

Summary of learning goals

- Students come to appreciate the power of algebra for generalising results from arithmetic. They connect arithmetic operations with algebraic notation and visualisations. The lesson begins with an observation made using arithmetic that students then justify and extend using algebra.

Australian Curriculum: Mathematics (Year 8)

ACMNA190: Extend and apply the distributive law to the expansion of algebraic expressions.

ACMNA191: Factorise algebraic expressions by identifying numerical factors.

ACMNA192: Simplify algebraic expressions involving the four operations.

Summary of lessons

Who is this sequence for?

- This lesson is designed to consolidate skills in algebra, including collecting like terms and expanding and factorising using the distributive law. It emphasises the importance of algebra for generalising and justifying arithmetic results. It is assumed that students have some familiarity with algebraic notation.

Lesson 1: Think of a Number (THOAN) – Linear Equations

- Think of a number (THOAN) activities show how algebraic reasoning can be used to explain and formulate problems in which chains of operations always result in the same outcome regardless of the number chosen. Students are introduced to THOANs involving simple linear operations, and use algebraic simplification to explain why the THOAN works. They are then asked to complete a THOAN when given some starting operations and, finally, to make and test their own.

Reflection on this sequence

Rationale

Approaching algebra as generalised arithmetic shows students the power of algebra for abstracting number. This focus on algebra as generalised arithmetic is typically under-represented in secondary mathematics in favour of more time spent on functions and equations.



reSolve mathematics is purposeful

- This task supports a rich interpretation and enactment of the Australian Curriculum: Mathematics, providing fun and engaging ways to understand the algebraic content of the Curriculum. The lesson explores mathematics as a creative and imaginative endeavour, emphasising the entertaining applications of mathematics and the understanding of the mathematical foundation beneath common ‘tricks’ that students are likely to have encountered before.



reSolve tasks are inclusive and challenging

- The task in this sequence activates existing knowledge, develops new knowledge and explores relationships between key ideas in the Australian Curriculum. The lesson allows students to determine the complexity of their problems themselves, which allows for a low floor and a high ceiling.



reSolve classrooms have a knowledge-building culture

- The task begins by inspiring curiosity and intrigue through a shared classroom experience that promotes higher-order thinking through the role of both teacher and student. Students build understanding through collaborative inquiry, action and reflection. The sequence encourages students to challenge their existing conceptions and to use their mistakes as a vehicle for further learning.

Think of a Number – Linear Equations

Y8

About this lesson

Think of a number (THOAN) tricks are sequences of operations that result (in this resource) in a return to the starting number regardless of what that number is. In this resource students examine a THOAN and model it visually and algebraically. This develops skills in algebraic operations, including expanding, factorising and collecting like terms. Students see that if the THOAN is to work for any number, it is essential to generate the expressions algebraically. Note that the Creating your own THOAN activity can be adapted for any year level and algebraic idea, including polynomials.

Australian Curriculum: Mathematics (Year 8)

ACMNA190: Extend and apply the distributive law to the expansion of algebraic expressions.

ACMNA191: Factorise algebraic expressions by identifying numerical factors.

ACMNA192: Simplify algebraic expressions involving the four operations.

Mathematical purpose

- This resource develops students' algebraic skills and provides visual cues to help them develop a deeper understanding of collecting like terms and the distributive law. The context of a THOAN highlights the importance of algebra in showing why a particular set of operations that returns to the starting number will work for any number. The resource gives students confidence in using algebra and a sense of power that they can develop their own number tricks.

Learning intention

- To understand how THOAN tricks work and to create a THOAN trick that works for any given number.



Time

Two to three lessons
of approximately 1 hour.



Vocabulary

- distributive law
- factorise
- like terms



Resources

- reSolve PowerPoint *1a THOAN 1*
- reSolve PowerPoint *1b THOAN 1 with images only*
- reSolve PowerPoint *1c THOAN 1 with algebra only*
- reSolve PDF *THOAN 1 Card Sort*

A bit of number 'magic'

Ask students to think of a number (THOAN). Although they may choose any number, it is best to have them choose one that will be easy to work with, such as a small integer. Then, either mentally or with the aid of a calculator, ask them to perform the following operations sequentially:

1. Multiply by 3.
2. Add 5.
3. Subtract the number you first thought of.
4. Subtract 3.
5. Divide by 2.
6. Subtract 1.

The answer should be the number they first thought of.

T Teacher notes:

- The algebraic reasoning is shown below but *do not show the algebra at this stage!*

Instructions	The algebra
1. Think of a number.	x
2. Multiply by 3.	$3x$
3. Add 5.	$3x + 5$
4. Subtract the number you first thought of.	$3x + 5 - x = 2x + 5$
5. Subtract 3.	$2x + 5 - 3 = 2x + 2$
6. Divide by 2.	$\frac{2x + 2}{2} = x + 1$
7. Subtract 1.	$x + 1 - 1 = x$

- We recommend that you guide the students through two or three more THOAN sequences, but use at least four or five operations each time to show that you are not simply reversing the operations. Although it is more impressive to 'make it up as you go' rather than using prepared examples, two other examples that could be used are:

Instructions	The algebra
1. Think of a number.	x
2. Subtract 5.	$x - 5$
3. Add twice the number you first thought of.	$x - 5 + 2x = 3x - 5$
4. Multiply by 2.	$2(3x - 5) = 6x - 10$
5. Subtract the number you first thought of.	$6x - 10 - x = 5x - 10$
6. Add 10.	$5x - 10 + 10 = 5x$
7. Divide by 5.	$\frac{5x}{5} = x$

Instructions	The algebra
1. Think of a number.	x
2. Multiply by 4.	$4x$
3. Add 11.	$4x + 11$
4. Subtract the number you first thought of.	$4x + 11 - x = 3x + 11$
5. Subtract 2.	$3x + 11 - 2 = 3x + 9$
6. Divide by 3.	$\frac{3x + 9}{3} = x + 3$
7. Add twice the number you first thought of.	$x + 3 + 2x = 3x + 3$
8. Divide by 3.	$\frac{3x + 3}{3} = x + 1$
9. Subtract 1.	$x + 1 - 1 = x$



Extending prompt:

- Try starting with a fraction or a negative number.

Trying your own trick

Ask the class to suggest the first three operations for a THOAN. Collect some suggestions on how they think they can get back to their original number.

For example, consider the following operations:

1. Think of a number.
2. Add 7.
3. Multiply by 4.
4. Subtract 11.

A student who started with 3 will reach 29, and might suggest subtracting 26. A student who started with 5 will reach 37, and might suggest subtracting 7 and dividing by 6. However, these suggestions will not work for everybody. The teacher will have mentally formulated the expression $4x + 17$ and will be able to suggest a set of operations such as:

1. Add the number you first thought of.
2. Subtract 7.
3. Divide by 5.
4. Subtract 2.

Assure the students that by the end of the lesson they will be able to make up their own THOAN magic and astound their friends and family!

Again, do *not* show the algebra at this stage!

Modelling the THOAN

Discuss with students the need to use a variable (rather than a specific number) to find a set of operations that will work for any number.



Resources: The image below is taken from slide 3 of reSolve PowerPoint 1a THOAN 1.

This shows one way of modelling the THOAN visually, with corresponding algebraic expressions.

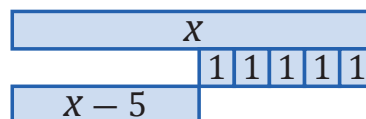
Think of a number.		x
Multiply by 3.		$3x$
Add 5.		$3x + 5$
Subtract the number you first thought of.		$3x + 5 - x = 2x + 5$
Subtract 3.		$2x + 5 - 3 = 2x + 2$
Factorise.		$2x + 2 = 2(x + 1)$
Divide by 2.		$\frac{2(x + 1)}{2} = x + 1$
Subtract 1.		$x + 1 - 1 = x$

The PowerPoint guides students through the modelling and algebraic formulation process. Each mouse click or right arrow press on slide 3 allows you to progress from the instruction to the model to the algebraic formulation, and then to the next instruction.

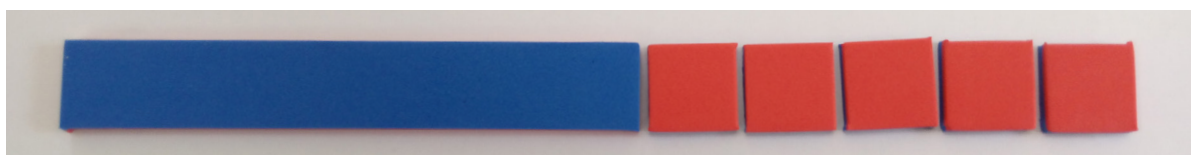
We recommend that the PowerPoint be used as a class demonstration, pausing after each element to ask students what the model will look like and what the algebraic expression will be.

The video reSolve THOAN 1 (see <https://youtu.be/rEdnoQhXHtE>) has the same animation with prerecorded timings.

Create two more THOANs that students can model. The two provided previously (in the last two tables in the Teacher notes) can be used, but be aware that using subtraction as the first step makes it harder to model. One way of modelling $x - 5$ is shown below.



It is also possible to model the algebra using algebra tiles. The image below shows $x - 5$.



Create your own THOAN

Have students create their own THOAN and test it with the class.

Students can also set up an Excel spreadsheet to test their THOAN.



Enabling prompts:

- For students struggling with the distributive law, use a THOAN such as:
 1. Think of a number.
 2. Multiply by 2.
 3. Add 6.
 4. Divide by 2.
- *What number do you need to add to get back to the number you first thought of? Why?*



Extending prompt:

- Encourage students to use more operations. (Although they will hardly need any encouragement!)

Discussion

Discuss with students the limitations of the modelling method. These might include the difficulty of dealing with negative numbers or with fractions. They may also include the limitation of the modelling to one dimension, such as for linear functions (although Lesson 2: Think of a Number – Binomial Equations of the reSolve Year 9 sequence Think of a Number moves into two dimensions, involving binomial expansions and the use of area diagrams).

Discuss the use of the distributive law and why it is important to factorise before dividing. Show how use of the visual models helps to illustrate expansion and factorisation using the distributive law. Make connections to the reSolve Year 8 resource Tens and Units, in which the connection is made between the distributive law and multiplication algorithms.

Further activities

Activity 1: Complete the THOAN

Give students some instructions that are the start of a THOAN; for example:

1. Think of a number.
2. Subtract 3.
3. Multiply by 5.
4. Add 5.

Ask students to complete the THOAN in two steps.

Ask: *Is there more than one way to do this? What is the connection between these two different methods?*

The result so far is $5x - 10$. So two possible ways to complete the THOAN are to add 10 and divide by 5, or divide by 5 and add 2. Note that there are other possibilities, such as subtracting 4, then multiplying by the original number and adding 10.

Ask students to work in pairs and try this for themselves, with one person giving three or four initial steps and the other completing the THOAN.

Ask students to complete a THOAN in three steps and compare the different answers.

Activity 2: A take-home THOAN

Suggest that students try this at home and explain to their family how it works.

Activity 3: A different end point

Ask students to create a THOAN for which the result is 5, no matter what number they started with.

Activity 4: Start with the diagrams

Give students a set of diagrams that give a visual representation of a THOAN and ask them to complete the list of instructions and the algebraic formulation.



Resources: The reSolve PowerPoint *1b THOAN 1 with images only* is an example.

Activity 5: Start with the algebra

Give students the algebraic formulations and ask them to complete the list of instructions and the visual representations.



Resources: The reSolve PowerPoint *1c THOAN 1 with algebra only* is an example.

Activity 6: Card sort

Use a set of cards that will result in a THOAN; mix them up and ask students to rearrange them to make it work.



Resources: The reSolve PDF *THOAN 1 Card Sort* is an example.