiler

Y4

About this lesson

This task explores arrays through the context of tiling a courtyard. Students are given the total cost of tiling a courtyard and use this to calculate the price for individual tiles. They then explore the cost of different tiling designs to determine if one is cheaper than another.

Australian Curriculum: Mathematics (Year 4)

ACMNA075: Recall multiplication facts up to 10 × 10 and related division facts.

ACMNA076: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder.

ACMNA082: Solve word problems using number sentences involving multiplication or division where there is no remainder.

Mathematical purpose

- To build multiplicative reasoning skills.
- The tiling array is made up of tiles of different sizes, so students' understanding of the array is extended to work with partially abstracted arrays.

Learning intention

- To find different ways to cover a square using large and small tiles and to find which way is cheaper.

Time

A lesson of approximately 1 hour.

Vocabulary

- arrays

Resources

- reSolve PowerPoint *1a The Tiler*
- Student Sheet 1 – The Tiler 1 (one per student)
- Student Sheet 2 – The Tiler 2 (one per student, optional)

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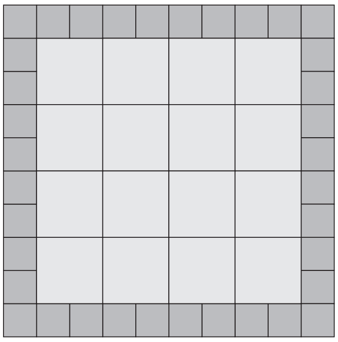
Tiling courtyards

Resources: Show the students slide 2 of the reSolve PowerPoint *1a The Tiler* and introduce the context: *A tiler is asked to pave a small courtyard. He draws up a plan of the following tiling design and calculates the cost of the tiles.*



The cost to buy the exact number of tiles needed:

Small tiles: $360

Large tiles: $640

Pose the question: *The courtyard's dimensions are such that only small tiles could be used or only large tiles could be used. Would it be cheaper to tile the courtyard with only small tiles or with only large tiles?*

Exploring courtyards

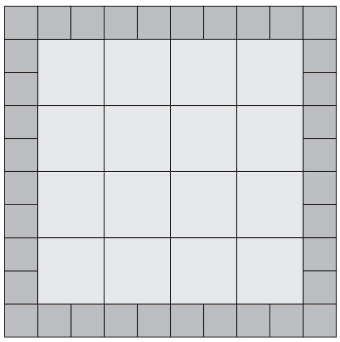
Resources: Distribute Student Sheet 1 – The Tiler 1 to each student and allow them to explore the question using their own strategies.

Enabling prompts:

- There are a number of steps to solving the question. It is useful to break down the question and isolate each part of the problem.
  - ◊ *What is the price of one small tile? What is the price of one large tile?*
  - ◊ *How many small tiles would be required to pave the entire area? What would be the cost of buying exactly the right number of small tiles needed?*
  - ◊ *How many large tiles would be required to pave the entire area? What would be the cost of buying exactly the right number of large tiles needed?*

Teacher notes:

Solving the problem for the first courtyard design (as shown):



a. The total cost of the small tiles is $360.
   i. There are 36 small tiles used.
   ii. This means that each tile costs $\$360 \div 36 = \$10$ each.
   iii. To pave the whole area in small tiles will take $10 \times 10 = 100$ tiles.
   iv. 100 small tiles $\times\ \$10 = \$1000$

b. The total cost of the large tiles is $640.
   i. There are 16 large tiles used.
   ii. This means that each tile costs $\$640 \div 16 = \$40$ each.
   iii. To pave the whole area in large tiles will take $5 \times 5 = 25$ tiles.
   iv. 25 large tiles $\times\ \$40 = \$1000$

c. To pave the courtyard using this design will cost $\$360 + \$640 = \$1000$.

Questions to ask the students:

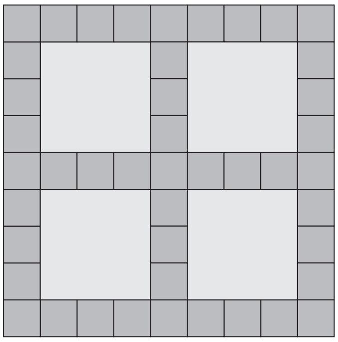
- *How did you work out the total number of small tiles and the total number of large tiles in the tiler's design?*
- *How did you work out the number of tiles needed to tile the whole area using only the small tiles?*
- *How did you work out the number of tiles needed to tile the whole area using only the large tiles?*

The aim of this task is to encourage the students to think more multiplicatively. Encourage students to look for groups to efficiently calculate the number of tiles rather than counting in 1s. Different ways to calculate the number of tiles are explored in the Reflection on the next page.

Pose the question: *Why is the cost for each courtyard design the same no matter which combination of tiles is used?*

Extending prompt:

Show students slide 3 of the reSolve PowerPoint *1a The Tiler*, showing the tiler's second courtyard design. For this design, the tiler uses different tiles and calculates the cost.



The cost to buy the exact number of tiles needed:

Small tiles: $450

Large tiles: $360

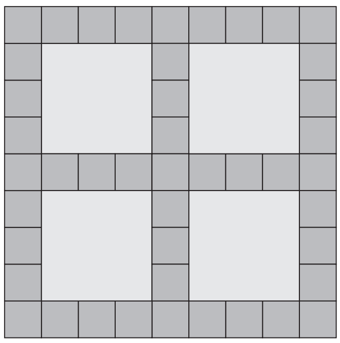
Pose the question: *The courtyard's dimensions are such that only small tiles could be used or only large tiles could be used. Would it be cheaper to tile the courtyard with only small tiles or with only large tiles?*

Teacher notes:

Solving the problem for the second courtyard design (as shown):

a. The total cost of the small tiles is $450.
   i. There are 45 small tiles used.
   ii. This means that each tile costs $\$450 \div 45 = \$10$ each.
   iii. To pave the whole area in small tiles will take $9 \times 9 = 81$ tiles.
   iv. 81 small tiles $\times$ $10 = $810

b. The total cost of the large tiles is $360.
   i. There are 4 large tiles used.
   ii. This means that each tile costs $\$360 \div 4 = \$90$ each.
   iii. To pave the whole area in large tiles will take $3 \times 3 = 9$ tiles.
   iv. 9 large tiles $\times$ $90 = $810



c. To pave the courtyard using this design will cost $\$450 + \$360 = \$810$.

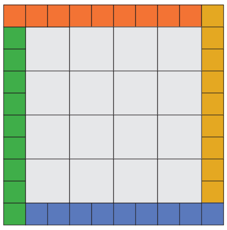
Again, pose the question: *Why is the cost for each courtyard design the same no matter which combination of tiles is used?*

Reflection

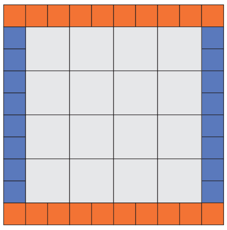
Select students to share their work.

Discuss the ways in which students calculated the number of small and large tiles. Some ways of seeing the number of tiles used in the tiler's design are shown.

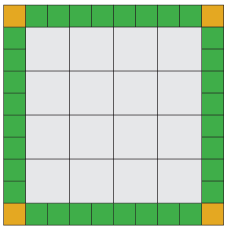
First courtyard design



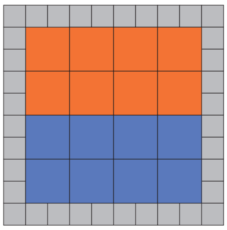
4 groups of 9
$\Rightarrow 4 \times 9 = 36$



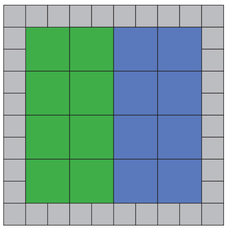
2 groups of 10 and 2 groups of 8
$\Rightarrow (2 \times 10) + (2 \times 8) = 20 + 16 = 36$



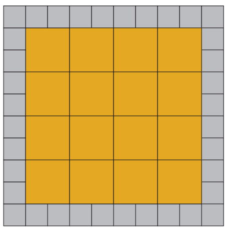
4 groups of 10 subtract 4, as each corner has been counted twice
$\Rightarrow (4 \times 10) - 4 = 40 - 4 = 36$



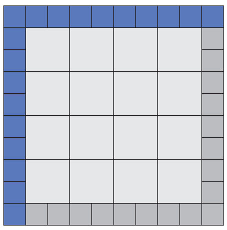
2 groups of 2 rows of 4
$\Rightarrow (2 \times 4) + (2 \times 4) = 8 + 8 = 16$



2 groups of 4 rows of 2
$\Rightarrow (4 \times 2) + (4 \times 2) = 8 + 8 = 16$



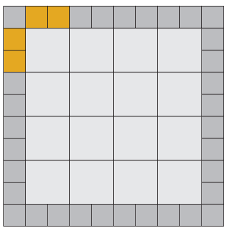
4 rows of 4
$\Rightarrow 4 \times 4 = 16$



10 small tiles fit in each row and column, forming a 10×10 array.

$10 \times 10 = 100$

100 small tiles will be needed to tile the whole area.



Each large tile is the size of 4 small tiles. This means an extra tile can be added to each row and each column, giving 5 in each row and column.

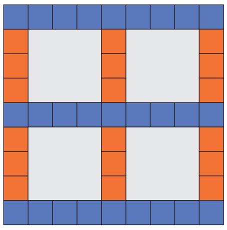
$5 \times 5 = 25$ large tiles

It could also be calculated using the knowledge that:

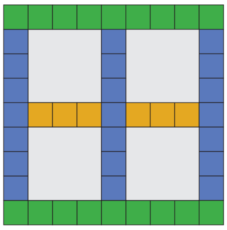
- 100 small tiles are needed
- 4 small tiles is the same as one large tile.

$100 \div 4 = 25$

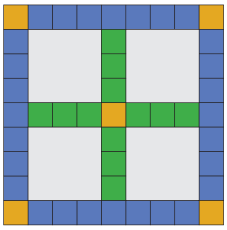
Second courtyard design



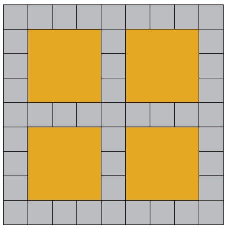
5 groups of 9
⇒ $5 \times 9 = 45$



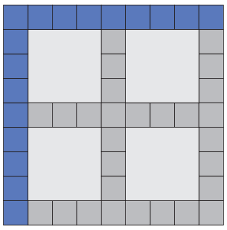
2 rows of 9, 3 columns of 7 and 2 groups of 3
⇒ $(2 \times 9) + (3 \times 7) + (2 \times 3) = 18 + 21 + 6 = 45$



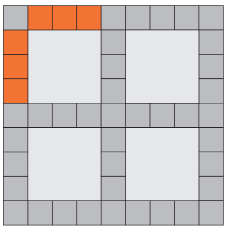
4 rows of 9 subtract 4, 2 rows of 7 subtract 1
⇒ $[(4 \times 9) - 4] + [(2 \times 7) - 1] = 32 + 13 = 45$



The 4 large tiles are easily subitised.
It can also be explained as $2 \times 2 = 4$.

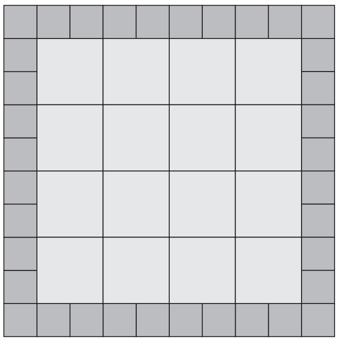


9 small tiles fit in each row and column, forming a 9×9 array.
$9 \times 9 = 81$
81 small tiles will be needed to tile the whole area.



Each large tile is the same size as 9 small tiles. This means an extra tile can be added to each row and each column, giving 3 in each row and column.
$3 \times 3 = 9$ large tiles
It could also be calculated using the knowledge that:
81 small tiles are needed
9 small tiles is the same as one large tile.
$81 \div 9 = 9$

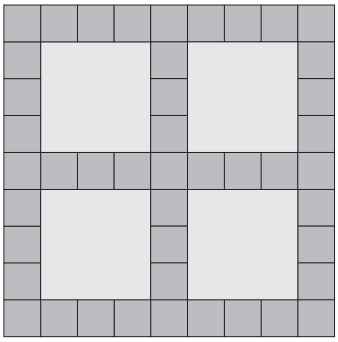
Pose the question: *For each courtyard design, why is the cost the same no matter which combination of tiles is used; that is, only small tiles or only large tiles or a combination of both?*



Small tiles: \$360
$\$360 \div 36 = \$10$
The small tiles are \$10 each.

Large tiles: \$640
$\$640 \div 4 = \$40$
The large tiles are \$40 each.

1 large tile is the same as 4 small tiles.
So, the cost of a large tile is 4 times more than that of a small tile.



Small tiles: \$450
$\$450 \div 45 = \$10$
The small tiles are \$10 each.

Large tiles: \$360
$\$360 \div 4 = \$90$
The large tiles are \$90 each.

1 large tile is the same as 9 small tiles.
So, the cost of a large tile is 9 times more than that of a small tile.

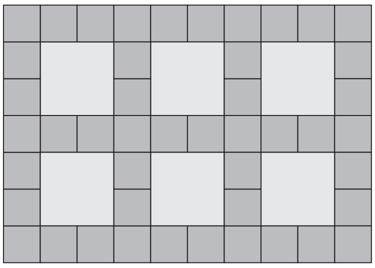
Further activities

Activity 1

Resources: Provide students with Student Sheet 2 – The Tiler 2.

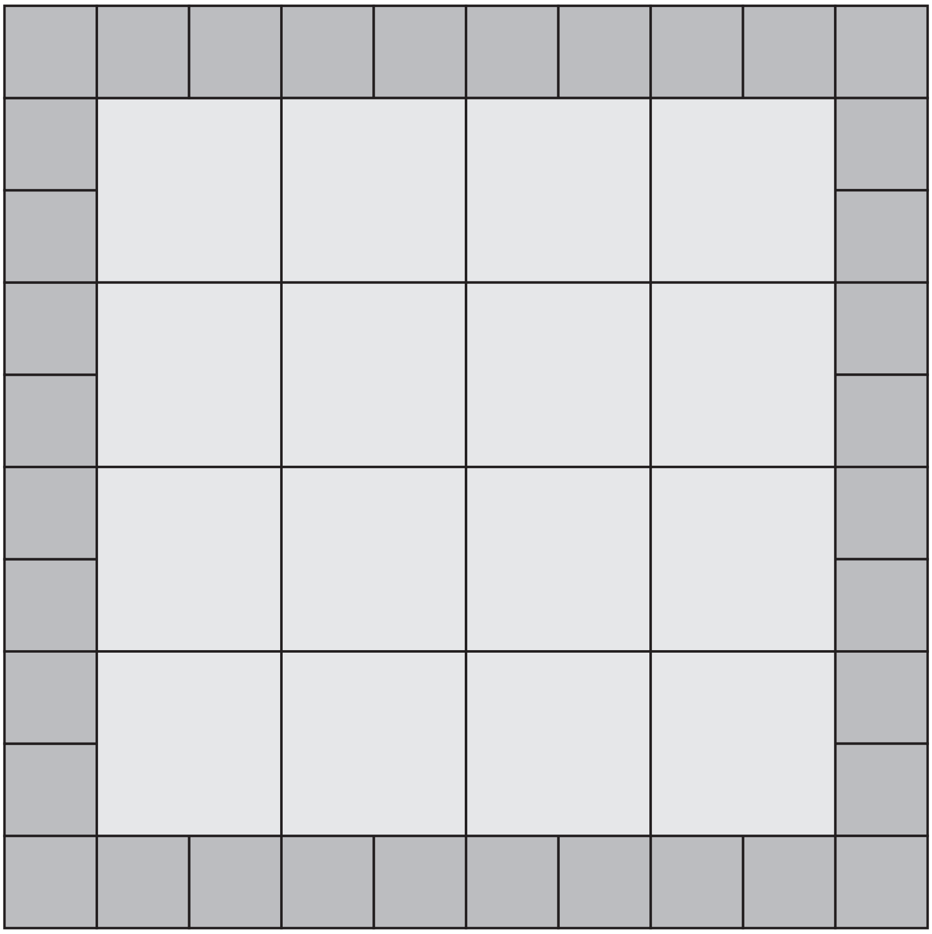
The total cost to buy the exact number of tiles for the following design is \$350.

- How much does a small tile cost?
- How much does a large tile cost?



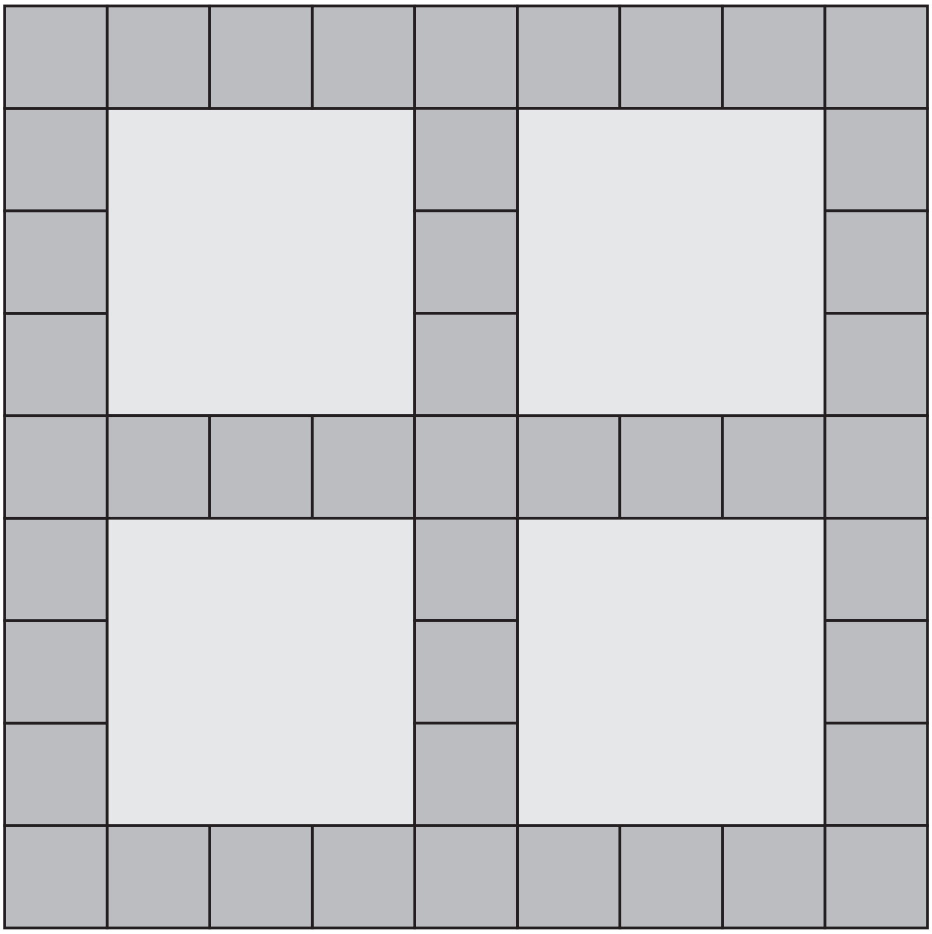
The Tiler 1

Name: ______________________________



The cost to buy the exact number of tiles needed:

- **Small tiles: $360**
- **Large tiles: $640**

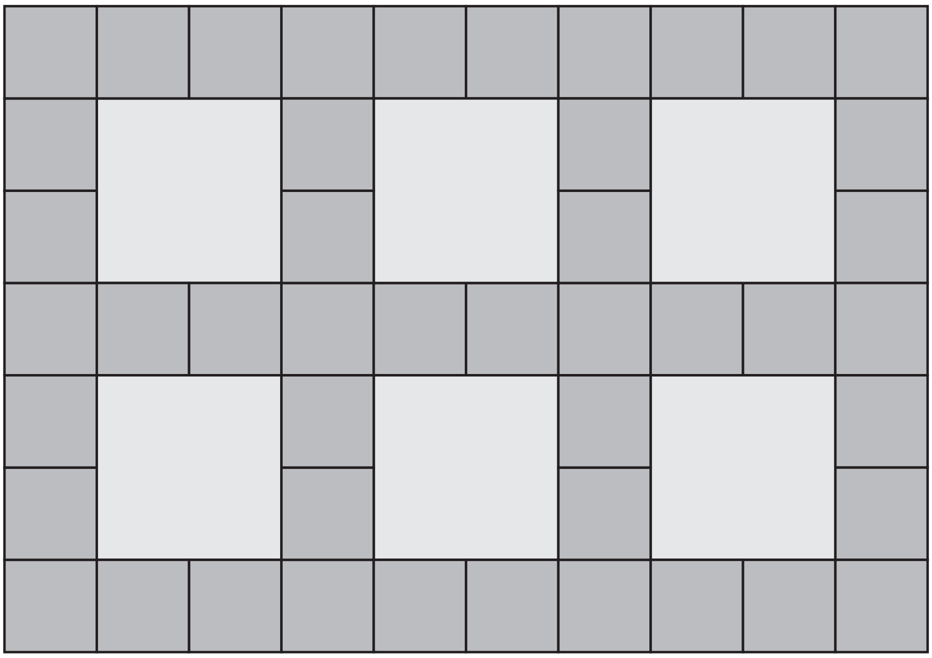


The cost to buy the exact number of tiles needed:

- **Small tiles: $450**
- **Large tiles: $360**

The Tiler 2

Name: ___________________________________



The cost to buy the exact number of tiles needed is $350:

- **What is the cost of one small tile?**
- **What is the cost of one large tile?**