Unit Overview: Introduction to Modelling

Summary of learning goals

This unit is designed to help students acquire a deeper understanding of mathematical modelling and the key processes involved. Through engaging with the modelling of two familiar queuing situations – traffic jams and waiting in line at a theme park – they develop an overview of modelling, captured in a diagram of the mathematical modelling cycle. The diagram is introduced as a structuring device for all the units of mathematical modelling. Its purpose is essentially metacognitive: to guide students through mathematical modelling endeavours and to highlight the special demands of each of its phases.

In doing this, students will use mathematical knowledge of proportional reasoning in space and time, especially ideas of rates of flow to experience the modelling steps of:

- Studying real world situations and representing the variables and their relationships mathematically.
- Selecting appropriate mathematical methods to use.
- Exploring the effects of systematically varying assumptions.
- Interpreting and evaluating predictions from their models.
- Communicating their reasoning clearly.

Australian Curriculum: Mathematics (Years 8 and 9)

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

- identifying direct proportion in real-life contexts

ACMNA 192: Simplify algebraic expressions involving the four operations. (Year 8).

Summary of lessons

Who is this unit for?

This unit introduces mathematical modelling, so it is suitable for all students. The mathematical skills required are below expected year level, so that students can concentrate on the modelling process. Working in well-supported groups, students need to be able to estimate rates such as the average number of metres (or cm) required per person standing in a queue and the number of people leaving a queue per minute, and finally combine these to get estimates for the rate at which the queue is moving.

Lesson 1: Modelling a traffic jam

Students approach modelling by considering how many vehicles there might be in a 200-metre section of a blocked highway. They first identify factors that might be taken into account. They then consider how to calculate how many vehicles, making some assumptions and either researching or estimating the values. They write a description of how to tackle the problem using their approach and how to develop a more sophisticated model. Groups work out a first solution and think about the effects of the assumptions they made.

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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Lesson 2: Modelling theme park queues

Building on the ideas developed in Lesson 1, students consider a structurally similar problem about queueing to get on a ride at a theme park. Initially they explore how many people might be expected to be in a fixed length of queue. They develop a model for time to get onto the ride by considering the length of the line and the rate of flow of people onto the ride (i.e. combining the number of people on the ride and the time taken for the ride).

Lesson 3: Understanding the modelling process

Students work through the modelling process as summarised in the modelling diagram. After reviewing their work from Lesson 2, they link their work on theme park queues to each stage to the modelling process, and reflect on how this helps them to analyse real problems and get realistic solutions.

Lesson 4: Improved Theme Park Model

Students plan a new ride to maximise flow of people through the ride whilst maintaining excitement. They investigate how aspects of the design of a ride affect the potential queuing time. They use this in combination with consideration of patterns of people joining lines to gain understanding of the net effect of the numbers of those leaving the line to go on the ride and those joining at the other end. They can improve their models of queuing time by including additional factors.

Lesson 5: Improved Traffic Jam Model

Students return to the traffic jam situation from Lesson 1, seeking to transfer to traffic flow the understandings gained in the theme park context. They look at what happens as the traffic jam clears, developing a more sophisticated model that considers inflow and outflow. They map their work onto the modelling diagram, reflecting on the factors involved and how they influence stages in the modelling process.

Reflection on this unit

Rationale

During their mathematics education, students encounter very many excellent 'standard' mathematical models. For example, they may use a linear function to model taxi fares, a differential equation to model population growth, and the compound interest formula to model the growth of an investment. However, their status as mathematical models of the real world is often assumed, rather than explored.

In these mathematical modelling experiences, however, there are two important differences which mirror how mathematics must be used outside of school. First, consideration of the real world is important, and so the assumptions that lie behind the standard models are highlighted. The compound interest formula, for example, only works for periods when the interest rate is constant. Second, accounting for all the factors in the real world is impossible and so mathematical modelling deliberately starts with simple models considering few factors and complexity is added if evaluation shows the real world answers are not good enough.

reSolve Mathematics is Purposeful

This unit demonstrates the process of using mathematics in the real world, so it highlights one of the ultimate purposes of learning mathematics. To highlight how the activity is purposeful, students are placed in the role of a consultant advising local traffic police or local radio (in the first scenario) and advising theme part management (second scenario). The product is a set of individual reports on various aspects of queue management prepared in a way that provides sensible and well-argued advice to these clients.

reSolve Tasks are Inclusive and Challenging

These tasks can involve all students by drawing on their different experiences and personal knowledge of the real world scenarios. Teachers can modify the scenarios slightly to give a stronger local flavour if this is desired. The real world knowledge is especially important in formulating the model and in evaluating it. The depth of mathematics used in the models can vary according to the capacity of the groups.

reSolve Classrooms Have a Knowledge Building Culture

Many different voices can be valued in formulating the mathematical models and evaluating their success. All students can contribute their different personal experiences to deciding the important factors to be included in the mathematical model, and in estimating real world parameters.