re(Solve) MATHS BY INQUIRY

Triangle Inequality

Y4

Summary of learning goals

- Students discover an important property about triangles while exploring the concepts of equality and inequality.
- Students learn that in any triangle the sum of the lengths of any two sides must be greater than the length of the remaining side. Students will represent these inequalities using is greater than and is less than symbols.

Australian Curriculum: Mathematics (Year 4)

ACMNA083: Find unknown quantities in number sentences involving addition and subtraction, and identify equivalent number sentences involving addition and subtraction.

Summary of lessons

Who is this sequence for?

- For students who understand a triangle as a polygon (i.e. a closed, plane shape, with three straight sides and three corners), and are familiar with non-equilateral triangles.
- It is important that students can identify and represent equality using the equals sign. They should also be comfortable using the is greater than (>) and is less than (<) symbols.

Lesson 1: Triangle Inequality

This task explores the triangle inequality theorem. Students use 20 matchsticks to create different triangles. Using 20 sticks, it is possible to make only eight different triangles. Students will learn that, in any triangle, the sum of the lengths of any two sides must be greater than the length of the third side. They are asked to represent inequalities using the is greater than and is less than symbols.

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Reflection on this sequence

Rationale

This task explores a simple geometric problem that builds on students' understanding of triangles. The triangle inequality states that the sum of the two sides needs to be greater than or equal to the third side. When side lengths are equal, a degenerate triangle is formed, which is an interesting discussion that can be had with students. The task also reinforces the concepts of equivalence and inequivalence. Often, equivalence is the focus of classroom exploration. This task shows that inequivalence is an important concept that can be used to solve mathematical problems.

reSolve mathematics is purposeful

- This task draws on multiple mathematical domains as it creates connections between geometry, data and algebra. It also builds on students' ability to reason mathematically and solve problems.
- Students discover for themselves the triangle inequality theorem, whereby the matchsticks materials provide a visual and concrete reason as to why it is true.
- Students learn to work systemically, and to record and order data so that patterns can be found and generalisations can be made.

reSolve tasks are inclusive and challenging

- This task is based on a concrete exploration, allowing access for all students.
- The low floor of this task is demonstrated by students constructing and recording different triangles that they can make using 20 matchsticks. The high ceiling of the task is realised when a generalisation is made about the lengths of sides necessary for a triangle to be formed.

reSolve classrooms have a knowledge-building culture

- This task is built around a whole class exploration. As students explore with the matchsticks, they become curious as to why some triangles can be built and others can't. This provides the opportunity for deeper thinking and the stimulating discussions.
- The richness of the task is attained as the reasoning of different students is made explicit to the class as a generalised statement about the side lengths of triangles is formed.

Acknowledgements

Graham P & Chick H, 2015, The 20 matchstick triangle challenge; an activity to foster reasoning and problem solving, *The Australian Mathematics Teacher*, 71(1).

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LESSON 1

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About this lesson

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Australian Curriculum: Mathematics (Year 4)

ACMNA083: Find unknown quantities in number sentences involving addition and subtraction, and identify equivalent number sentences involving addition and subtraction.

Mathematical purpose

- Students learn that in any triangle the sum of the lengths of any two sides must be greater than the length of the remaining side. Students will represent these inequalities using the is greater than and is less than symbols.

Learning intention

- To discover what makes a triangle a triangle.

Time

It is recommended that this task is presented over two 1 hour lessons.

Resources

- matchsticks or ice-cream sticks (20 per student)

Vocabulary

- is greater than symbol >
- is less than symbol <

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Creating triangles

Resources: Provide each student with 20 matchsticks or ice-cream sticks.

Pose the question: *I made a triangle using exactly 20 sticks. The sticks were all joined together so there were no gaps. None of the sticks were broken. What might my triangle look like?*

Allow students to explore the different triangles that they can make. Have them choose a method to record the possibilities. The mode of recording will be refined during the task.

Teacher note:

- If students use a ruler or other straight edge (e.g. the side of their books) to check that the lines of their triangles are straight, the task becomes much easier to monitor — otherwise keep an eye out for 'fake triangles' with slightly curved edges!

After the students have found several different triangles, **pose the questions:** *How many different triangles can you make with 20 sticks? How will you know if you have found all of them?*

At this point students should consider a way of collecting *and organising* their data that will make it easier to show they have found all possible solutions.

Possible student response:

- One effective way to organise the data is to record the side lengths in a table, sorted in ascending order. This will enable all triangles with a side length of 5, for example, to be grouped together, as illustrated in this table.

Side 1	Side 2	Side 3
2	9	9
3	8	9
4	8	8
4	7	9
5	6	9
5	7	8
6	7	7
6	6	8

Teacher note:

- There is opportunity in this activity to discuss the transformations of triangles in order to show that some triangles created are the same. For example, a triangle with sides measuring 9, 8 and 3 is the same as a triangle with sides 3, 9 and 8. This is also an example of the commutative property with regard to the sum of the side lengths.

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Making generalisations

Ask students: *Can you explain why it is possible to make some triangles and why it is impossible to make other triangles?* Discuss as a group which triangles have/have not been made and what rules might govern which triangles can be made.

- The triangle inequality theorem states that in any triangle the sum of the lengths of any two sides must be greater than the length of the third side.
- Students should be encouraged to record their own generalised statement using their own words.

Follow-up discussion questions:

- *What is the longest side that can be made with 20 sticks? Why?*
  - The longest side is nine sticks. It is not possible to make a side with 10 or more sticks. For a triangle with one side length of 10, the sum of the other two sides would have to be equal to 10; and for a triangle with one side longer than 10, the sum of the other two sides would have to be less than 10.
- *What is the shortest side possible? Why?*
  - The shortest side is a length of 2 sticks. If one side has a length of 1 stick, then the sum of the two shortest sides can never be larger than the length of the longest side (this is easiest explained via demonstration).

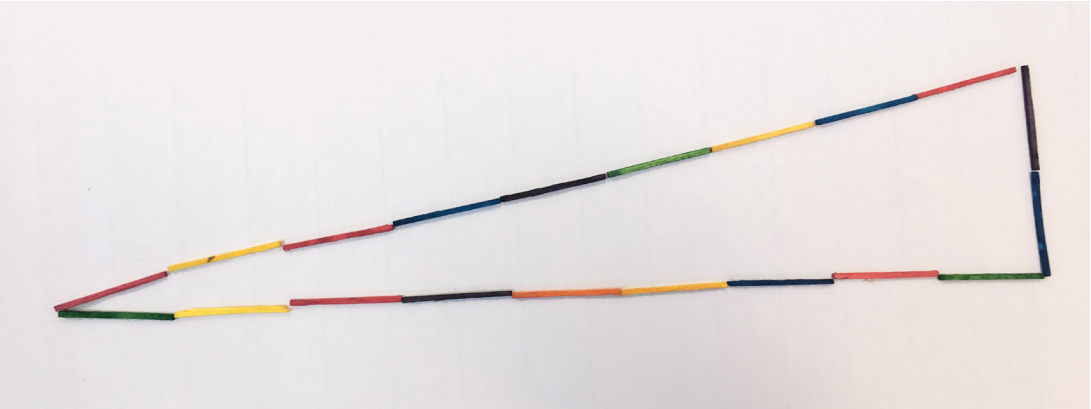
Reflection

Share students' solutions for the different triangles that can be made with 20 sticks, including the way that they recorded and organised their data, and the generalised statements that they generated.

As a class explore the triangle inequality theorem: the sum of the lengths of any two sides must be greater than the length of the remaining side.

Possible student responses:

- It is possible to make a triangle with side lengths of 2, 9 and 9 units. This is because $2 + 9 > 9$.



$$2 + 9 > 9$$

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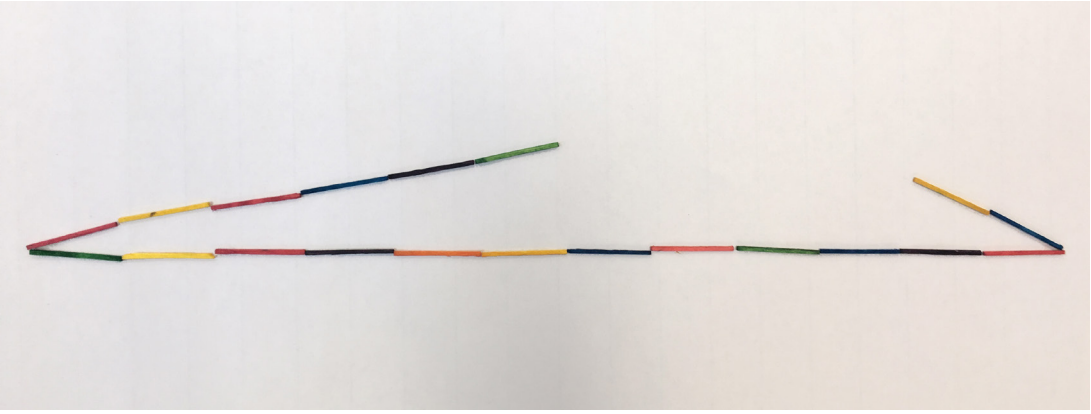
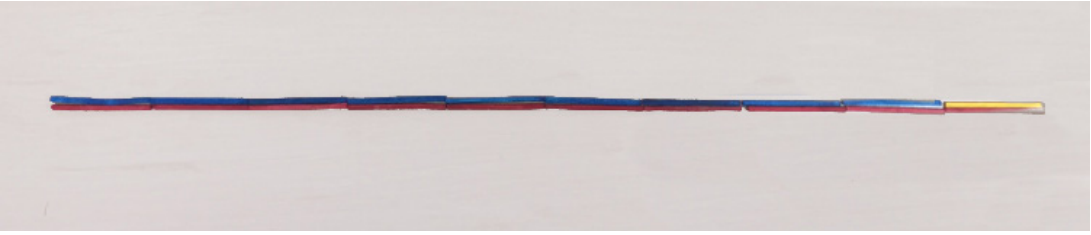
- It is not possible to make a triangle with side lengths 12, 2 and 6 because $2 + 6 < 12$. Nor is it possible to make a triangle with side lengths of 10, 9 and 1 because $10 = 9 + 1$.



$$12 > 2 + 6$$



$$10 = 1 + 9$$

Teacher note:

- As an interesting side note: the shape above with side lengths 1, 9 and 10 is called a 'degenerate triangle', and is constructed from three points along the same line. For our purposes today we will not count it as a triangle!

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Further activities

Activity 1

Provide students with three dice. The dice can be six-sided (marked 1–6) or 10-sided (marked 1–10). Students roll the dice and use the numbers rolled as the three side lengths for a possible triangle. Students record whether a triangle can be made from these three lengths.

- For example, students roll three 10-sided dice and get 6, 3 and 10.
- Students record that the triangle cannot be made because $6 + 3 < 10$.

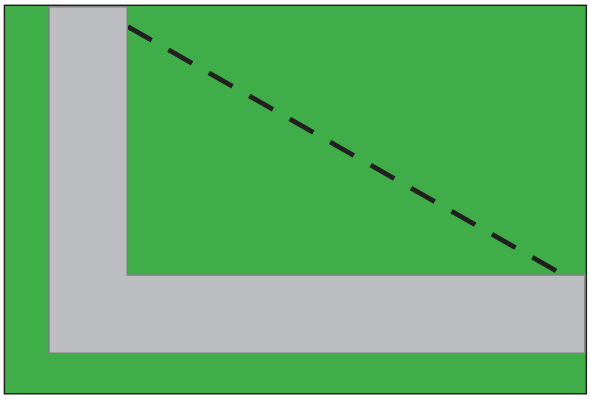
Enabling prompt:

- Students can use sticks to see if the triangle can be constructed.

Activity 2

Consider: *Where do we see the triangle inequality theorem in action?*

A good example is watching people cut across the grass rather than walking on paths. We will naturally take the shortest route.



Activity 3

Make different quadrilaterals using 30 matchsticks. What generalisations can be made about the side lengths?

In a quadrilateral, the sum of three sides must be greater than the fourth side.

All quadrilaterals are made up of two triangles. As such, the quadrilateral inequality can be shown by applying the triangle inequality twice.

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